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Articles

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Antibacterial Activity of the seed of *Dialium guineense* against Selected Enteric Bacteria

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Abstract: This study was aimed at evaluating the antibacterial activity and screening the phytochemical composition of the seed of *Dialium guineense*. The aqueous, methanolic and ethanolic extracts of the seed of *Dialium guineense* were analyzed against some clinical isolates. The phytochemical composition and antibacterial sensitivity testing were carried out using standard methods. The clinical isolates were *Escherichia coli*, *Proteus mirabilis*, *Klebsiella pneumoniae* and *Salmonella typhi*. Broad spectrum antibiotics were used as positive control for the antibiotic sensitivity test. Phytochemical screening of the extracts revealed the presence of saponin, tannins, alkaloid and glycoside while antimicrobial activity test at various concentrations for ethanolic and methanolic extracts showed significant results against the selected enteric bacteria with the exception of aqueous extracts which showed no antimicrobial activity against any of the isolates. The highest zone of inhibition (13.33mm) was obtained against *S. typhi* using methanolic extract while *E. coli* had a zone of inhibition of 10.67mm using ethanolic extract. *P. mirabilis* showed the lowest zone of inhibition

(3.67mm) using ethanolic extract. Ciprofloxacin showed the highest sensitivity to the test organisms while Gentamicin showed the lowest sensitivity. MIC results for the methanolic and ethanolic seed extract against the microbial isolates varied. However, a higher concentration above 225 mg/ml would be required for bactericidal activity, From these findings, the seed of *Dialium guineense* is a potential source of bioactive compounds and may have implications in the management of infectious diseases caused by some enteric bacteria.

Keywords*:* Dialium guineense, *phytochemicals, solvent extraction, enteric bacteria, antimicrobial activity.*

Introduction

Medicinal plants represent a rich source of antimicrobial agents. A wide range of medicinal plant parts is used for extract as raw drugs and they possess varied medicinal properties. Although, hundreds of plant species have been tested for antimicrobial properties, the vast majority of them have not been adequately evaluated, considering the vast potentiality of plants as sources of antimicrobial drugs with reference to antibacterial and antifungal agents. It's been reported globally about the antimicrobial properties of various medicinal plants and their use for medicinal purposes [1, 2, 3,]. Quite a number of plants have been used traditionally for medicinal purposes as they contain a variety of compounds of known therapeutic values[4, 5, 6].It is expedient that extracts from these medicinal plants showing antimicrobial activities at specific sites other than those used by synthetic drugs will be active against drug-resistant microbial pathogens. However, information the activity of such medicinal plants is inadequate [7]. Plants have ability to synthesize diverse aromatic secondary metabolites, and these groups of compounds show antimicrobial action and serve as plant defense mechanisms against pathogenic microorganisms [8, 9].*Dialium guineense* (Wild) belongs to the family of Fabaceae, commonly called black velvet or velvet tamarind. The fruit pulp is edible and sweet, with some quantities of tannins and ascorbic acid. It is quite a good source of protein and minerals as reported by Arogba *et al*. [10]. *D. guineense* is called "Awin" among the Yoruba speaking Nigerians. It is also known as "Icheku" among the Igbos in the Eastern part of Nigeria and as "Tsamiyarkurm" among the Hausa speaking Nigerians [9, 11]. Nwosu [11] reported that the bark and leaves of *D. guineense* have shown some medicinal properties and are used against several diseases. The usefulness of *D. guineense*as an anti-ulcer agent has been reported [12]. The antimicrobial activities of the fruit pulp extracts of *D. guineense* against some clinical isolates have been elucidated [13]. According to Bero *et al*. [14] the leaves and stem bark are used for the treatment of some infections such as diarrhoea, severe cough, bronchitis, stomach aches, malaria fever, jaundice and haemorrhoids.

Some researchers have authenticated activities of the leaves and stem bark of *D. guineense* which include its antibacterial and analgesic activities [15, 16,], as well as antioxidant properties [17]. Globally, infectious

diseases have been a main cause of death and diverse kinds of disability which accounts for about 23% of worldwide disease as opined by Murray and Lopez [18]. Lomovskaya and Bostian [19] have suggested that improvement of the efficacy of available antibiotics might be a reasonable and sustainable option due to the challenge between the slow development of new drugs and the fast emergence of resistant strains. This may raise some hope rather than making the future management of infectious diseases look bleak. Although, various works have been done to investigate the antimicrobial and phytochemical screening of *D. guineense* plant using its leaves, bark, and roots but fewer or no researches have been recorded in investigating the potential antimicrobial effects of the plant seeds.

This work is therefore aimed at screening the phytochemicals and conducting antimicrobial activity tests on the seed of *Dialium guineense.*

Materials and Methods

Plant Collection

The fruits of *Dialium guineense* were purchased at Ketu in Lagos State, Nigeria. The seeds were separated from the fruit and were identified at the herbarium section of the Department of Plant Biology, University of Ilorin, Nigeria.

Collection of Microbial Isolates

The bacteriall isolates were obtained from the Department of Microbiology and Parasitology, University of Ilorin Teaching Hospital, Ilorin, Kwara State, Nigeria. The pure isolates collected were as follows; *Salmonella typhi*, *Escherichia coli*, *Klebsiella pneumoniae* and *Proteus mirabilis*. The isolates were sub-cultured and the culture was maintained at 4 ^{o}C. However, further sub-culturing was done to keep the organisms viable.

Extraction of Plant Material

Forty grams of *D. guineense* powdered sample was soaked in 1000ml of sterile distilled water, 160 ml of 95 % ethanol and methanol respectively for 24 hours at room temperature on orbital shaker at 160 rpm. The content was filtered using muslin cloth and evaporated to dryness using water bath at 60 ^{o}C. The extracts were collected and stored at refrigerator temperature until when needed.

Antimicrobial Sensitivity Method

Antimicrobial activity study of the crude extracts of ethanol, methanol and aqueous extract of *Dialium guineense* seed was carried out on *Escherichia coli, Proteus mirabilis*, *Klebsiella pneumoniae* and *Salmonella typhi*. The agar well diffusion method of Collins *et al*. [20] with slight modification was adopted for this assay. The antibiotic discs of Ofloxacin, Gentamicin and Ciprofloxacin were placed aseptically on seeded plate of the isolates with the aid of a sterile pair of forceps.

Determination of Minimum Inhibitory Concentration (MIC)

The MIC of the plant extracts were determined by using the broth dilution method [20]. One ml of 24 hours culture of test organisms (10^{7}cfu/ml) adjusted to McFarland turbidity standard was incubated in test tubes with varying concentrations of 125, 150, 175, 200 and 225 mg/ml of plant extracts in normal saline at 37 ^{o}C for 24 hours. The concentration with the lowest dilution and no detectable bacterial growth was considered as the minimum inhibitory concentration (MIC) [21].

Determination of Minimum Bactericidal Concentration (MBC)

The minimum bactericidal concentration was determined by first selecting plates that showed no growth during MIC determination and incubated for another 24 hours at 37 ^{o}C. The minimum bactericidal concentration was considered as the lowest concentration that could not produce a single bacterial colony [20, 21].

Qualitative Phytochemical Screening

The method described by Adebayo and Sofowora [22] was used to test the presence of saponins, tannins, phenolics, alkaloids, steroids and glycoside.

Statistical Analysis

Data obtained were expressed as mean and standard deviation of triplicates and were statistically analysed using SPSS statistical package of version 16.0. The results obtained were statistically analysed by ANOVA. Values were considered significant at $p<0.05$

Results

Antibacterial Activity of the Extracts.

Ethanolic and methanolic extracts showed significant result against the selected enteric bacteria with the exception of aqueous extracts which showed no zones of inhibition on any of the isolates.

S. tyhpi showed the highest inhibition to the crude methanolic extract of *D. guineense* seed with 13.33mm zone of inhibition followed by *E.coli* with 11.33mm zone inhibition. *P. mirabilis* showed the lowest zone of inhibition to the crude methanolic extract with 3.67mm zone of inhibition. The methanolic extract however showed the highest zones of inhibition on the enteric bacteria. Also, the ethanolic crude extract showed a different range of inhibition on the isolates with *E. coli* being the most sensitive with 10.67mm diameter zone of inhibition followed by *S. typhi* with 10.00 mm diameter zone while *K. pneumoniae* showed the lowest zone of inhibition with 2.67mm (Figure 1).

