Covenant Journal of Physical & Life Sciences (CJPL) Vol. 8 No. 1, June 2020
ISSN: *p. 2354 – 3574 e. 2354 – 3485* ***DOI:*** *xxxxxxxxxxxxxxx*

An Open Access Journal Available Online

Estimating the Parameters of GARCH Models and Its Extension: Comparison between Gaussian and non-Gaussian Innovation Distributions

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Received: 17.05.2020 Accepted: 10.06.2020
Date of Publication: June, 2020

Abstract**:** Innovation distributions play significant role in determining the fitness as well as forecasting performance of volatility models. Several studies aimed at comparing the performance of volatility have been carried out but most of the studies focused on the use of Gaussian innovation distribution. Hence, this study compares the performance of GARCH models and its extensions using five innovation distributions, one Gaussian distribution (normal distribution) and four non- Gaussian innovation distributions(Student –t distribution, generalized error distribution, skewed Student- t and skewed generalized error distribution). Data on the daily closing prices of Zenith bank (04/01/2007 to 31/12/2019) and ETI (04/01/2007 to 31/12/2019) were obtained from cashcraft website and then converted to daily returns. Hence, using these five innovation distributions, the parameters of GARCH(1,1), TGARCH(1,1), EGARCH(1,1), IGARCH(1,1) and GJR- GARCH(1,1) were estimated. The performances of these models were compared in terms of fitness using AIC and forecasting performance based on Root Mean Square Error. Result of analysis revealed that GARCH models and its extensions estimated using non- Gaussian innovation distributions outperformed other innovation distributions both in terms of fitness and forecasting accuracy. Result also shows that among the non-Gaussian innovation distributions considered, the skewed generalized error distribution performed better than other non-Gaussian innovation distributions. The TGARCH (1,1)-sged and E-GARCH

(1,1)-sged were recommended as the best model for predicting the volatility in ETI and Zenith bank stocks respectively.
Keywords: Gaussian distribution, non-Gaussian distribution, innovation distributions, volatility.

1.0 Introduction

Over the years, volatility modelling has gained the attention of researchers especially those in financial time series [10; 12; 14; 15; 17]. This is because volatility is the major indices used to evaluate investment. Several volatility models have evolved overtime, one of which is Autoregressive Conditional Heteroscedasticity model (ARCH) proposed by [1]. The ARCH though was observed to capture true volatility, it was observed that a higher order of ARCH is needed and to overcome the problem of model parsimony, the generalized form of ARCH model called Generalized Autoregressive Conditional Heteroscedasticity model (GARCH) was proposed by [2]. The introduction of GARCH model helped to reduce the number of estimated parameters from infinity to just two. Due to the limitations of ARCH and GARCH models which is their inability to capture volatility modelling which is one of the major properties of asset returns, other forms of volatility models were proposed some of which include Exponential GARCH (E-GARCH) by [3], the Integrated Generalized Autoregressive Conditional Heteroscedastic model (IGARCH) by [4]), Glosten, Jagannathan and Runkle – Generalized Autoregressive Conditional Heteroscedastic model (GJR-GARCH) by [5], Absolute Value Generalized Autoregressive Conditional Heteroscedastic (AVGARCH) of [6], Asymmetric Power Autoregressive Conditional Heteroscedastic (APARCH), Fractional Integrated Exponential Generalized Autoregressive Conditional Heteroscedastic model (FIEGARCH (p,d,q)) by [7], the Hyperbolic Autoregressive Conditional Heteroscedastic (HYGARCH (p,d,q)) by [8], Asymmetric Power Autoregressive Conditional Heteroscedastic (APARCH) model by [9] among other models were proposed.

Furthermore, in order to estimate the parameters of these heteroscedastic models, various distribution of error innovation have been proposed. This is because as suggested by [10] the distribution of error distribution plays significant role in estimating the parameters of the heteroscedastic model. Notable among these innovation distributions are the normal distribution, Student- t distribution, generalized error distribution among others. Also, efforts have also been made by researchers ([11] and [12]) at estimating the parameters of volatility models using any of these distributions of error innovation. But the major gap in these studies was that their conclusion were derived by most of these studies based on normal distribution and given the recent developments in Nigeria most importantly government policies it very important to carry out a more recent study on this subject using other non-Gaussian innovation distribution (Student-t and generalized error distribution, skewed Student- t and skewed generalized error distributions). Hence, this study therefore compares the performance of GARCH models and its extensions using normal, Student- t distribution and generalized error distribution.

