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Magnetic Spin Susceptibility of quasi-particles in metals using the Landau Fermi Liquid Theory

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In this work, the magnetic spin susceptibility of quasi-particles in metals were computed for some metals based on the modified Landau Fermi Liquids Theory using the electron density parameter. The results showed that for each metal, the Landau magnetic spin susceptibility of quasi-particles is higher than the computed magnetic spin susceptibility of quasi-particles and experimental values. This may be due to the fact that the Landau parameter must have been over estimated in its application. The computed magnetic spin susceptibility of quasi-particles is in good agreement with the experimental values of metals available with a remarkable agreement at $F_0^a \geq -9$. The better estimation of the magnetic spin susceptibility of quasi-particles using the modified Landau Fermi Liquid theory were compared with available experimental values. This show that the introduction of the electron density parameter in the Landau Fermi Liquid theory is promising in predicting the contribution of quasi-particles to the bulk properties of metals. The magnetic spin susceptibility of quasi-particles for transition metals is higher than most of the magnetic spin susceptibility of quasi-particles for alkali metals. This suggests that the magnetic spin susceptibility is considerably higher for most transition metals due to the incomplete inner electronic shells as more quasi-particles can be excited which enhances their susceptibility than the alkali metals.

Keywords: Quasi-particles, Electron density parameter, Magnetic spin susceptibility, Fermi Liquids, Landau parameter F_0^a.

1.0 Introduction

The Landau's theory of Fermi liquids is a basic fundamental paradigm in many-body physics that has recorded a remarkable success in solving the properties of a wide range of interacting fermion systems, such as liquid helium-3, nuclear matter, and electrons in metals. The Landau theory of Fermi liquids gives a good understanding of weakly correlated, gapless Fermi systems at low temperatures, such as ^{3}He atoms in the normal liquid state and travelling electrons in metals [1, 2, 3]. It provides the understanding of metals in terms of weakly interacting quasi-particles which is the basis effects of interaction between electrons on the metallic state. It also gives account of puzzling observation that despite strong interactions between the constituent fermions, many Fermi systems behave essentially as free Fermi gases, except for the renormalization of their physical properties which is captured by dimensionless quantities known as Landau parameters. These Landau parameters describe how the elementary excitations of the Fermi Liquid that is the quasi-particles and quasiholes interact with one another [4, 5].

The instabilities in spin and charge channels for Landau parameters in the non-degenerate extended using Hubbard model with intersite coloumb and exchange interaction was investigated by Lhoutellier *et al*., [6]. The inverse propagator was determined using spin rotational invariant slave boson approach. It derived the spin Landau parameter F_0^a of the non-degenerate Hubbard model towards ferromagnetism by $F_0^a = -1$ for the divergence of the magnetic susceptibility for half-filled band uncovers intrinsic. It results showed instability in the strongly correlated metallic regime for any lattice in two or three dimensions.

As discussed by Lundgren and Maciejko, [7], the d-dimensional boundaries of (d + 1)-dimensional topological phases of matter give new types of many-fermion systems that are topologically distinct from conventional systems. It constructed phenomenological Landau theory for the two-dimensional helical Fermi liquid on the surface of a 3-dimensional time reversal invariant topological insulator. In the presence of rotation symmetry, interactions between quasi-particles are described by ten independent Landau parameters per angular momentum channel. As a result of this nontrivial Berry phase, projection can increase or lower the angular momentum of the quasiparticle interactions to the Fermi surface. It also accounted for the equilibrium properties, criteria instabilities, and collective mode dispersions.

Chubukov, *et al.* [8], considered the non-analytic temperature dependences of the specific heat coefficient, CT=T, and spin susceptibility, sT, of 2D interacting fermions beyond the weak-coupling limit. It demonstrated within the Luttinger-Ward formalism that the leading temperature dependences of CT=T and sT are linear in T, and are described by the Fermi liquid theory. It concluded that the temperature dependences and are universally determined by the states near the Fermi level.

Rodriguez-Ponte *et al*. [9] studied Fermi liquids with a Fermi surface that lacks continuous rotational invariance and, in the presence of an arbitrary quartic interaction. The results gives generalized static susceptibilities which measured linear response of a generic order parameter to a perturbation of the Hamiltonian. These results were applied to spin and charge susceptibilities. Based on this, a proposal for the definition of the Landau parameters in non-isotropic Fermi liquid was made.