Minimum Inhibitory Concentration

About five-fold dilutions was done from the crude methanolic and ethanolic extracts with the concentration of 225 mg/ml, 200 mg/ml, 175 mg/ml, 150 mg/ml and 125 mg/ml for each extract respectively. The MIC (minimum inhibitory concentration) of the extracts was also determined. All the organisms showed different MIC concentrations of each extract. *P. mirabilis* showed low MIC range at 150 mg/ml using methanolic extract and 200 mg/ml using ethanolic extract. However, the MIC of both the methanolic and ethanolic extract against *E. coli* and *S. typhi* was 225 mg/ml. The MIC value for *K. pneumoniae* was 200 and 225 mg/ml for ethanolic and methanolic seed extract respectively. Using ANOVA to analyse the data obtained, there was no significant difference at $P < 0.05$.

Minimum Bactericidal Concentration (MBC)

All the plates that showed MIC after 18-24hours of incubation were plated out for MBC. Minimum bactericidal concentration was evaluated in order to ascertain the bactericidal effect of the methanolic and ethanolic extracts on the enteric bacteria. Both methanolic and ethanolic extracts at the various concentrations used did not show any bactericidal activity. This

implies that a higher concentration well above the MIC would be required for bactericidal activity. For all concentrations plated out there was growth of organisms after 18-24hours of incubation.

Antibiotics Sensitivity Testing

Using some broad antibiotics as reference drugs, the sensitivity of the antibiotics, Ofloxacin (5 µg), Ciprofloxacin (30 µg) and Gentamicin (10 µg), were investigated on Muller-Hinton agar plates (MHA). Reading was taken after incubation for 18-24hours at 37 ^{o}C. Among all, Ciprofloxacin showed the highest zones of inhibition to isolate *K. pneumoniae*. Gentamicin showed the lowest range of inhibition zones to the organisms with zero effect i.e. no zones of inhibition on *E .coli* and *P. mirabilis* but about 4.0mm and 0.67mm on *K. pneumoniae* and *S. typhi* respectively. *K. pneumoniae* showed high sensitivity to Ofloxacin having a zone of 14.00mm. Zones of inhibition of 11.67mm and 11.00 mm was obtained for *E. coli* and *S. typhi* respectively. *P. mirabilis* expressed very low sensitivity with a zone of 4.33 mm (Figure 2).

Qualitative Phytochemical Screening of *D. guineense* extracts.

The ethanolic, methanolic and aqueous extracts of the seed of *D. guineense* showed positive test for all the screened phytochemicals except for steroid which showed a negative result for all extracts. Table 3 showed the phytochemical screening result for each extract.

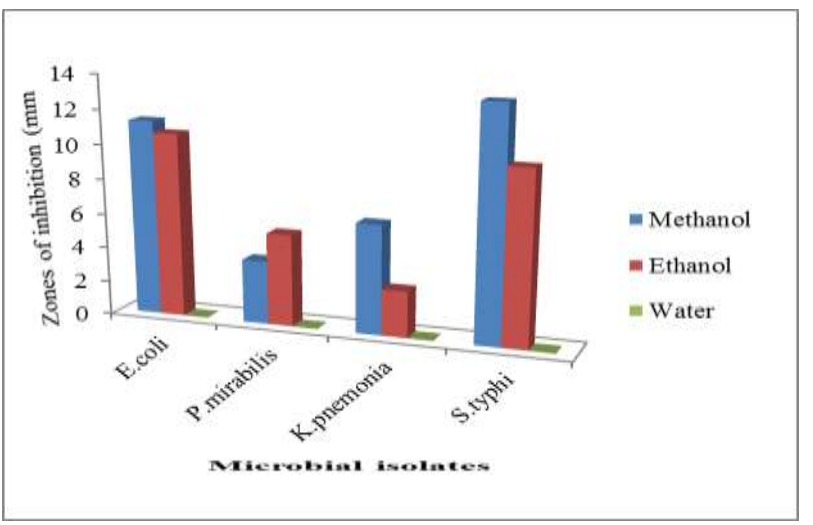

Figure 1: Zones of inhibition of the crude extracts of *D. guineense* on the bacterial isolates

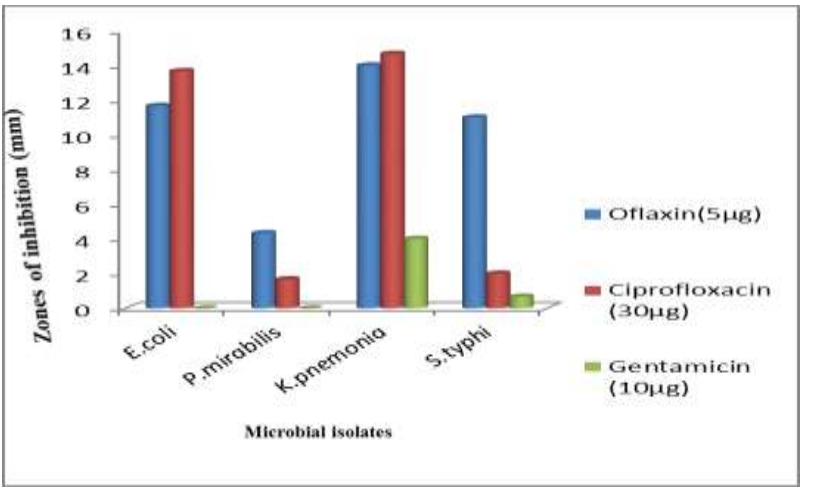

Figure 2: The effect of antibiotics on the bacterial Isolates.

Table 1: MIC and MBC of the ethanolic and methanolic seed extract of *D. guineense*

Enteric Bacteria	Ethanolic extract MIC	Conc. mg/ml MBC	Methanolic extract MIC	Conc. mg/ml MBC
Escherichia coli	225	-	225	-
Proteus mirabilis	200	-	150	-
Klebsiella pneumoniae	200	-	225	-
Salmonella typhi	225	-	225	-

Table 2: Qualitative phytochemical screening of the ethanolic, methanolic and aqueous seed extracts of *D. guineense*

Phytochemicals	Ethanolic extract	Methanol extract	Aqueous extract
Saponin	+++	+	++
Tannin	+++	+++	+
Alkaloid	++	++	+++
Steroid	-	-	-
Glycoside	+++	++	++

Key Notes:
+ → Positive, ++ → More positive, +++ → Most positive, - → Negative

Discussion

The seed of *Dialium guineense* had a significant effect on all test bacteria. However, the crude aqueous extract did not have any antimicrobial effect on enteric bacteria such as *Escherichia coli*, *Proteus mirabilis*, *Klebsiella pneumoniae* and *Salmonella typhi*. Orji *et al*. [16] explains that the active ingredient in the plant bark and leaf are more soluble in ethanol than water. This may be the reason why aqueous extract had no effect on the enteric isolates. However, the evaluated phytochemicals of the aqueous extract of the seed of *Dialium guineense* revealed the presence of saponins, tannins, alkaloids and glycosides, although, there is a relatively low presence of tannin and absence of steroids. This supports the trends of Gideon *et al*. [17] who said flavonoids, alkaloids, tannins and saponins are present in the leaf and bark of *D. guineense* of ethanolic and aqueous extract. Akinpelu *et al*. [9] also supported the fact that there are active components present in *D. guineense*. He reported that the phenolic compounds from medicinal herbs and dietary plants play important role in health; in addition to enhancing antimicrobial activities in these plants. Udoh *et al*. [23] reported that the sensitivity of aqueous extract of *Lasianthera africana* possess antimicrobial effect against *Salmonella typhi*, *Escherichia coli* and *Proteus vulgaris*.

However, the ethanolic and methanolic crude extracts showed a greater range of inhibition to the microbial isolates with *E. coli* and *S. typhi* showing the highest sensitivity and *K. pneumoniae* and *P. mirabilis* being the least sensitive. The sensitivity of these extracts has been revealed from the result of the phytochemical screening of ethanolic and methanolic extracts. Ethanol and methanol extracts showed a positive result for the components tested except for steroid which showed a negative result for both extracts and also for the aqueous extracts. Fat and lipids are not present in the seed of *Dialium guineense*. This could be the reason why steriod was negative for all extracts. This is in line with Arogba *et al*. [24], who said that the edible part (pulp) of ripe *D. guineense* fruit is sweet but acidic and relatively poor in protein and oil with fairly low levels of ascorbic acid and tannin. However, the seed had also been reported to be mildly acidic, poor in oil but fairly good source of protein and minerals. This could also be evident from this study as ethanolic, methanolic and aqueous extract showed a negative composition of steroid. The findings of this study do not agree with Orji *et al*. [16] who reported the antimicrobial properties of the crude aqueous leaf extracts of *D. guineense* against *S. aureus* and *K. pneumoniae* but showed the presence of flavonoids, alkaloids, tannin and saponin in the aqueous extracts. This is in conformity with the findings from this present study as the aqueous extract of *D. guineense* seed had these phytochemicals. Akinpelu *et al*. [9] also reported the bioactivity of the methanolic crude leaf extract of *D. guineense* on fourteen environmental strains of *Vibro* species. Their findings are in conformity with this study in line with the antimicrobial activities of the methanolic and ethanolic extracts of the seed of *D. guineense* against *Escherichia coli*, *Salmonella typhi*, *Proteus mirabilis* and *Klebsiella pneumoniae*. Orji *et al*. [16] also

reported the presence of alkaloids, flavonoids, tannins and saponins in the ethanol extract of the stem bark of *D. guineense*. The fruit pulp of *D. guineense* has been reported by Arogba *et al*. [10] as antiulcer and as a vitamin supplement.