Several studies have been conducted on volatility modelling. [13] modeled and forecast the volatility of the Malaysian stock markets. The study made of high – frequency data so as to enhance the comparison of volatility forecast. The study focused on three volatility model which are GARCH(1.1), EGARCH(1.1) and NAGARCH(1.1) which were estimated using six distributions of error innovation. These include the normal, skew normal, student t, GED and NIE (GED – Generalized error distribution, NIG-normal inverse Gaussian distributions). The result suggested that heavy tailed error distribution gave a better variance forecasts comparing to using normal distribution. The study therefore concluded that the successful forecast of volatility depends largely on the choice of the error distribution rather than the choice of the GARCH model. Similarly, [14] estimated stock market volatility using asymmetric GARCH models. The asymmetric GARCH models used in the study include the GJR-GARCH, APARCH, and EGARCH. The study was carried out in Tel Avis Stock Exchange (TASE) in Israel and three distributional form of error innovation were used namely normal, student – t, and skewed-student-t. The study also quantified the day of the week effect and the leverage effect on volatility of Tel. Avis Stock Exchange in Israel. The result revealed that asymmetric GARCH model with fat tailed densities improved overall estimation of measuring conditional varies. The study also found that the skewed student-t distribution is the most successfully distribution for forecasting the volatility of TASE indices.

Also, [15] modeled the market volatility with APARCH model. The study discussed the APARCH model and its ability to forecast the conditional volatility of standard and poor 500 stock market daily closing price index and MSCI Europe Index under the various density functions normal distribution, student-t distribution and skewed student-t distribution. The study found that skewed student-t distribution is the most efficient distribution under APARCH model. The APARCH model under skew student-t distribution has a larger likelihood and smaller error compared to other distribution.

Similarly, [16] modeled the volatility in the Nigerian Stock using the new class of volatility models precisely Generalized Autoregressive Score (GAS), Exponential GAS (EGAS) and Asymmetric Exponential GAS (AEGAS). These models were applied to data on the Nigeria All Share Index (ASI) from January 3, 2006 to July 22, 2014. Parameters of these models were estimated using the Quasi Maximum Likelihood (QML) approach, and in-sample conditional volatility forecasts from each of the models were evaluated using the minimum loss function approach. The findings showed that the EGARCH-Beta-t innovation outperformed IGARCH- Student-t innovation. In Pakistan, [17] evaluated and forecasted the volatility of stocks in Karachi stock exchange, Pakistan. Data were collected between the between 1998 and 2011. The study fitted various forms of volatility models to the data. The study considered three distribution of error innovations namely Gaussian distribution, generalized error distribution, Student- t distribution and. The findings revealed the superiority of Student-t distribution over other innovation distributions considered. The findings also showed that asymmetric volatility models were better than symmetric volatility models. [18] carried a study which compared the performance of GARCH, EGARCH and GJR in estimating financial volatility. Data used in the study was a 1278 daily closing value of USD – INR exchange rate from 11th June, 2007 to 20th August, 2013. Models considered were GARCH model, EGARCH and GJR models with afferent order of autoregressive and moving average. The normal (Gaussian) distribution form was used. Result revealed that among the twenty GARCH types of model, GARCH specifications, particularly GARCH (1, 1) specification was measured to be better than advanced EGARCH and GJR – GARCH Specifications.

In Nigeria, [19] modeled the volatility in Nigerian stock market using Nigeria All Share Index (ASI) between 02/10/2001 and 29/03/2018. The study considered five volatility models; GARCH(1,1), APARCH (1,1), GJR-GARCH(1,1), IGARCH(1,1) and EGARCH(1,1) which were estimated using skewed normal, skewed Student- t distritbution and skewed generalized error distributions. Result revealed evidence of volatility clustering and high persistence of volatility. Result also showed that among the competing models, APARCH(1,1)- skewed normal distrbution outperformed other volatility models.

Also, [22] examined the persistence of shock, symmetric and asymmetric responses in Nigerian stock market using one symmetric volatility model [GARC(1,1)] and two asymmetric volatility models [EGARCH(1,1) and TARCH(1,1)] which were estimated using normal, Student-t, skewed Student- t, generalized error distribution and skewed generalized error distribution. The study obtained All Share Index data between 3rd July, 1999 and 12th June 2017 and January 1985 to March 2017 respectively. Result showed evidence of volatility clustering and high persistence of volatility shocks with explosive tendency. Only few of these studies carried out in Nigeria considered modeling volatility using non Gaussian innovation distribution. The few that considered skewed innovation distribution do not consider the price of individual stock listed on the Nigeria Stock Exchange (NSE) but rather make use of All Share Index as a proxy for stock price. These identified gaps served as a motivation for this study.

2.0 Methods

2.1 Study Data

Data used in conducting this study were the daily closing price of Zenith bank (04/01/2007 to 31/12/2019) and ETI (04/01/2007 to 31/12/2019). These data were accessed through the official website of Cashcraft which is one of the leading stock broking firms in Nigeria (www.cashcraft.com). The R statistical package was used in data analysis.

