



# Selected Methods of Balanced Incomplete Sequence Crossover Designs

**Adebara L.\*, Muhammad K. and Rotipin A. J.**

Department of Mathematics and Statistics, The Federal Polytechnic, Ado-Ekiti (FEDPOLAD),

Corresponding author: [lanreadebara@gmail.com](mailto:lanreadebara@gmail.com)

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**Abstract:** Crossover design is a design in which sequences receive different treatments one at a time at another period. Three construction methods of Balanced Incomplete Sequence Crossover Design (BISCOD) are selected for review in this paper. The first and third methods used the primitive root of every non-zero element of the multiplicative root, the prime number of treatment, and the second method used Balanced Incomplete Block Design (BIBD) for construction of any number of treatments, either prime or non-prime. Results show that each treatment is preceded by each other treatment once for both first and third methods, and for the second method, each treatment is preceded by each other treatment once and itself an unequal number of times. The conclusion shows that both first and third are used to construct a design for the prime number of treatments, while the second method constructs design for any whole number.

**Keywords:** Crossover, Residual, Incomplete, Primitive root, Treatment

## 1.0 Introduction

Crossover design is a design in which sequences receive different treatments one at a time at different periods. This design can also be called change over design, and it useful in clinical trials and pharmaceutical investigation. The significance is that it gives direct and residual effects in the immediately following periods. Sharma et al., Residual effect is the effect of a treatment that persists to the current period's next immediate period. To effectively eliminate the residual effect, a washout

period is introduced to separate any two periods of treatment by an interval of time long. Several authors have presented the construction method of Balanced Incomplete Sequence Crossover Design (Biscod), [1], [2], [4], [5], discussed universally optimal (Uo) for crossover designs selected. We shall consider three selected construction methods for  $r$  in this paper.

## 2.0 Mathematical Model

Sprott noted the Bose construction

method of Balanced Incomplete Block Design (Bibd) with parameters  $v$  treatment,  $b$  block,  $r$  replication,  $k$  block size with an initial block(I) used and  $i$  ranges from 0 to  $m-1$ . Sprott used the above initial block to construct BISCOD, where  $x$  is a primitive root of  $GF(v)$ . [10], reviewed the work of [9], for the number of treatments  $v=7$ , the design was constructed for each above initial block with mode  $v$ , and the last period is repeated in the pre-period (0) for design to be balanced.

Where

$$v = mk + 1,$$

$$b = m(mk + 1),$$

$$r = mk,$$

$$k = v - 1/2$$

$$I = (x^i \quad x^{i+m} \quad x^{i+2m} \dots, \quad x^{i+(k-1)m})$$

Each treatment is preceded by each other treatment once Patterson and Lucas introduced another construction method for  $k$  treatment to block BIBD with parameters  $v, b, r, k, \lambda$  and  $b$  block arranged in  $b$  rows. Then come BISCOD with parameters  $v, p=k, n$ .

**Table 2. Biscod for 3Treatments, 2Period**

Period	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	1	2	1	3	1	3	2	3	2	3
2	2	1	1	2	3	1	1	3	3	2	2	3

Each treatment is preceded by each other treatment once and itself an unequal number of times. Mihilesh and Archana introduced another construction method for BISCOD, which is universally optimal for the first residual effect using BIBD with parameter  $v=4m+1, b=2(4m+1), r=4m, k=2m, \lambda=2m-1$ . Two initials blocks algorithms with different values of positive integer  $m$  and primitive root  $x=2$  for the number of treatments  $v$ , which is prime, are given to obtain block elements.

(1)

Example 1: Let  $v=5, m=2, x=2$  to obtain BIBD with two initial blocks.

$$i=0, x^i \quad x^{i+m} = 2^0, 2^{0+2} = 1, 4 \text{ Mod } 5 = 1, 4$$

$$i=1, x^i \quad x^{i+m} = 2^1, 2^{1+2} = 2, 8 \text{ Mod } 5 = 2, 3$$

(1,4) and (2,3) are the two initial blocks, other blocks obtained through the arrangement of cyclic development for Biscod:

**Table 1. Biscod for 5 Treatments, 2 Period**

Period	1	2	3	4	5	6	7	8	9	10
0	4	5	1	2	3	3	4	5	1	2
1	1	2	3	4	5	2	3	4	5	1
2	4	5	1	2	3	3	4	5	1	2

Example 2: Consider BIBD with parameters  $v=3, b=3, r=2, k=2$  and  $\lambda=1$  in  $b$  rows

$$1 \quad 2$$

$$1 \quad 3$$

$$2 \quad 3$$

$K(\text{number of element per block}) = (v-1)/2$

$$I_1 = (x^0, x^2, x^4, \dots, x^{4m-2})$$

(2)

$$I_2 = (x, x^3, x^5, \dots, x^{4m-1})$$

**(3) 2.1 Construction procedure (Cp):**

- (i) choose any initial blocks of the above
- (ii) obtain the  $(v-1)$  initial sequences from the above block by multiplying with elements which are non-zero of  $GF(v)$  and
- (iii) by developing the initial sequences

mod (v), we get a balanced crossover design which is universally optimal (Uobcod) for the first order with parameter  $v= 4m+1, N=4m(4m+1), k= 2m$ .

Example 3: For  $v=5, m=2, x=2, k= 2$

For (2),  $x^0 = x^{4m-2}$  at  $m=0.5$

$$x^2 = x^{4m-2} \text{ at } m=1$$

$$I_1 = x^0, x^2 = 2^0, 2^2 = 1, 4$$

For (3)  $x = x^{4m-1}$  at  $m=0.5$

$$x^3 = x^{4m-1} \text{ at } m=1$$

$$I_2 = x, x^3 = 2, 2^3 = 2, 8$$

$$\text{Mod } 5 \ I_1 = 1, 4 \ I_2 = 2, 3 \ \text{The two}$$

initial blocks are (1,4) and (2, 3)

### 2.2 Construction

Consider first initial block (1,4) and multiply with non zero elements to obtain (v-1) initial sequences with mod 5

$$\text{Mod } 5 \text{ for } 4 \text{ initial sequences} = \begin{matrix} 1 & 2 & 3 & 4 \\ & 4 & 3 & 2 & 1 \end{matrix}$$

Develop above 4 initial sequences cyclically to obtain UOBISCOD for first order residual effect  $v=5, p=k=2, n=20$

**Table 3. Biscod for 5Treatments, 2Period**

<b>Period 1</b>	1	2	3	4	5	2	3	4	5	1	3	4	5	1	2	4	5	1	2	3
<b>Period 2</b>	4	5	1	2	3	3	4	5	1	2	2	3	4	5	1	1	2	3	4	5

### 3.0 Result

The result showed that table 1 and table 3, which were tables for the first and third designs, respectively, showed that each treatment was replicated 6 and 8 times. Each treatment was preceded by each other treatment once, respectively. Table 2 showed that each treatment was replicated six and preceded by each other treatment once and itself was unequal. It was also discovered that both first and third designs were constructed through the primitive root, and last period treatments were repeated in the pre-period of the first design. Only parameters of BIBD was used by the second design to produced b row

### 4.0 Discussion

From the findings, both first and third construction methods are for the number of prime treatments that take primitive root to generate every non-zero element. Both methods consider design construction for prime numbers. For the first design to be balanced, the last period

was repeated in the pre-period. The second method considers design construction for any whole number.

### 5.0 Conclusion

To construct a design for the number of prime treatments using primitive root, either the first or third method can be used, but the second method should be used for any number of treatments, either prime or non-prime. A third should be used to measure the residual effect in the next immediate period after the treatment application period.

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