The range of MICs in this study is between 150-225mg/ml for all the isolates. Also, Orji *et al*. [16] reported MIC values for the aqueous and ethanolic extract of leaf and stem bark of *Dialium guineense* for *S. aureus* and *K. pneumoniae* to be 200mg/ml. This variation could be as a result of the variety of strains of microbial isolates used, varying phytochemical components of plant parts and the extraction methods used as well.

The present study has revealed evidence of the sensitivity of the microbial isolates to some of the broad-spectrum antibiotics while some were resistant. *E. coli* and *P. mirabilis* showed resistance to gentamicin while *K. pneumoniae* and *S. typhi* were susceptible. However, the isolates were susceptible to ofloxacin and ciprofloxacin, but *P. mirabilis* and *S. typhi* showed low zones of inhibition to ciprofloxacin. Ofloxacin is a broad-spectrum antibiotic of the class quinolones and due to its mode of action on the bacterial isolates, it was able to inhibit their growth.

In conclusion, the result of this study has revealed that the methanolic and ethanolic seed extract of *D. guineense* has antimicrobial activity against some microbial isolates which are enteric Also the seed extracts were shown to possess significant quantities of phytochemicals such as saponin, tannin, alkaloid, steroid and glycosides. This study also agrees with the findings of Ajiboye *et al*. [13] who also reported antimicrobial activity of the fruit pulp of *D. guineense* on some microbial isolates. Therefore, the seed of *D. guineense* can be employed as a potential antimicrobial agent. However, further research should be tailored towards investigating the different parts of *D. guineense* such as leaf, root, bark, stem and fruit pulp in relation to their phytochemical components and their effects on clinical isolates.

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Antibacterial activity of *Jatropha curcas* against Isolates of Clinical Origin

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***Abstract*:** The antibacterial activities of ethanol, methanol and aqueous extracts of *Jatropha curcas* leaves were observed in vitro against *Pseudomonas aeruginosa, Salmonella typhi, Escherichia coli, Staphylococcus aureus* and *Klebsiella pneumoniae* was analyzed using agar well diffusion method. The pattern of zone of inhibition varied with different plant extract, the solvent used, and the organism tested. The antibacterial activities of the methanolic extract were significantly higher ($p<0.05$) and the most susceptible organism was *Pseudomonas aeruginosa* at the lowest concentration. The Minimum Inhibitory Concentration (MIC) exhibited by *Pseudomonasaeruginosa* in the ethanolic extract and aqueous extract was 5mg/ml and 75mg/ml respectively. *Jatropha curcas* proved to be effective over the use of antibiotics by inhibiting the activity of *Pseudomonas aeruginosa* which was resistant when tested with standard antibiotics. The antibacterial activity of the extract could be enhanced if the components are purified. This plant therefore holds a promise as a potential source of new drug for treating infections caused by these clinical pathogens.

Keywords: Antibacterial, *Jatropha curcas*, clinical pathogens

Introduction

The antimicrobial characteristics of medicinal plants are due to compounds like alkaloids, flavonoids, phenolic compounds, tannins, resins, gum, fatty acids, saponins and steroids. Many of the plant materials used in traditional medicine are generally proved more

effective andrelatively cheaper than modern medicine [1]. For ages, thousands of species of medicinal plants used globally have contributed many ingredients to help fight diseases and illnesses. Over 80% of the world's population particularly in developing world relies on medicinal plants as sources of medicine for their primary health care [2]. Phytochemicals have been attracting much interest as natural alternatives to synthetic compounds because the antimicrobial properties are of great importance in curative treatments. *Jatropha curcas* is becoming a very useful economic resource both in agriculture, phytomedicine development and development of new lead compounds [3, 4].

In spite of various researches as regards the antimicrobial activity of plant extracts, little has been developed in comparism to modern medicine. The pharmacological activities of medicinal plants provide clues to synthesize less expensive antimicrobial chemicals that are relatively safe to man and limit the supply of synthetic chemicals. These plants are used as antimicrobial agents and several works have been carried out by scientist to find out a scientific basis [5] and some of these plants include *Jatropha curcas, Moringa oleifera*, *Senna occidentalis*. Anti-microbial agents are widely employed to cure bacterial diseases [6]. Current social trends in health care show a definite movement towards the use of natural remedies like medicinal plants and away from chemotherapeutic regimens [7].

Medicinal plants like *Jatropha curcas* have played a major role in the treatment of various diseases including bacterial and fungal infections. The extracts of many *Jatropha* speciesdisplayed potent cytotoxic, anti-tumor and anti-microbial activities in different assays. The latex of *J. curcas* also showed anti-bacterial activity against *Staphylococcus aureus* [8], however the antimicrobial activity of the other parts has not been fully investigated. The aim of this study is therefore to investigate the effectiveness of *Jatropha curcas* against some selected microorganisms of clinical origin which are established to cause infections.

Materials and methods

Collection and Identification of plant Sample

Jatropha curcas leaves were collected from Farm, Sango, in Ilorin west Local Government area of Kwara State, Nigeria. It was identified properly and authenticated at the Botany unit of the Department of Plant Biology, University of Ilorin. The leaves were washed with distilled water and air dried. The dried leaves were grinded into powder and stored in a tightly covered container.

Collection of test organisms

Pure culture of *Pseudomonas aeruginosa, Salmonella typhi, Escherichia coli*, *Staphylococcus aureus* and *Klebsiella pneumoniae* were obtained from the Medical Laboratory of the Microbiology and Parasitology unit of the University of Ilorin Teaching Hospital and used as test organisms.

Preparation of plant extracts

The fresh leaf sample was sundried and ground into fine powder and kept in a plastic container until use

Extraction

Aqueous and ethanolic extraction of the plant material was prepared as

described by [9]. The aqueous and ethanolic extracts of the plant material was carried out by suspending 25g of the finely ground leaf in 125ml of 95% ethanol and 250mls of distilled water respectively. A preliminary test has shown that the extract shared greater activity at 80^0C than at 28^0C, so the aqueous extractionwas done at 80^0C in a water bath for 11/2 hours. The ethanolic extraction was done at 28+ 1^0C for 120 hours by subjecting it to agitation on rotator shaker at 200 rpm. The resulting aqueous extract suspension were filtered with Whatman filter paper and evaporated to dryness at 45^0C in an oven.

Antimicrobial susceptibility test of some standardized antibiotics

The antibiotics used for this study had been prepared into multiple discs kit containing different antibiotics. The plate diffusion technique was used for the antibiotic sensitivity test. Broth cultures of the organisms were swabbed on sterile Mueller Hinton agar plates. The multiple antibiotic discs were then placed on the agar surface after solidification and pressed using sterile forceps to ensure complete contact with agar. The plates were incubated at 37^oC for 24hrs and after incubation the diameter of the inhibition zones were measured and recorded [7]. The antibiotics used and their corresponding concentrations are as follows: Gentamycin (10 µg), Tetracycline (30 µg), Erythromycin (5 µg), Ceftriaxone (30 µg), Cloxacillin (5 µg), Nitrofurantoin (300 µg), Cotrimoxazole (25 µg), Augmentin (30 µg), Ofloxacin (5 µg), Amoxicillin (25 µg), Nalidixic acid (30 µg), Cefuroxime (30 µg) and Ceftazidime (30 µg).

Phytochemical screening of extracts* of *Jatropha curcas

The phytochemical screening of the extracts was done on the aqueous extracts and the powdered specimens using standard procedure as described by [8] and [10]. The following qualitative tests were carried out as follows:

Test for tannins

About 0.5 gram of dried powdered samples was boiled in 20mL of water and filtered. A few drops of 0.1% ferric chloride was added and observed for brownish green or blue black colouration.

Test for phlobatannins

The aqueous extract of each sample was boiled with 1% aqueous hydrochloric acid. Observation of deposits of a red precipitate was taken to indicate the presence of phlobatannins.

Test for saponins

Two grams of each powdered sample was boiled in a water bath and filtered. The boiled samples were one milliliter of filtrate was mixed vigorously to form a stable persistent froth. A formation of emulsion was observed after mixing the froth with about 3 drops of olive oil to indicate the presence of saponins.

Test for flavonoids

The method described by [10]was used to determine the presence of flaxonoids. Five milliliters of diluted ammonia solution were added to a portion of the aqueous filtrate of each plant extract followed by the addition of concentrated H_2SO_4. A yellow coloration which disappeared after some minutes indicates the presence of flavonoids.