2.2 Generation of daily return series from price

The daily returns series for each stock were generated from price using the daily price using the formula below:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right), \text{ t = 2, n} \qquad (1)$$

where, P_t is the closing price at day t (present day) while P_{t-1} is the daily closing price at the day t-1(previous day).

2.3 Normality of the result series

To assess the normality of the daily return series of the selected stock, the Jacque Bera test was used. The Jacque Bera test is given by:

$$JB = \frac{n}{6}\left[\rho^2 + \frac{(\eta-3)^2}{4}\right] \qquad (2)$$

where, ρ is the skewness and η is the kurtosis. The test statistics is approximately χ_2^2 and the null hypothesis is rejected if the probability value is less than .05.

2.4 Stationarity test for the daily return series

The stationarity of the daily return series were tested using Augmented Dickey Fuller Test (ADF) with the null and alternative hypotheses stated below:

The null hypothesis is: $H_0 : \theta = 1$

The alternative hypothesis is $H_0 : \theta \prec 1$

The test statistic,

$$\text{t-ratio} = \frac{\hat{\theta}-1}{Std(\theta)} = \frac{\sum_{r=2}^{n} p_{t-1}e_t}{\hat{\sigma}^2\sqrt{\sum_{r=2}^{n} p_{t-1}^2}} \qquad (3)$$

where, $$\hat{\theta} = \frac{\sum_{r=2}^{n} p_{t-1}p_t}{\sum_{r=2}^{n} p_{t-1}^2} \qquad (4)$$

and, $$\hat{\rho}^2 = \frac{\sum_{r=2}^{n}(p_t - \hat{\theta}p_{t-1})^2}{n-1} \qquad (5)$$

where, n is the sample size which in the observations for returns. The null hypothesis is rejected if the probability value is less than 0.05(p<0.05).

2.5 Innovation Distributions considered in the study

Normal distribution/ Gaussian distribution

$$f(z_t) = \frac{1}{\sqrt{2\pi}} e^{-z_t/2} \qquad -\infty < z_t < \infty \tag{6}$$

Student t-distribution (Non- Gaussian distribution)

$$f(z_t) = \frac{\Gamma\left(\frac{\rho+1}{2}\right)}{\sqrt{\rho\pi\Gamma\left(\rho/2\right)}}\left(1+\frac{z_t^2}{\rho}\right)^{-\left(\frac{\rho+1}{2}\right)}, \quad -\infty < z_t < \infty \tag{7}$$

where, ρ denote the number of degrees of freedom and Γ is the Gamma function.

Generalized Error Distribution (GED) (Non- Gaussian distribution)

$$f(z_t, \mu, \sigma, \rho) = \frac{\sigma^{-1}\rho e^{\left[-0.5\left|\left(\frac{z_t-\mu}{\sigma}\right)\lambda\right|^{\rho}\right]}}{\lambda 2^{\left[1+\left(\frac{1}{\rho}\right)\right]\Gamma\left(\frac{1}{\rho}\right)}} \quad -\infty < \eta_t < \infty \tag{8}$$

$\rho > 0$ is the degree of freedom or tail thickness parameters and

$$\lambda = \sqrt{2^{\left(-\frac{2}{\rho}\right)\Gamma\left(\frac{1}{\rho}\right)} \Big/ \Gamma\left(\frac{3}{\rho}\right)} \tag{9}$$

If $\rho = 2$, the GED will give the normal distribution.

Skewed Student t-distribution

$$f(z_t, \mu, \sigma, \phi, \lambda) = \begin{cases} bc\left[1+\frac{1}{\phi-2}\left(\frac{b\left(\frac{\eta_t-\mu}{\sigma}\right)+a}{1-\lambda}\right)^2\right]^{-\frac{\rho+1}{2}}, & z_t < -\frac{a}{b} \\ bc\left[1+\frac{1}{\phi-2}\left(\frac{b\left(\frac{\eta_t-\mu}{\sigma}\right)+a}{1+\lambda}\right)^2\right]^{-\frac{\rho+1}{2}}, & z_t \geq -\frac{a}{b} \end{cases} \tag{10}$$

where, ϕ and λ represent the shape and skewness parameters respectively.

$$a = 4\lambda c\left(\frac{\phi-2}{\phi-1}\right), \; b = 1+3\lambda^2 - a^2, \; c = \frac{\Gamma\left(\frac{\phi+1}{2}\right)}{\sqrt{\pi(\phi-2)\Gamma\left(\frac{V}{2}\right)}} \tag{11}$$