2.0 Theory and calculations.

2.1 Magnetic Spin susceptibility of quasi-particles.

Susceptibility is generally defined as the ratio of the induced magnetization to the inducing magnetic force. Determination of magnetic spin susceptibility is defined by [10],

$$M = \chi H = \mu_B \left(\delta n_\uparrow - \delta n_\downarrow \right), \qquad (1)$$

the change of the quasi-particle distribution function due to an external magnetic field applied in z-direction was computed. The quasi-particle energies changes because of the field and the change in the distribution function,

$$\delta\varepsilon_{p\sigma} = -\mu_B \sigma_z H + \sum_{p'\sigma'} f_{p\sigma,p'\sigma'} \delta n_{p'\sigma'}, \qquad (2)$$

with $\sigma_z = \pm 1$. The change in the distribution function is given by,

$$\delta n_{p\sigma} = \frac{\partial n^0_{p\sigma}}{\partial \varepsilon_{p\sigma}} \left(\delta\varepsilon_{p\sigma} - \delta\mu \right), \qquad (3)$$

the change of the chemical potential $\delta\mu$ is proportional to H^2 and can be neglected in the calculation of the linear susceptibility. For $T = 0$ and **p** on the Fermi surface, equation (3) reduces to,

$$\delta n_{p\sigma} = -\delta\varepsilon_{p\sigma}, \qquad (4)$$

from equation (2), it is seen that $\delta\varepsilon_{p\sigma}$ and $\delta n_{p\sigma}$ are independent of the direction of **p** and of opposite sign for spin up and spin down particles, Therefore, equation (2) becomes,

$$\delta\varepsilon_{p\sigma} = -\mu_B \sigma_z H + 2 f_0^a \delta n_\sigma \qquad (5)$$

where, $\delta n_\sigma = \sum_p \delta n_{p\sigma}$ (6)

δn_σ is the change in the total number of particles per unit volume of spin σ. Summing equation (5) over **p** and using equation (4), we get,

$$\delta n_\sigma = \frac{1}{2} N(0) \left(\mu_B \sigma_z H - 2 f_0^a \delta n_\sigma \right) \qquad (7)$$

The net spin polarization is given by,

$$\delta n_\uparrow - \delta n_\downarrow = 2\delta n_\sigma \sigma_z = \mu_B \frac{N(0) H}{1 + F_0^a} \qquad (8)$$

and the total magnetization is given by,

$$\mu_B \left(\delta n_\uparrow - \delta n_\downarrow \right) = \mu_B^2 \frac{N(0) H}{1 + F_0^a} \qquad (9)$$

The magnetic spin susceptibility of quasi-particles becomes,

$$\chi = \mu_0 \mu_B^2 \frac{N(0)}{1 + F_0^a} \qquad (10)$$

which is again the same result as for a free Fermi gas divided by a factor due to the interaction.

where, $$F^{a}\left(\theta\right)=-\frac{1}{2}U\left(2p_{F}\sin\frac{\theta}{2}\right) \quad (11)$$

The Landau parameter $F_0^a\left(\theta\right)$ is the dimensionless spin-antisymmetric Landau parameter, which characterize the effect of the interaction on the quasi-particle energy spectrum. The coefficient F_0^a has anti-symmetric in the spin index and spherical symmetric (L = 0). It is the Landau amplitude related to the exchange interaction, and for strongly correlated systems $F_0^a \geq -0.9$. The denominator is the change with respect to the non-interacting result. Usually, F_o^a is negative, and the magnetic susceptibility is enhanced. If F_o^a becomes positive, it means that it diverges [2, 11, 12, 13]. This corresponds to a ferromagnetic instability and the ground state without field changes qualitatively by creating a spontaneous magnetization. This is how the Fermi liquid description creates its own stability criteria.

Recall that the quasi-particle density is given as,

$$N\left(0\right)=\frac{m^{*}P_{F}}{\pi^{2}\hbar^{3}} \quad (12)$$

Also, the Fermi momentum of the quasi-particle at the Fermi level is given as,

$$P_{F}=\hbar k_{F}=\hbar\left(\frac{9\pi}{4}\right)^{\frac{1}{3}}\frac{1}{r_{s}} \quad (13)$$

By inserting equation (13) into equation (12) we obtained,

$$N\left(0\right)=\frac{m^{*}}{\pi^{2}\hbar^{2}}\left(\frac{9\pi}{4}\right)^{1/3}\frac{1}{r_{s}} \quad (14)$$

Then, inserting equation (14) into (10), the modified Landau Fermi liquid theory's expression for the magnetic spin susceptibility of quasi-particles in terms of the electron density parameter (r_s) was obtained as,

$$\chi=\frac{\mu_{0}\mu_{B}^{2}}{1+F_{0}^{a}}\frac{m^{*}}{\pi^{2}\hbar^{2}}\left(\frac{9\pi}{4}\right)^{\frac{1}{3}}\frac{1}{r_{s}} \quad (15)$$