Test for steroids

Two milliliters of acetic anhydride was added to 0.5g ethanol extract of each sample with 2mL of H_2SO_4. A colour change of violet to green is indicative of the presence of steroids.

Test for terpenoids

The Salkowski test was used to test for the presence of terpenoids. Five milliliter of each aqueous extract was mixed with 2mL of chloroform and 3mL of concentrate H_2SO_4. A reddish brown colouration of the interface was used to indicate the presence of terpenoids.

Test for cardiac glycosides

Five milliliter of each aqueous extract was treated with 2mL of glacial ascetic acid which contains one drop of ferric chloride solution. One milliliter of H_2SO_4 was later added. The formation of a brown ring indicates the presence of cardenoids.

Quantitative Determinations

To determine the quantity of each of the constituents, 2g of each powdered sample was first defatted with 100mL of diethyl ether for 2h using a soxhlet apparatus.

Determination of Minimum Inhibitory Concentration (MIC) of the Extract:

The broth dilution method was used to determine MIC. Varying concentrations of the extracts were used which, ranged from 5mg/ml – 200mg/ml each concentration contain 0.1ml was added to each 9ml of nutrient broth containing 0.1ml of standardized test organism of bacterial cells. The tubes were incubated aerobically for 24hours at 37°C. Controls were equally set up by using solvent and test organisms without the extract.

Effect of varying concentrations

Varying concentrations of the extracts were prepared in sterile test tubes. Six different concentrations were prepared i.e. 150mg/ml, 100mglml, 75mg/ml, 50mg/ml, 25mg/ml and 5mg/ml. To prepare these concentrations, dilution factor was first determined, and the concentrations were obtained by diluting each of the extract with the corresponding solvent in a known fraction. The effect of these concentrations on the test organisms was checked for by inoculating the test organisms on the appropriate media. Six equidistance holes were bore on the inoculated plates using a sterile cork-borer. Then each of the concentrations was dispensed making use of a sterile syringe, into the hole (well labeled). The plates were left on the work bench for 10-15 minutes to allow proper diffusion. The plates were then incubated for 24 hours at 37˚C. After incubation, concentrations that have zone of clearance around it was observed and recorded in 'mm'.

Antimicrobial assay of extracts of *Jatropha curcas* using agar well diffusion method

Muller Hilton agar (MHA) was used in carrying out the assay by appropriately inoculating the test organisms which were already pre-adjusted to the 0.5 McFarland's turbidity standard in peptone water. The inoculums were spread all over the surface of the media. Agar well diffusion method was employed. This was done by boring three equidistant holes on the media with the use of sterile cork borer. Appropriate quantity of the three different extracts was dispensed into the holes. The plates were left for 10-15 minutes for diffusion incubated at 37˚C for 24 hours. After incubation,

the diameter of zones of inhibition were measured using a meter rule and was recorded in standard unit.

Results

In Table 1, some of the bacteria isolates were sensitive to one or two of the antibiotics used. The isolates used were basically multi-drug resistant. The Gram-positive organism was sensitive only to Ofloxacin but it was resistant to Amoxicillin, Cotrimoxazole, Nitrofurantoin, Nalidixic acid, Augmentin, Tetracycline and Gentamycin.The Gram negative bacteria were sensitive to Gentamycin, Ofloxacin, and Erythromycin and were resistant to Ceftazidime, Cefuroxime, Gentamycin, Ceftriaxone, Erythromycin, Cloxacillin and Augmentin.

Table 2 shows positivity of the phytochemical screening carried out on the ethanolic plant extracts. As shown in Table 3, the MIC values of methanolic extract against *S. aureus* and *S. typhi* were 75mg/ml and 150mg/ml respectively. The MIC values of ethanolic and aqueous extracts obtained against *P. aeruginosa*were5mg/ml and 75mg/ml respectively. *Escherichia coli* was susceptible to all the extracts. *S. typhi* showed sensitivity to two of the extracts and resistant to one. *S. aureus* was sensitive to ethanolic extract but resistant to methanolic and aqueous extract. *K. pneumoniae* and *P. aeruginosa* were resistance to all the extracts (Figure 1). Figure 3 shows the zone of inhibition of the extracts of *Jatropha curcas* on the isolates at different concentrations. *S. typhi* and *S. aureus* showed sensitivity to varying concentrations of methanolic extract while *E. coli, K. pneumoniae and P. aeruginosa* were resistant. For both ethanolic and aqueous extracts, *P.aeruginosa*was the most susceptible organism with sensitivity at all concentrations. The other organisms showed no zone of inhibition, hence, were resistant

Table 1: Antibiogram profile of the test organisms used against standard antibiotics

Organisms	AMX	AUG	GEN	COT	OFL	NAL	TET	NIT	CFX	CAZ	CTR	ERY	CXC
S. aureus	R	R	R	R	S	R	R	R	-	-	-	-	-
S.typhi	-	-	S	-	S	-	-	-	R	R	R	S	R
P.aeruginosa	-	-	R	-	R	-	-	-	R	R	R	R	R
E.coli	-	-	S	-	S	-	-	-	R	R	R	S	R
K.pneumoniae	-	-	R	-	S	-	-	-	R	R	R	R	R

Key: R indicates resistance S indicates susceptibility
GEN- Gentamycin,TET- Tetracycline, ERY- Erythromycin, CTR- Ceftriaxone,CXC- Cloxacillin,NIT-Nitrofurantoin, COT- Cotrimoxazole,AUG- Augmentin, OFL- Ofloxacin, AMX-Amoxicillin, NAL- Nalidixic acid, CFX- Cefuroxime, CAZ- Ceftazidime

Table 2: Phytochemical screening of ethanolic extract of *Jatropha curcas*

PHYTOCHEMICALS	RESULTS
Tannins	Present
Saponins	Present
Flavonoids	Present
Anthraquinones	Present
Terpenoids	Present
Alkaloids	Present

Key: Present indicates presence of the phytochemicals in the extract

Table 3: Minimum Inhibitory Concentration (MIC) of the extracts against the isolates

Isolates	Varying concentrations of extracts (mg/ml)						MIC
	150	100	75	50	25	5	
Ethanolic extract							
P. aeruginosa	-	-	-	-	-	-	5
Methanolic extract							
S. aureus	-	-	-	+	+	+	75
S. typhi	-	+	+	-	+	+	150
Aqueous extract							
P. aeruginosa	-	-	-	+	+	+	75

Key - Not turbid
+ Turbid

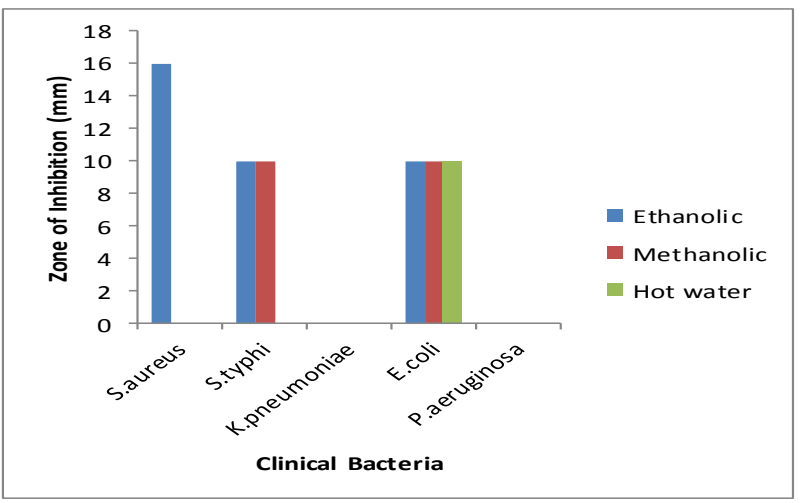
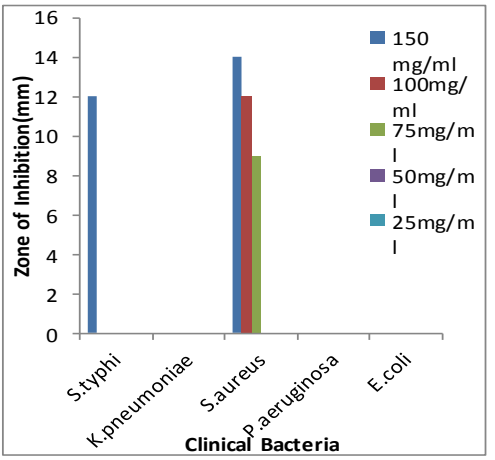
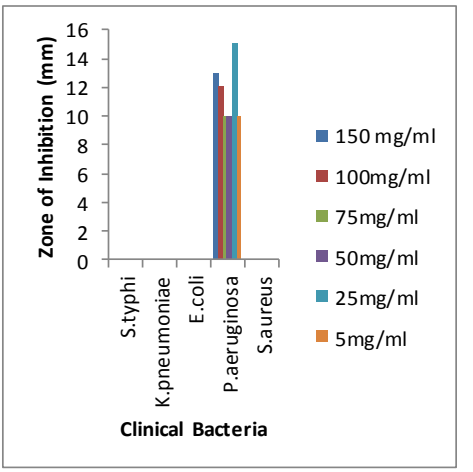


Figure 1: Susceptibility of the clinical isolates to the extracts of *Jatropha curcas*



a.



b.