Skewed Generalized Error Distribution

$$f(z_t / \rho, \varepsilon, \theta, \delta) = \frac{\rho}{2\theta\Gamma\left(\frac{1}{\rho}\right)} \exp\left[-\frac{|z_t - \delta|^{\rho}}{[1 + sign(z_t - \delta)\varepsilon]^{\rho}\theta^{\rho}}\right]$$

$$\theta > 0,\ -\infty < z_t < \infty,\ \rho > 0,\ -1 < \varepsilon < 1 \qquad -\infty < z_t < \infty \qquad (12)$$

where,

$$\theta = \Gamma\left(\frac{1}{\rho}\right)^{0.5} \Gamma\left(\frac{3}{V}\right)^{-0.5} S(\varepsilon)^{-1},$$

$$\delta = 2\varepsilon S(\varepsilon)^{-1},$$

$$S(\varepsilon) = \sqrt{1 + 3\varepsilon^2 - 4A^2\varepsilon^2},$$

$$A = \Gamma\left(\frac{2}{\rho}\right)\Gamma\left(\frac{1}{\rho}\right)^{-0.5} \Gamma\left(\frac{3}{\rho}\right)^{-0.5}$$

where, $\rho > 0$ is the shape parameter, ε is a skewness parameter with $-1 < \varepsilon < 1$.

2.6 GARCH model and its extensions considered in the study

(i.) The Generalized Autoregressive Conditional Heteroscedasticity model (GARCH).

$$R_t = \mu + \varepsilon_t, \quad \eta_t{}^2 = \omega + \sum_{j=1}^{p} \beta_j \eta_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2. \quad \varepsilon_t = \eta_t z_t \qquad (13)$$

$\omega, \beta_j,\ \alpha_i \geq 0$ and for stationarity, $\alpha_i + \beta_j < 1$, ω is constant term, β_j is GARCH term while α_i is the ARCH term, η_t is the volatility, R_t is the returns and is ε_t the residuals.

(ii.) Threshold Generalized Autoregressive Conditional Heteroscedasticity model (TGARCH).

$$\eta_t{}^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j \eta_{t-j}^2 + \sum_{k=1}^{w} \gamma_k I_{t-i} \varepsilon_{t-1}^2,\ \varepsilon_t = \eta_t z_t \qquad (14)$$

where, ω is constant term, α_i is ARCH term while β_j is the GARCH term, $\omega \geq 0$, α_i and $\beta_j \geq 0$, η_t is the volatility and I_{t-i} is an indicator variable.

$$I_{t-i} = \begin{cases} 1, \varepsilon_{t-i} \prec 0 \\ 0, \varepsilon_{t-i} \geq 0 \end{cases}$$

(iii.) Glosten, Jagannathan and Runkle Generalized Autoregressive Conditional Heteroscedasticity model (GJR-GARCH).

$$\eta_t{}^2 = \omega + \sum_{i=1}^{p}\left(\alpha_i \varepsilon_{t-1}^2 + \gamma_1 I_{t-i}\varepsilon_{t-1}^2\right) + \sum_{j=1}^{q}\beta_j \eta_{t-i}^2. \quad \varepsilon_t = \eta_t z_t \qquad (15)$$

$$I_{t-i} = \begin{cases} 1, \varepsilon_{t-i} \prec 0 \\ 0, \varepsilon_{t-i} \geq 0 \end{cases}$$

ω is constant term, α_i is ARCH term while β_j is the GARCH term, γ_1 is the leverage term, $\omega \geq 0$, α_i and $\beta_j \geq 0$ and η_t is the volatility.

(iv.) Exponential Generalized Autoregressive Conditional Heteroscedasticity model (EGARCH)

$$R_t = \mu + \varepsilon_t, \ \ln(\eta_t^2) = \omega + \sum_{i=1}^{p}\alpha_i\left[\lambda\varepsilon_{t-i} + \gamma\left\{\left|\varepsilon_{t-i}\right| - \sqrt{\frac{2}{\pi}}\right\}\right] + \sum_{j}^{q}\beta_j \ln(\eta_{t-j}^2), \ \varepsilon_t = \eta_t z_t$$

(16)

where, ω is constant term, α_i is ARCH term while β_j is the GARCH term and, γ is the leverage term and η_t is the volatility.

(v.) Integrated Generalized Autoregressive Conditional Heteroscedasticity model (IGARCH)

$$R_t = \mu + \varepsilon_t, \ \eta_t{}^2 = \omega + \sum_{j=1}^{q}\beta_j \eta_{t-j}^2 + \sum_{i=1}^{p}\alpha_i \varepsilon_{t-i}^2. \quad \varepsilon_t = \eta_t z_t \qquad (17)$$

where, ω is constant term, α_i is ARCH term while β_j is the GARCH term and

$$\sum_{j=1}^{p}\alpha_j + \sum_{i=1}^{q}\beta_j = 1 \ .$$

3.0 Results

Table 1: Descriptive statistics for daily prices and returns series of Zenith banks and ETI stocks