3.0 Results and Discussion

3.1 Magnetic Spin susceptibility of quasi-particles.

Figure 1(a and b) show the variation between computed magnetic spin susceptibility of quasi-particles in metals and Landau Parameter F_0^a for some metals. Figure 2 (a and b) show the variation between Landau magnetic spin susceptibility of quasi-particles in metals and Landau Parameter F_0^a for some metals. The computed magnetic spin susceptibility and the Landau magnetic spin susceptibility of quasi-particles decreased as the Landau Parameter F_0^a increased for all the metals investigated. It is again observed from the figure for each metal, the Landau magnetic spin susceptibility of quasi-particles is higher than the calculated magnetic spin susceptibility of quasi-particles. The difference may be due to the value of electron density parameter which was used in the computation of the calculated magnetic spin susceptibility of quasi-particles but was not accounted for in the Landau magnetic spin susceptibility of quasi-particles. This may also suggest that magnetic spin susceptibility depends on other properties of metals, such as band structure energy, crystal binding and the nature of bonds between the quasi-particles in the metals [14, 15, 16]. Also, it was observed that the Landau magnetic spin susceptibility of quasi-particles in metals are not in good agreement with experimental values. The Landau Fermi liquid theory overestimated the magnetic spin susceptibility of quasi-particles. This seems to suggest that the Landau parameter must have been over estimated in its application. The computed magnetic spin susceptibility of quasi-particles is in good agreement with experimental values in all the ranges of the electron density parameter. This suggests that the modified version can effectively account and predict the magnetic spin susceptibility and magnetic properties of quasi-particles in metals.

From Tables 1 and 2, it was noticed that the magnetic spin susceptibility of quasi-particles for noble metals is higher than most of the magnetic spin susceptibility of quasi-particles for alkali metals. This is due to the d-block electrons that have filled electron shell which lies high up in the conduction band of noble metals. The magnetic spin susceptibility is considerably higher for most transition metals due to the incomplete d-orbital shells as more quasi-particles can be excited which enhances their susceptibility than the alkali metals [11, 17]. Also, transition metals have high value of conduction electron concentration. This shows that more quasi-particles could be formed when excited. The high values of the magnetic spin susceptibility of quasi-particles for transition metals could also be attributed to their electron density parameter that lies within the high density region $r_s \leq 3$.

4.0 Conclusion

The expression for the modified Landau Fermi Liquid Theory in terms of the electron density parameter was used to compute the magnetic spin susceptibility of quasi-particles in metals and the computed values are compared with Landau values and experimental values available. The computed magnetic spin susceptibility and the Landau magnetic spin susceptibility of quasi-particles show same trend with Landau Parameter F_0^a that is, as the Landau Parameter F_0^a increased for all the metals investigated, magnetic susceptibility of Quasi-particles decreased. Thus, the Landau magnetic spin susceptibility of quasi-particles in metals are not in good agreement with experimental values. The Landau Fermi liquid theory overestimated the magnetic spin susceptibility of quasi-particles while the computed magnetic spin susceptibility of Quasi-particles shows good agreement with available experimental values. This suggests that the modified Landau Fermi Liquid Theory can effectively account and predict the magnetic spin susceptibility and magnetic properties of quasi-particles in metals.

Table 1: Calculated Magnetic Spin Susceptibility of Quasi-particles in terms of the electron density parameter (r_s)

Metals	m^*	r_s (a.u)	Calculated values of Magnetic Spin Susceptibility quasi-particles at different Landau parameter(F_0^a) $10^6\chi$									Exp. Susceptibility $10^6\chi$
			-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9	
Li	2.3	3.28	0.19	0.21	0.24	0.29	0.34	0.43	0.57	0.86	1.71	2.0
Na	1.3	3.99	0.09	0.10	0.11	0.13	0.16	0.20	0.27	0.40	0.80	1.1
K	1.2	4.96	0.06	0.07	0.08	0.10	0.12	0.15	0.20	0.30	0.59	0.85
Rb	1.3	5.23	0.06	0.07	0.08	0.10	0.12	0.15	0.20	0.30	0.61	0.8
Cs	1.5	5.63	0.07	0.08	0.09	0.11	0.13	0.16	0.22	0.33	0.65	0.8
Cu	1.3	2.67	0.13	0.15	0.17	0.20	0.24	0.30	0.44	0.60	1.19	1.24
Ag	1.1	3.02	0.09	0.11	0.13	0.15	0.18	0.22	0.30	0.45	0.89	0.9
Au	1.1	3.01	0.09	0.11	0.13	0.15	0.18	0.22	0.30	0.45	0.89	1.4
Mg	1.3	2.66	0.13	0.15	0.17	0.20	0.24	0.30	0.40	0.60	1.19	1.2
Ca	1.8	3.27	0.15	0.17	0.19	0.22	0.27	0.34	0.45	0.67	1.35	2.1
Zn	0.85	2.30	0.10	0.11	0.13	0.15	0.18	0.23	0.30	0.45	0.90	0.38
Cd	0.74	2.59	0.07	0.08	0.10	0.12	0.14	0.17	0.23	0.35	0.70	0.39