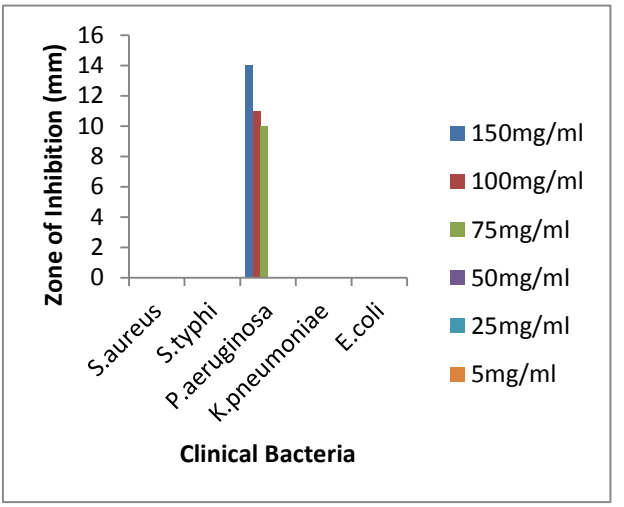


Figure 2: Susceptibility of the clinical isolates to concentrations of extracts *Jatropha curcas* a) Methanolic extract b) Ethanolic extract c) Aqueous extract

Discussion

Antibiogram profiling of the test organisms showed the organisms used were multi-resistant. The Gram-positive bacterium (*Staphylococcus aureus*) was sensitive only to Ofloxacin but it was resistant to Amoxicillin, Cotrimoxazole, Nitrofurantoin, Nalidixic acid, Augmentin, Tetracycline and Gentamycin. This is an indication of broad spectrum antibiotic compounds.This may be attributed to the fact that the standard antibiotics as a conventional antibiotic, is a refined and purified product, while extracts of herbal medicines are a mixture of various plant constituents some of which can interfere with antimicrobial activity and are subjected to

degradation and decomposition on storage [11].The Gram negative bacteria (*S. typhi, E.coli, K. pneumoniae* and *P. aeruginosa*) were sensitive to Gentamycin, Ofloxacin, and Erythromycin and were resistant to Ceftazidime, Cefuroxime, Gentamycin, Ceftriaxone, Erythromycin, Cloxacillin and Augmentin. Positivity of the phytochemical screening carried out on the ethanolic plant extracts tend to agree with the report of El-Mahmood and Doughari [12] that linked the antimicrobial properties of the plant to the presence of the bio-active secondary metabolites like alkaloids, tannins, saponins, flavonoids, phenols, and glycosides. This finding agrees with previous works of El Di-wani *et al*. [13] who reported the presence of saponins in *Jatropha curcas* leaf. Although absence of alkaloids in Jatropha curcas leaf extracts had also been reported by Kubmarawa *et al*. [14] although Igbinosa *et al*. [15] and Akinpelu *et al*. [16] observed the presence of alkaloids in *J. curcas* stem bark and leaves extracts respectively. These compounds have been associated with medicinal uses for centuries and were reported as the most efficient, therapeutically significant plant substance [17, 18] and exert antibacterial activity through different mechanisms [19, 20].

The Minimum Inhibitory Concentration was carried out only on the organisms that showed zone of inhibitions for ethanolic, methanolic and aqueous extracts respectively at different concentrations. *Pseudomonas aeruginosa* exhibited susceptibility to lesser concentrations of ethanolic extract (5mg/ml for MIC value). *P. aeruginosa* exhibited susceptibility to lower concentrations of aqueous extract (75mg/ml for MIC). *Staphylococcus aureus* and *Salmonella typhi* exhibited susceptibility to lesser concentrations of the extract (75mg/ml for *Staphylococcus aureus* and 150mg/ml for *Salmonella typhi*). There was no MBC value since there was no growth after extracts of *Jatropha curcas* leaves were plated and incubated. This indicates that there was no cidal effect.

Escherichia coli showed susceptibility to all the extracts while *Klebsiella pneumoniae* and *Pseudomonas aeruginosa* were resistant. *Pseudomonas aeruginosa* was the only organism that showed sensitivity at varying concentrations to ethanolic extract and aqueous extract respectively, the other organisms were resistant. *Salmonella typhi* and *Staphylococcus aureus* were sensitive at different concentrations of methanolic extract. E.*coli, K. pneumoniae and P. aeruginosa* were resistant. High MIC values are indication of low activity while low MIC values are indication of high activity because it is the minimum or the smallest concentration capable of inhibiting the growth of organism. In *Salmonella typhi* and *Staphylococcus aureus* the MIC was obtained as 150mg/ml and 75mg/ml respectively indicating that lower concentrations other than this concentration cannot exert any antimicrobial effect on the organism but can still exhibit at higher concentration as seen in the stock concentration (200mg/ml).

The ability of all the extracts to extricate the phytochemicals and exert antimicrobial activities on one or more of the test organisms shows that they are good solvent for extraction but

methanolic extract extricating more phytochemicals than ethanolic and aqueous extract. This is due to the fact that methanol is a solvent that dissolves all type of compounds either polar, semi polar and non-polar. This gives it the ability to extract the antimicrobial agent from the leaves more than the other solvents.

Conclusion

This study has revealed ofloxacin, gentamycin and erythromycin to be effective against the clinical isolates used. It has further confirmed that the plant extracts could be used for the treatment of various infections and may serve as a good source of novel bioactive compounds and also the presence of many secondary metabolites in the leaves of *Jatropha curcas.*

Public enlightenment programme to educate people on the hazards of misuse of antibiotics will be beneficial as these antibiotics are a group of drugs that can easily be possessed by many people. Individuals on drug prescription should be educated on the need to follow doses strictly and Government should pass a policy against the unnecessary use and handling of antibiotics.

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Modeling Tuberculosis (TB) Using Higher Order Markov Model

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Abstract: This paper focused on the higher-Order Markov Model whose number of states and parameters are linear with respect to the order of the model and as well as classifying it. Model for Efficient estimation methods of the parameters was developed and the model was applied to solve the application of DOTS in the treatment of tuberculosis health problem. Numerical examples with applications are given to illustrate the power of our proposed model. It was discovered that the second order Markov model was best fit base on the values of the AIC and BIC result obtained.

Keywords: Directly Observed Treatment (DOT), transition matrix, AIC, BIC, optimization, Higher Order Markov Model

1.0 Introduction

Research has shown that Tuberculosis (TB) remains a major global health problem. It causes ill health among millions of people and it is ranked as the second leading cause of death from an infectious disease worldwide after human immunodeficiency virus (HIV) [1]. Tuberculosis (TB) is an infectious disease caused by *Mycobacterium tuberculosis* and can affect different body organs [2, 3]. TB can result from a rapidly progressive disease following recent infection with Mycobacterium tuberculosis or from reactivation of a past latent TB infection. TB is largely transmitted by aerosols produced from coughing in individuals with active pulmonary disease. TB surveillance and preventing further spread of the disease requires full understanding of the biological factors affecting TB, and also finding mathematical patterns explaining the mechanism of TB transmission through the community [4]. The objectives of this study is to apply the use of Directly Observed Treatment short- course (DOTS) to monitor and control the epidemic, the probability of being in a given state at

a given point in time, the expected number of transitions between states and finally find the best model for DOTS applications. This strategy was expected to bring about a major change in controlling the disease. Directly Observed Treatment short-course (DOTS) has been found to be an effective means of administering anti-TB drugs, significantly reducing the rates of relapse and drug resistance as well as improving the treatment compliance rate [5, 6].

Many statistical models have been explored to control the TB epidemic. Markov processes are not suitable for modeling all disease types and answering all disease related questions due to the complexity that is involved in the modeling of some diseases [7]. Evaluation of treatment outcome is central to the assessment of effectiveness of tuberculosis [8]. Treatment outcomes for TB patients (excluding patients treated for RR-TB or MDR-TB, that is multi-drug resistance to tuberculosis drugs) are classified as successful (cure or treatment completed) or poor (default, treatment failure or death) as defined by the [9].