Statistic	**Zenith bank**		**ETI**	
	Daily price	Daily returns	Daily price	Daily returns
n	3131	3130	3131	3130
Mean	22.0806	-0.00009	36.3125	-0.00052
Maximum	68.9700	0.051335	300.9800	0.65321
Minimum	9.00000	-0.176257	6.00000	-0.69897
Standard deviation	10.9943	0.011426	58.51499	0.024400
Skewness	1.8189	-1.652441	2.65007	-8.63473
Kurtosis	5.9658	26.48196	8.65986	573.2944
Jacque Bera	2873.9560	73336.51	7843.879	4245505
p-values	0.0000	0.000	0.0000	0.0000

Table 2: Augmented Dickey Fuller (ADF) test result summary and test of heteroscedasticity

Stocks	ADF Test Statistic	Probability values	Comment	ARCH test F-stat.	p-value
Zenith	-45.045	0.0001	Stationary at level	5.18183	0.0229
ETI	-46.410	0.0001	Stationary at level	324.1762	0.0000

Table 1 presents the descriptive summary for both the daily prices and the returns of the two selected stocks. Result showed that the mean returns of the two selected stocks were negative meaning that the stock recorded loss within the periods under study. The skewness obtained for ETI (-8.63473) and Zenith bank stock (-1.652441) were both negative indicating that the returns of these stocks decreased more than it increased hereby corroborating the results of the mean returns. The Jacque Bera test showed p-value less than 0.05 both for the daily prices as well as the daily returns of the two selected stock meaning that the daily prices and the daily returns of these stocks do not follow normal distribution. The ADF test result revealed that these returns are stationary ($p<0.05$) (Table 2). The ARCH effect was found to be significantly present in the returns series ($p<0.05$). This therefore necessitated the need to subject the daily returns series of ETI and Zenith bank stocks to volatility models. The result also revealed evidence of volatility clustering in both stocks which is an indication that large changes in volatility were followed by large changes in volatility while small changes in volatility were also followed by small changes in volatility. The leverage effect which measures whether there is a negative relationship between asset returns and volatility was found to be significant in both stocks ($p<0.05$)(Tables 3 and 4). The result also revealed that in terms of fitness and forecasting performance based on LL, AIC and RMSE, volatility model estimated using the non Gaussian innovation distributions(Student-t, generalized error distribution, skewed Student- t and skewed generalized error distributions) were found to outperform that of the Gaussian innovation distribution both in terms of fitness performance and forecasting performance(Table 5). Result also reveals that among the non- Gaussian innovation distributions considered, the skewed generalized error distribution outperformed other non- Gaussian innovation distributions both in terms of fitness and forecasting performance.

Table 3a: Parameter estimates and fitness for GARCH (1,1), TGARCH(1,1) and EGARCH(1,1) using normal, Student-t, generalized error distribution, skewed Student-t and skewed generalized error distribution for Zenith bank returns.

Model		μ	ω **(p-value)**	α_1 **(p-value)**	β_1 **(p-value)**	γ_1 **(p-value)**	**LL**	**AIC**	**ARCH test for diagnostic checking**
GARCH (1,1)	norm	-0.000095 (0.18597)	0.000005 (0.0000)	0.198940 (0.0000)	0.784670 (0.0000)	-	9940.362	-6.3491	0.9841
	std	0.00000 (0.9999)	0.00000 (0.0000)	0.18883 (0.0000)	0.80821 (0.0000)	-	10243.24	-6.5420	0.9858
	ged	0.00000 (0.9999)	0.00000 (0.54556)	0.13288 (0.0000)	0.86358 (0.0000)	-	10279.67	-6.5653	0.8810
	sstd	0.00000 (0.93636)	0.00000 (0.9999)	0.18629 (0.0000)	0.81072 (0.0000)	-	10243.09	-6.5413	0.9857
	sged	0.000000 (0.98449)	0.000005 (0.0000)	0.164108 (0.0000)	0.786454 (0.0000)	-	**10292.07**	**-6.5726**	0.9248
TGARCH (1,1)	norm	-0.000087 (0.00156)	0.001377 (0.029192)	0.164227 (0.0000)	0.819651 (0.0000)	-0.048584 (0.299116)	9974.613	-6.3697	0.4167
	Std	0.000000 (0.99955)	0.000001 (0.56158)	0.183007 (0.0000)	0.857547 (0.0000)	0.035299 (0.29330)	10527.41	-6.7223	0.9858
	ged	0.000000 (0.99992)	0.007107 (0.00000)	0.165637 (0.0000)	0.880555 (0.0000)	-0.429430 (0.00000)	**10745.77**	**-6.8618**	0.9857
	sstd	0.000000 (0.99940)	0.000001 (0.61839)	0.163696 (0.0000)	0.868822 (0.00000)	0.015927 (0.61683)	10442.81	-6.6676	0.9859
	sged	0.000000 (0.99993)	0.004978 (0.0000)	0.195998 (0.0000)	0.854360 (0.000000)	-0.161770 (0.002415)	10514.04	-6.7131	0.9857
EGARCH (1,1)	norm	-0.000213 (0.0000)	-0.748281 (0.0000)	0.004178 (0.7581)	0.914749 (0.00000)	0.336637 (0.0000)	9963.755	-6.3634	0.4248
	Std	0.000004 (0.44142)	-0.404546 (0.0000)	-0.03685 (0.0400)	0.954585 (0.0000)	0.384687 (0.0000)	10251.26	-6.5465	0.8387
	ged	0.00000 (0.99982)	-0.44615 (0.0000)	-0.04405 (0.0956)	0.94813 (0.0000)	0.49146 (0.0000)	10360.31	-6.6155	0.8563
	sstd	-0.000067 (0.0000)	-0.362049 (0.0000)	-0.03866 (0.0291)	0.959294 (0.0000)	0.382385 (0.0000)	10252.73	-6.5468	0.8392
	sged	0.000000 (0.99582)	-0.447791 (0.0000)	-0.06924 (0.0213)	0.947852 (0.0000)	0.505887 (0.0000)	**10360.31**	**-6.6162**	**0.8737**