Table 2: Landau Magnetic Spin Susceptibility of Quasi-particles

Metals	m^*	K_f ($10^{10}m^{-1}$)	Landau values of Magnetic Spin Susceptibility quasi-particles at different Landau parameter(F_0^a)									Exp. Susceptibility $10^6\chi$
			-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9	
Li	2.3	1.12	5197	5846	6681	7795	9354	11692	15590	23384	46769	2.0
Na	1.3	0.92	1692	1903	2175	2538	3045	3807	5075	7613	15226	1.1
K	1.2	0.75	1273	1432	1637	1910	2292	2864	3819	5729	11458	0.85
Rb	1.3	0.70	1287	1448	1655	1931	2317	2896	3862	5793	11585	0.80
Cs	1.5	0.65	1379	1552	1773	2069	2483	3103	4138	6206	12413	0.80
Cu	1.3	1.36	2501	2814	3215	3751	4502	5527	7503	11254	22508	1.24
Ag	1.1	1.20	1867	2101	2401	2801	3361	4201	5602	8402	16805	0.90
Au	1.1	1.21	1883	2118	2421	2824	3389	4236	5648	8472	16945	1.40
Mg	1.3	1.36	2501	2814	3215	3751	4502	5627	7503	11254	22508	1.2
Ca	1.8	1.11	2826	3180	3634	4239	5087	6359	8479	12718	25437	2.1
Zn	0.85	1.58	1900	2137	2443	2850	3420	4274	5699	8549	17098	0.38
Cd	0.74	1.40	1465	1649	1884	2198	2638	3297	4396	6595	13189	0.39

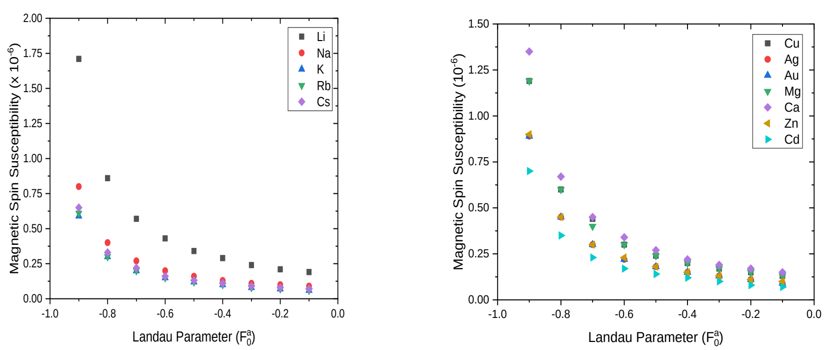

Figure 1: Variation of Calculated Magnetic Spin Susceptibility of quasi-particle of (a) Alkali Metals with Landau parameter (F_0^a) (b) Transition Metals with Landau parameter (F_0^a)

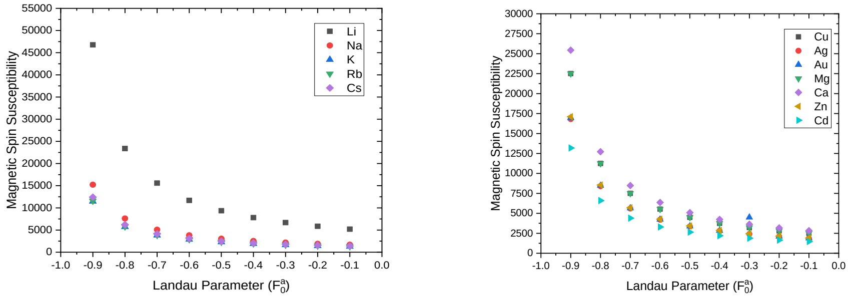

Figure 2: Variation of Landau Magnetic Spin Susceptibility of quasi-particle of (a) Alkali Metals with Landau parameter (F_0^a) (b) Transition Metals with Landau parameter (F_0^a)

5.0 Conflict of Interest

The authors declare that there are no conflicts of interest regarding the publication of this work.

6.0 References

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