Statistical methods like regression techniques, time series analysis, statistical process control and Bayesian methods have been used to monitor the epidemiologic surveillance of infectious diseases [10]. [11] introduces a warning threshold for detecting the unexpected incidences of Tuberculosis (TB) using a Hidden Markov Model (HMM) and it was concluded that the warning threshold constructed based on the Periodic Autoregressive Model can be regarded as a useful alternative for HMM in detection of the weeks with unexpected incidence of TB, therefore it was suggested for monitoring TB surveillance. Treatment outcomes of patients are classified as successful (cure or treatment completed) or poor (default, treatment failure or death) as defined by [12]. Markov chain concerns about a sequence of random variables, which correspond to the states of a certain system, in such a way that the state at one time epoch depends only on the one in the previous time epoch. Higher order markov model is used to model treatment outcome of tuberculosis as DOTS application.

Higher Order Markov Model (HOMM) has been used in analysis and prediction of time series demonstrating the effectiveness of Markov chain model and it has been applied to price and sales volume for beef prediction problem by Tie Liu. Markov chains have been used to model categorical data sequences and this can be found in [13] and [14]. Markov chain model of order higher than one that involves only one parameter for each extra log variable was suggested by [15]. This was extended to qth order marginalized transition model (MTM) by [16] and [17].

[18] generalized the [15] model by allowing $Q = \{q_{ij}\}$ to vary with different lag and then developed effective method for parameter estimation. Higher-order Markov Model, each data point X(n) in a categorical data sequence takes value in the set $M \equiv \{1, 2, \ldots, m\}$ and m is finite i.e. a sequence has m possible categories or states. The total number of independent parameter to be estimated in kth order Markov chain is $m^k (m-1)$.The number of independent

parameters increases geometrically as the order increases, thereby discourages people from using higher order Markov chain. See Table 1

Raftery proposed a higher order Markov chain model which involves one additional parameter for each lag. The model can be written as

$$P\left(X^{(n)} = j_0 \mid X^{(n-1)} = j_1, \dots\dots, X^{(n-k)} = j_k\right) = \sum_{i=1}^{k} \lambda_i q_{ij} \quad 1.3$$

Where,

$$\sum_{i=1}^{k} \lambda_i = 1 \quad 1.4$$

And $Q = [q_{ij}]$ is transition matrix with column sum equals one, such that

$$0 \le \sum_{i=1}^{k} \lambda_i q_{j_0 j_1} \le 1, \qquad j_0 j_i \in \mathrm{M} \quad 1.5$$

A more general higher-order Markov chain model is obtained by allowing Q to vary with different lags. Here we assume that the weight λ_i is non-negative. It should be noted that it can be written as

$$X^{(n+k+1)} = \lambda_i Q X^{(n+k+1-i)} \quad 1.6$$

Where $QX^{(n+k+1-i)}$ is the probability distribution of the states at time$(n+k+1-i)$. Using (1.4) and Q is a transition probability matrix, we consider each entry of $X^{(n+k+1)}$ which is between 0 and 1 and also parameter λ_i is non-negative. The additional constraint should be added to guarantee that $X^{(n+k+1)}$ is the probability distribution of the states.

Raftery's model can be generalized as follows:

$$X^{(n+k+1)} = \lambda_i Q_i X^{(n+k+1-i)}$$

The total number of independent parameters in the new model is $k + km^2$. We note that if $Q_1 = Q2 = \cdots = Q_K$, then the above equation is just the Raftery's model.

In the model we assume that $X^{(n+k+1)}$ depends on $X^{(n+1)}, (i = 1,2,\dots,k)$ via the matrix Q_i and weight λ_i. One may relate Q_i to the i-step transition matrix of the process and this can be used to estimate Q_i. It is assumed that each Q_i is a non-negative stochastic matrix with column sums equal to one.

2.1 Estimation of the model Parameters

Given an observed data sequence $\{X^n\}$, where $\{X_{(n)}\}$ can be written in a vector form as:

$$\{X^{(1)}, X^{(2)}, X^{(3)}, \dots\dots\dots\dots\dots, X^T\}$$

Where T is the length of the sequence and $X_I \in \mathrm{DOM}(\mathrm{A})$. $\mathrm{DOM}(\mathrm{A})$ is categorical data if it finite and unordered.

One can find the transition frequency F_{ij} in the sequence by counting the number of transitions from state *i* to state *j* in one step. Also one can construct the one step transition matrix or the sequence $\{X_{(n)}\}$ as follows:

$$\mathrm{F} = \begin{pmatrix} F_{11} & \dots & F_1 m \\ \vdots & \vdots & \vdots \\ Fm_1 & \dots & Fmm \end{pmatrix} \quad 2.1$$

From F, one can get the estimates for P_{ij} as follows:

$$\mathrm{P} = \begin{pmatrix} P11 & \cdots & P1m \\ \vdots & \vdots & \vdots \\ Pm1 & \cdots & Pmm \end{pmatrix} \quad 2.2$$

Where,

$$p(x) = \begin{cases} \frac{F_{ij}}{\sum_1^m F_{ij}}, & if \sum_1^m F_{ij} > 0 \\ 0, & if \sum_1^m F_{ij} = 0 \end{cases}$$

And the estimators in p(x) satisfies

$$E(q_{ij}^k = q_{ij}^k E(f_{ij}^k)$$

The following proposition by [19], helps in estimating the parameters in HOMM.

Proposition 1. The matrix P has an eigenvalue equal to one and all the eigenvalues must have modulus less than or equal to one.

Proposition 2. (Perron-Frobenius theorem). Let A be a non-negative and irreducible square matrix of order m. Then

1. A has positive real eigenvalue, λ, equal to its spectral radius i.e. $\lambda = max|\lambda_k(A)|$, where $\lambda_k(A)$ denotes k^{th} eigenvalue of A
2. To λ there corresponds an eigenvector **x** its entries being real and positive, such that $\boldsymbol{Ax} = \lambda \boldsymbol{x}$
3. λ is a simple eigenvalue of A

using these two propositions, one can see that there exists a positive vector $\boldsymbol{x} = [x_1, x_2, \dots\dots\dots\dots\dots\dots\dots\dots x_m]^T$ such that $P\boldsymbol{x} = \boldsymbol{x}$ if P is irreducible. The vector **x** in normalized form is called the stationary probability of vector P. Therefore, x_i is the probability that the system is in state *i*. As a result of the proposition (1) and (2) above $P\boldsymbol{x} = \boldsymbol{x}$ and $Q = \sum_{i=0}^{n} \lambda_i P_i$, then the proposition (1) gives a sufficient condition for sequence $\{X^n\}$ to converge to stationary distribution X. As $X^n \to \bar{X}$ as n goes to infinity, then $\bar{X}$ can be estimated from sequence $\{X^{(n)}\}$. Therefore, the proportionality of occurrence of each state can then be denoted by $\hat{X}$ i.e.

$$\sum_i^k \lambda_i Q_i \hat{X} \approx \hat{X}$$

And this is one of ways to estimate parameter λ.

$\lambda = (\lambda_1, \lambda_2, \dots\dots\dots, \lambda_k)$.

One of the ways to estimate the parameter λ is to consider the minimization problem through certain vector norm, $\| \cdot \|$, using $\| \cdot \|_1$ which leads linear programming equation, $\| \cdot \|_2$ which leads to quadratic programming problem and $\| \cdot \|_\infty$ which avoids gross discrepancies with data as much as possible.

Definition of Matrix-Norm

From Wolfram Mathworld,

Let A be a square or real matrix, a matrix norm $\|A\|$ is a non-negative number associated with A with the following properties:

1. $\|A\| > 0 \; when \; A \neq 0 \; and \; \|A\| = 0 \; iff \; A = 0$
2. $\|kA\| = |k|\|A\| \; for \; any \; scalar \; k$
3. $\|A + B\| \leq \|A\| + \|B\|$
4. $\|AB\| \leq \|A\| \; \|B\|$

Let $\lambda_1, \lambda_2, \dots . \lambda_n$ be the eigenvalues of A, then

$$\frac{1}{\|A^{-1}\|} \leq |\lambda| \leq \|A\|$$

The matrix *p*-norm is defined for a real number $1 \leq p \leq \infty$ and a matrix A by

$$\|A\|_p = \max_{X \; s.t. \; |X|_p = 1} |Ax|_p$$

where $|x|_p$ is a vector norm. It is tasking to compute *p*-norm for $p > 1$ because it is a non-linear optimization problem with constraints so mathematical software are used.

The maximum absolute column sum norm $\|A\|_1$ is defined as

$$\|A\|_1 = \max_j \sum_{i=1}^{n} |a_{ij}|.$$

The spectral-norm, $\|A\|_2$, which is the square root of the maximum eigenvalue of $A^H A$, where A^H is the conjugate transpose of A. therefore

$\|A\|_2 = (maximum\ eigenvalue\ of A^H A)^{1/2}$ and this is always referred to as matrix norm.