Bolded values are the highest value of likelihood function and the least value of AIC, norm-normal distribution std- Student-t distribution, ged- generalized error distribution, sstd- skewed Student- t distribution and sged- skewed generalized error distribution.

Table 3b: Parameter estimates and fitness for IGARCH (1,1) and GJR-GARCH(1,1) using normal, Student-t, generalized error distribution, skewed Student-t and skewed generalized error distribution for Zenith bank returns.

Models		μ	ω **(p-value)**	α_1 **(p-value)**	β_1 **(p-value)**	γ_1 **(p-value)**	**LL**	**AIC**	**ARCH test for diagnostic checking**
IGARCH (1,1)	norm	-0.00010 (0.0803)	0.000005 (0.0000)	0.210754 (0.0000)	0.789246 (0.0000)	-	9939.165	-6.3490	0.9626
	std	-0.000001 (0.99969)	0.000001 (0.9999)	0.192966 (0.9866)	0.807034 (0.0000)	-	10237.22	-6.5388	0.9841
	ged	0.00000 (0.9999)	0.00000 (0.88321)	0.14003 (0.0000)	0.85996 (0.0000)	-	**10285.82**	**-6.5699**	**0.7637**
	sstd	0.000001 (0.96596)	0.000000 (0.99896)	0.191053 (0.0000)	0.808947 (0.0000)	-	10237.59	-6.5384	0.9841
	sged	0.00000 (0.99979)	0.00000 (0.90048)	0.13647 (0.0000)	0.86353 (0.0000)	-	10285.73	-6.5692	0.8732
GJR-GARCH (1,1)	norm	-0.000097 (0.18393)	0.000005 (0.0000)	0.198140 (0.0000)	0.784314 (0.0000)	0.002646 (0.9219)	9940.367	-6.3485	0.9817
	std	0.00001 (0.98774)	0.000000 (0.99999)	0.196905 (0.0000)	0.790381 (0.0000)	0.019792 (0.3634)	10250.17	-6.5458	0.9857
	ged	0.000000 (0.9999)	0.000000 (0.8502)	0.130632 (0.0000)	0.858021 (0.0000)	0.020688 (0.2846)	10284.78	-6.5673	0.8886
	sstd	0.000001 (0.91335)	0.000000 (0.9999)	0.166659 (0.0000)	0.811290 (0.0000)	0.037696 (0.0691)	10244.29	-6.5414	0.9288
	sged	0.000000 (0.99998)	0.000000 (0.86014)	0.129816 (0.0000)	0.858727 (0.0000)	0.020853 (0.2792)	**10284.84**	**-6.5679**	0.8843

Bolded values are the highest value of likelihood function and the least value of AIC, norm- normal distribution std- Student-t distribution, ged- generalized error distribution, sstd- skewed Student- t distribution and sged- skewed generalized error distribution.

Table 4a: Parameter estimates and fitness for GARCH (1,1), TGARCH(1,1) and EGARCH(1,1) using normal, Student-t, generalized error distribution, skewed Student-t and skewed generalized error distribution for ETI stock.

Models		μ	ω **(p-value)**	α_1 **(p-value)**	β_1 **(p-value)**	γ_1 **(p-value)**	**LL**	**AIC**	**ARCH test for diagnostic checking**
GARCH (1,1)	norm	-0.000387 (0.0000)	0.000004 (0.0000)	0.165005 (0.0000)	0.833994 (0.0000)	-	9435.445	-6.0265	0.9780
	Std	0.00000 (0.98889)	0.00000 (0.9999)	0.33017 (0.0000)	0.6530 (0.0000)	-	12002.60	-7.6630	0.9998
	ged	-0.000517 (0.0000)	0.000001 (0.33939)	0.050000 (0.0000)	0.900000 (0.0000)	-	6302.196	-4.0238	0.7701
	sstd	0.00000 (0.98548)	0.00000 (0.9999)	0.33066 (0.0000)	0.65224 (0.0000)	-	**12002.83**	**-7.6657**	**0.9998**
	sged	-0.000517 (0.0000)	0.000001 (0.29726)	0.050000 (0.0000)	0.900000 (0.0000)	-	6302.196	-4.0231	0.9547

TGARCH (1,1)	norm	-0.000028 (0.80816)	0.000110 (0.00000)	0.167073 (0.00000)	0.859810(0.0000)	-0.225313 (0.00000)	9504.962	-6.0696	0.9530
	Std	0.00000 (0.999234)	0.00000 (0.336918)	0.34381 (0.0000)	0.68964 (0.0000)	0.11260 (0.0000)	**12070.22**	**-7.7081**	**0.9721**
	ged	-0.000517 (0.0000)	0.000001 (0.0000)	0.050000 (0.0000)	0.900000 (0.0000)	0.050000 (0.0000)	6301.908	-4.0223	0.8166
	sstd	0.000000 (0.994474)	0.000000 (0.454084)	0.331444 (0.0000)	0.702610 (0.0000)	-0.045771 (0.0000)	12029.33	-7.6814	0.9998
	sged	-0.000517 (0.0000)	0.000001 (0.00000)	0.050000 (0.0000)	0.900000 (0.0000)	0.050000 (0.0000)	6301.908	-4.0217	0.9547
EGARCH (1,1)	norm	0.000021 (0.68489)	-0.408695 (0.0000)	0.108233 (0.0000)	0.948265 (0.0000)	0.251125 (0.0000)	9521.375	-6.0808	0.9518
	std	0.00000 (0.9997)	-0.61282 (0.0000)	0.65907 (0.0000)	0.93291 (0.0000)	1.05076 (0.0000)	12028.55	-7.6815	0.9546
	ged	0.00000 (0.9997)	-0.61282 (0.0000)	0.65907 (0.0000)	0.93291 (0.0000)	1.05076 (0.0000)	14821.73	-9.4663	0.9546
	sstd	0.000007 (0.46381)	-0.170598 (0.0000)	0.682461 (0.0000)	0.969759 (0.0000)	1.969245 (0.0000)	10592.17	-6.7637	0.9684
	sged	-0.000016 (0.0000)	-0.709868 (0.0000)	0.346432 (0.0000)	0.908134 (0.0000)	0.494257 (0.0000)	**14821.75**	**-9.4669**	0.9546

Bolded values are the highest value of likelihood function and the least value of AIC, norm-normal distribution std- Student-t distribution, ged- generalized error distribution, sstd- skewed Student- t distribution and sged- skewed generalized error distribution.

Table 3b: Parameter estimates and fitness for IGARCH (1,1) and GJR-GARCH(1,1) using normal, Student-t, generalized error distribution, skewed Student-t and skewed generalized error distribution for ETI stock.

Models		μ **(p-value)**	ω **(p-value)**	α_1 **(p-value)**	β_1 **(p-value)**	γ_1 **(p-value)**	**LL**	**AIC**	**ARCH test for diagnostic checking**
IGARCH (1,1)	norm	-0.000134 (0.50405)	0.000000 (0.14590)	0.025732 (0.00000)	0.974268 (0.0000)	-	9000.746	-5.7494	0.9797
	Std	0.00000 (0.94640)	0.00000 (0.93444)	0.25578 (0.0000)	0.74422 (0.0000)	-	10783.19	-6.8877	**0.9546**
	ged	-0.000517 (0.00000)	0.000001 (0.00000)	0.050000 (0.0000)	0.950000 (0.0000)	-	6763.717	-4.3193	0.9645
	sstd	0.00000 (0.73544)	0.00000 (0.94191)	0.30018 (0.0000)	0.69983 (0.0000)	-	**10800.25**	**-6.8979**	0.9479
	sged	-0.000517 (0.0000)	0.000001 (0.0000)	0.050000 (0.0000)	0.950000 (0.0000)		6763.717	-4.3187	0.9645
GJR-GARCH (1,1)	Norm	-0.000235 (0.0000)	0.000004 (0.0000)	0.232767 (0.0000)	0.830418 (0.0000)	-0.12837 (0.0000)	9466.374	-6.0456	0.9922
	std	-0.000232 (0.0000)	0.000007 (0.0000)	0.232763 (0.0000)	0.830416 (0.0000)	-0.12822 (0.0000)	9486.362	-6.0556	0.9977
	ged	-0.000517 (0.0000)	0.000001 (0.002169)	0.050000 (0.0000)	0.900000 (0.0000)	0.05000 (0.0000)	5959.125	-3.8039	0.9858
	sstd	0.00000	0.00000	0.30562	0.66246	0.04273	**11985.01**	**-7.6537**	**0.9721**