The maximum absolute row sum norm is defined by

$$\|A\|_\infty = \max_i \sum_{j=1}^{n} |a_{ij}|$$

$\|A\|_1, \|A\|_2, \|A\|_\infty$ Satisfies the inequality $\|A\|_2^2 \leq \|A\|_1 \|A\|_\infty$ [20]

Let us consider $\|\,.\,\|_1$ which leads to solve linear programming problem

$$\min_\lambda \sum_i^k \left\| \lambda_i Q_i \hat{X} - \hat{X} \right\|$$

s.t =

$$\begin{cases} \sum_i^k \lambda_i = 1 \\ \lambda \geq 0 \quad i = 1, \ldots, n \qquad \forall i \end{cases}$$

And considering $\|\,.\,\|_\infty$ norm, will lead to

$\min_\lambda \max_i |[\sum_i^k \lambda \text{ iQi}\hat{X} - \widehat{X}]|_i$

s.t =

$$\begin{cases} \sum_i^k \lambda_i = 1 \\ \lambda \geq 0 \quad \text{i} = 1, \ldots, \text{n} \qquad \forall_i \end{cases}$$

where $[.]_i$ denotes the *ith* entry of the vector and the optimization problem leads existence of stationary distribution X, while minimization problem can be formulated as a linear programming problem as follows:

$\min_\lambda w$ subject to

$$\begin{pmatrix} w \\ w \\ \vdots \\ w \end{pmatrix} \geq \hat{X} - \left[\hat{Q}_1\hat{X} \mid \hat{Q}_2\hat{X} \mid \ldots \mid \hat{Q}_n\hat{X}\right] \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_k \end{pmatrix}$$

$$\begin{pmatrix} w \\ w \\ \vdots \\ w \end{pmatrix} \geq -\hat{X} + \left[\hat{Q}_1\hat{X} \mid \hat{Q}_2\hat{X} \mid \ldots \mid \hat{Q}_n\hat{X}\right] \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_k \end{pmatrix}$$

$$w \geq 0, \sum_i^k \lambda_i = 1, \ and \ \lambda_i \geq 0, \quad \forall_i$$

Materials and Methods

The data was collected from a survey on "Appraisal of Directly Observed Treatment short- course (DOTS) and Tuberculosis Eradication in Secondary Healthcare facility in Southwest, Nigeria" at West African Post Graduate College of Pharmacist, Yaba, Lagos. The questionnaire contains 42 items which was divided into 4 subsections (A – D). Section A (made up of 9 items) consisted of questions on socio-demographic characteristics of individuals – age (years), sex, marital status, religion, education. Section B was on the knowledge of DOTS. Sections C was on application of DOTS while Section D was on impact of DOTS. This study focuses on the application of DOTS with state of the patients (success, failure). The individuals attending the Out-patients Department of the hospital for tuberculosis treatment were used for the study. Convenient sampling technique was used. Data was collected three times a week for the period of one and a halved month. R programming language was used to analyze the data using Higher Order Markov Model.

Results

This chapter presents the analysis and interpretation of the data on "Appraisal of Directly Observed Treatment short course (DOTS) strategy and Tuberculosis Eradication in a Secondary Healthcare Facility in Southwest, Nigeria" used in the study. The result of the analysis is presented using tabular presentations. Data were collected on 250 patients suffering from Tuberculosis. There are two possible states in the Markov chain, which are 1 and 2. States 1 and 2 represent success and failure respectively. Lambda: [1] = 1. The lambda is one showing adequacy of order 1. The steady state vector v satisfies the equation vP = v. That is, it is an eigenvector for the eigenvalue λ = 1. If the probability in P remain the same over a long run, it will get to a stage where the vector will be stable i.e. no change occur and it will be in equilibrium stage. At this stage, the system is said to be in steady state and the steady state vector is called stationary vector.

vP = v

The steady state probability is of the form:

1	2
0.1732523	0.8267477

Table 1: The probability transition matrix of the different states summing up to one for success and failure for order 1. The probability of moving from success to success is 0.56, the probability of moving from success to failure is 0.09, and the probability of moving from failure to success is 0.44 while the probability of moving from failure to failure is 0.91. Lambda: [1] = 1. The lambda is one showing adequacy of order 1. The Table 3 shows that the mean transition from state one to one is 34.6,.the mean transition from state one to two is 5.80, the mean transition from state two to one is 26.70, while the mean transition from state two to two is 56.5. The higher expectation from state one (success) to state two (failure) is due to relapsed in the application of DOTS process during application of treatment i.e. the conditional expectation of success given failure.

The Table 4 is the probability transition matrix of the different states summing up to one for success and failure for order 2. The probability of moving from success to success is 0.47, the probability of moving from success to failure is 0.11, and the probability of moving from failure to success is 0.53 while the probability of moving from failure to failure is 0.89. [1] 0.5 0.5 .The lambda is one showing adequacy of order 2.

Table 5 is the mean transition from state one to one is 28.8, the mean transition from state one to two is 7.0, the mean transition from state two to one is 32.9, while the mean transition from state two to two is 54.8. The higher expectation from state one (success) to state two (failure) is due to relapsed in the application of DOTS process during application of treatment i.e. the conditional expectation of success given failure.

Calculating the AIC and BIC with the following log likelihood:

$LL^{(1)}$ = -95.66233

$LL^{(2)}$ = -62.3243

$LL^{(3)}$ = -45.4162

The Table 6 & 7 shows AIC and BICwhich indicates that the best model that appears to give an excellent fit is the second order Markov model since it has a lower AIC and BIC when

compared with the first order. This is because the third order was invalid due to the inability of the lambda value to sum up to one. Hence, the best model is the second order Markov model with AIC (132.649) and BIC (134.219). The model has parameters $\lambda 1 = 0.5$ for the first lag, $\lambda 2 = 0.5$ for the second lag.

Discussion

This study focuses on the application of DOTS with state of the patients (success, failure). Directly Observed Treatment short-course (DOTS) has been found to be an effective means of administering anti-TB drugs, significantly reducing the rates of relapse and drug resistance as well as improving the treatment compliance rate [5, 6]. [11] introduces a warning threshold for detecting the unexpected incidences of Tuberculosis (TB) using a Hidden Markov Model (HMM) and it was concluded that the warning threshold constructed based on the Periodic Autoregressive Model can be regarded as a useful alternative for HMM in detection of the weeks with unexpected incidence of TB, therefore it was suggested for monitoring TB surveillance. This research uses higher order markov model on the application of DOTS with state of the patients (success, failure). This will help to the determine future condition of patients and the efficient control of Tuberculosis by concentrating on the initial conditions of TB patients and focus on other factors that can improve the condition of patients because the conditional probability of being in the current state depends on the previous state. This will also help in reducing cost and making decision on policies on DOTS.

5.0 Conclusion

The Higher Order Markov Model has helped in obtaining vital information on the observable states of patients and has served as an efficient and effective tool for classifying the patients based on their observable states. The information obtained from this model can be used by the health organization to enable them operate maximally in combating the menace and deadly effect of tuberculosis. A quick look at the results for different models reveals that with order increasing for a specific model the number of estimated parameters increases rapidly. It is not surprising, therefore, that higher order models do a comparatively good job in fitting data structures and stating the best model with a lower AIC and BIC in the second order when compared with the first order. Hence, the best fitted model is the second order Markov model with AIC (132.649) and BIC (134.219). The model has parameters $\lambda_1 = 0.5$ for the first lag, $\lambda_2 = 0.5$ for the second lag.