	(0.98141)	(0.9999)	(0.0000)	(0.0000)	(0.18062)			
sged	-0.000517	0.000001	0.050000	0.900000	0.050000	5959.125	-3.8033	0.9536
	(0.00000)	(0.004027)	(0.0000)	(0.0000)	(0.0000)			

Bolded values are the highest value of likelihood function and the least value of AIC, norm- normal distribution std- Student-t distribution, ged- generalized error distribution, sstd- skewed Student- t distribution and sged- skewed generalized error distribution.

Table 5: Fitness and forecasting performance for GARCH models estimated using normal, Student- t, generalized error distribution, skewed Student-t and skewed generalized error distribution

Stocks	**Volatility models**		**Normal**	**STD**	**GED**	**SSTD**	**SGED**
ETI	GARCH(1,1)	RMSE	0.024399	0.024395	0.024392	0.024320	**0.024318**
	TGARCH(1,1)	RMSE	0.024499	0.024499	0.024482	0.024418	**0.024314**
	EGARCH(1,1)	RMSE	0.024418	0.024399	0.024392	**0.024380**	0.024385
	IGARCH(1,1)	RMSE	0.024419	0.024410	0.024412	0.024319	**0.024412**
	GJR-GARCH(1,1)	RMSE	0.024400	0.024300	0.0244310	**0.0244210**	0.0244220
Zenith bank	GARCH(1,1)	RMSE	0.011424	0.011422	0.011420	**0.011414**	0.011418
	TGARCH(1,1)	RMSE	0.011455	0.011453	0.011440	**0.011430**	0.011433
	EGARCH(1,1)	RMSE	0.011441	0.011439	0.011433	0.011432	**0.011410**
	IGARCH(1,1)	RMSE	0.011460	0.011455	0.011453	**0.0114300**	0.011420
	GJR-GARCH(1,1)	RMSE	0.011477	0.011472	0.011453	0.0114500	**0.011428**

MSE- Mean Square Error, RMSE- Root Mean Square Error. Bolded values are the least Root Mean Square Error (RMSE)

4.0 Discussions

This study found that GARCH model and its extensions estimated using non-Gaussian innovation distribution gave better results in terms of fitness and forecasting performance than those estimated under the assumption of normally distributed innovation distribution. The advantage of the non-normally distributed error innovation over normally distributed error innovation could be as a result of the fact that the normal distribution does not have the ability to capture the leptokurtosis (excess kurtosis) that is usually exhibited by asset returns. This finding could also be due to the fact that the non- Gaussian innovation distributions have fatter tail than the normal distribution. This finding agrees with that of the finding by [11] on estimation of GARCH models for Nigerian rates under non- Gaussian innovations were the non- Gaussian distribution precisely student- t distributions and generalized error distribution were found to be superior than the Gaussian distribution when estimating parameters of GARCH models.

Also, this finding is in line with that of the finding by Atoi(2014) on the volatility of Nigerian stock market using GARCH models which found that the non- Gaussian distributions (Student- t distribution and generalized error distribution) gave better fitness and forecasting ability than the normal distribution. This finding agrees with that of [21] on characteristic responses of symmetric and asymmetric volatility shocks in the Nigerian Stock Market where the heavy tailed distributions were found to better capture the volatility than Gaussian distribution. But this finding is not in line with that of the finding by [16] in Sweden OMXS 30 where TGARCH model with normal distribution for error innovation was found to be superior to both the student-t distribution and generalized error distribution. This disparity in finding could be due to the period where the former study was conducted and that of the present study as the volatility behaviour in 2011 may not be the same as that of 2019 due to the present realities.

5.0 Conclusion and Recommendation

This study has examined the performance of GARCH model and its extensions estimated using Gaussian and non- Gaussian distributions. The empirical analysis using daily closing prices of ETI and Zenith bank between 04/01/2007 to 31/12/2019 showed that the non- Gaussian distributions outperformed Gaussian innovation distributions. This study therefore recommends the use of non- Gaussian distributions when estimating parameters of GARCH models and its extensions. Also, among the competing volatility models, TGARCH (1,1)-sged and E-GARCH (1,1)-sged were recommended as the best model for predicting the volatility in ETI and Zenith bank stocks respectively.

Acknowledgment

The researchers appreciate the Cashcraft Asset Management Limited for making data on daily prices of these stocks available on their website.

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