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Table 1: Number of Independent parameters by order in HOMC

Number of State(m)	Order(k)	Markov Chain
2	1	2
	2	4
	3	8
	4	16
3	1	6
	2	18
	3	54
	4	162
4	1	20
	2	100
	3	500
	4	2500

Table 2: The probability transition matrix of order 1

	1	2
1	0.5555556	0.09313725
2	0.4444444	0.90686275

Table 3: Mean transitions of order1

	1	2
1	34.58334	5.797794
2	27.66666	56.452206

Table 4: The probability transition matrix of order 2

	1	2
1	0.4666667	0.1133005
2	0.5333333	0.8866995

Table 5: Mean transitions of order 2

	1	2
1	28.81667	6.996306
2	32.93333	54.753694

Table 6: Model selection with AIC

Order	Log likelihood	k	AIC
1	-95.6623	2	195.3247
2	-62.3243	4	132.6486
3	-45.4162	8	106.8324

Table 7: Model selection with BIC

Order	Log likelihood	2Loglikelihood	k	klog(n)	BIC
1	-95.6623	191.3247	2	4.792399	196.1170587
2	-62.3243	124.6486	4	9.570788	134.2193878
3	-45.4162	90.8324	8	19.14158	109.9739756

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Modeling Solar Radiations Series in Nigeria using ARIMA-GARCH Models

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Abstract: Modeling solar radiation is a necessity for the utilization of the benefits it brings to mankind. Time series analysis has proved to stand out amidst other statistical tools when estimating and forecasting solar radiations and their variations. In this paper, a mixture of the Autoregressive Moving Average (ARMA) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) time series models were implemented on the solar radiation series for three (3) representative meteorological stations in Nigeria namely; Ibadan, Sokoto and Port Harcourt to capture and model the conditional mean and volatility that may exist in the series. After subjecting the models to some evaluation metrics for model adequacy, the results gave appropriate ARMA models for the stations and indicated the presence of volatility in the radiations series. Furthermore, a-week-ahead forecasts were conducted for these stations using the ARMA-GARCH model combination which gave close convergence with the actual radiations for year 2016.
Keywords: Models, Solar radiation, ARMA, GARCH, Volatility

1. Introduction

In most developed countries, the use of solar energy as an alternative source for generating power is gaining an edge over other sources, despite its maintenance expenses. It is vital to understand the behaviour of solar energy for proper utilization [1]. Solar radiation is the radiant energy transferred from the sun to the surface of the earth. Solar energy warms our planet and gives us our everyday wind and weather. Without the sun's radiant energy, the earth will gradually cool

and become encased in a layer of ice [2]. The sun is an unending source of natural energy that when compared with other forms of renewable energy, has the potential for a broad range of applications due to its accessibility. The closer the earth is to the sun, the more the intensity of solar energy it receives. Some factors that affect the amount of solar radiation the earth's surface receives are the geographic region, time of day, time of year, local landscape and local climate condition [3]. Solarimeters are the instruments used to measure solar radiation [2]. Nigeria has the potential for a wide range of application of solar energy due to the availability of sunshine throughout the year, which can in turn impact positively on her economy. This is true because, every hour the earth receives more energy from the sun than is consumed by mankind in a year [4. 5] found that there is an estimated 3,000 hours of sunshine annually and on an average per day, Nigeria receives as high as 20 Ms/m2 of solar radiation, depending on the time of the year and location [6]. In the western region of Africa, Nigeria is located between latitude 4oN and 13o N and longitude 3oE and 15oE. An insight as to how a particular geographical location encounter variations in solar energy distribution, would surely lead one to discovering that the solar energy received in the states makes up Nigeria, possesses different meteorological data which accounts for these variations [1]. Though the measurement of solar radiation is not having total coverage for all locations in most developing nations such as Nigeria, meteorological indicators like sunshine hours, temperature, relative humidity and rainfall to name but a few, are use to extrapolate the solar energy reaching the earth's surface [7]. Knowing that for various states in Nigeria, there are varying solar radiation intensities, for instance, there is higher intensity of sun-rays in the Northern part compared to the southern part of Nigeria, which are the differences that were considered to improve the accuracy of the models.

In the research community, Autoregressive Moving Average (ARMA) methods are widely used time series models when compared with other models like Artificial Neural Network Models, Markov Chains, Fuzzy networks, etc. [8]. The ARMA model is able to extract the useful statistical properties of many regions, and can easily take on the well-known Box-Jenkins methods [9]. In addition, these models are very flexible; therefore, they can be used in various types of time series modelling with different orders. Finally, it offers a regular pervasiveness at individual phases (identification, estimation and diagnostic checks) for a suitable model. In ARMA models, one of the greatest difficulties is the need for enormous amount of data [10]. Forecasts are essential in monitoring solar systems, energy systems sizing, and optimization and utility applications. Utilities and independent system operators utilize forecast information to manage generation and distribution. Hypothetically, there is no stochasticity in solar irradiance; hence, deterministic models are frequently incorporated to model this dataset. At ground level, the achievement of seasonal ARIMA models are ascribed to their abilities to capture the stochastic component of the irradiance

series due to the effects of the unstable atmospheric conditions [11]. Relative to other electricity generating sources, solar powered systems produces electricity that are more prone to instability, which suggests the challenges present when integrating solar energy into traditional electricity systems [12]. In the utilization of solar radiation, one of the critical difficulties is modeling solar radiation [13].Although accurate prediction of the mean solar radiation can be provided from various techniques proposed by professionals, the turbulence (volatility or heteroscedasticity) of solar radiation is often missing [14].

In this paper, some time series statistical tools that have been extensively utilized in finance and financial decisions were applied to solar energy so as to better estimate the mean and volatility (variations) in solar radiation received in Nigeria. Although countless researchers in Nigeria who are more of physicists and engineers have developed some good models for estimating global radiation, there is little or no attention on modelling and forecasting solar radiation using time series tools especially S/ARMA, GARCH models for mean and volatility of solar radiation series. Time is an important factor in virtually every aspect of life and human endeavours, which have made researchers from various works of life, explore all areas ranging from economy, business, archaeology, engineering, academia etc. As a result of this, time series analysis has grown to be relevant in all of these fields. Among the most effective approaches for analysing time series data is the model introduced by Box and Jenkins, Autoregressive Integrated Moving Average (ARIMA). For instance, in a study by [15], an ARIMA model was developed in MATLAB environment for simulating and forecasting the rainfall data for the study area Krisnanaga, India using the Box-Jenkins methodology. The rainfall data covered the period of 1971 to 2010, where the first thirty (30) years i.e. from 1971 to 2000 of the data was used for the model development and the remaining ten (10) years i.e. from 2001 to 2010 of the data was used to verify the developed model. From the study, it was found that the ARIMA model is suitable for forecasting monthly rainfall over the study area and further suggested that the model could be used for forecasting the monthly rainfall for up-coming years. Suitable solar data modeling and reliable forecasting of solar radiation is vital for design, performance forecast and monitoring of solar energy conversion systems. One category of models used effectively to achieve this are the short-memory Box-Jenkins seasonal/non-seasonal Autoregressive Integrated Moving Average (S/ARIMA) stochastic models [16, 17, 18].

Also, [19] applied Box-Jenkins method to average solar radiation data that covered the period of 31st May to 14th October, 2007 for Bangi, Malaysia and discovered that the non-seasonal autoregressive model of order 1 i.e. is adequate after using Ljung-Box statistics for diagnostic checking. In the study, they reported that there were missing measurements in the data on 4th to 8th of July, 5th, 6th and 15th of August and these were replaced with the value derived from the average of the data in the same week.

Meanwhile, an analysis of the international variability of solar radiation and sunshine hours for Brazil was done by [20] to generate statistical parameters for model checking which was to be used as an input data for synthetic time series generation. The AR-1 was the suggested approach for monthly solar radiation synthesis time series generation with auto-correlation coefficient varying from 0.30 to 0.40 for the localities in the north of Brazil and 0 for the other regions.

Generally, it is well-known in time series analysis that the ARMA-GARCH models are used in finance for modeling the mean and volatility [21, 22], yet these models have not received much attention in the energy community except for wind-speed forecasting [23, 24, 25]. Recently, [14] conducted an empirical investigation of solar radiation series using ARMA-GARCH models. Representative dataset from two china stations were incorporated into six different ARMA-GARCH models to model and predict the mean and volatility of monthly time series which out-performed the traditional point forecasting models like the simple Artificial Neural Network (ANN), because ANN was a poorer model in dealing with volatility of solar radiation data. In their work, the results reported that the ARMA-GARCH (-M) models are effective in radiation series estimation. The remaining part of this paper is organised as follows. Section 2 reviews the general ARIMA and GARCH methodologies. Section 3 describes in details the representative meteorological sites under investigation. Section 4 uses the daily solar radiation time series from the sites to describe the appropriate ARIMA-GARCH models for estimating the mean and volatility that exist in the series. Finally, in Section 5, the summaries of the results from the study were given with a brief remark to conclude the paper.

2. Method

2.1 Foundations for ARMA Models

A stationary time series is said to be an autoregressive moving average process of order p and q written as ARMA (p, q), if it satisfies the difference,

$$S_t - \emptyset_1 X_{t-1} - \cdots - \emptyset_p S_{t-p} = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} \quad (1)$$

$\{S_t\}$ are the solar radiation series, $\{\varepsilon_t\}$ are white noise (shocks) for the solar radiation process and the coefficients $\emptyset's \text{ and } \theta's$ are such that the model is stationary and invertible. For stationarity, the roots of $\Phi(B)$ must lie outside the unit circle i.e. $|B| > 1$ while the invertibility condition is that the roots of $\theta(B)$ must lie outside the unit circle

A general non-seasonal $ARIMA(p, d, q)$ *model is* $\Phi(B)\nabla(B)^d X = \theta(B)\epsilon$

where $\nabla(B) = I - B.$

$$\Phi(B) = 1 - \emptyset_1 B - \emptyset_2 B^2 - \ldots - \emptyset_p B^p$$

And

$$\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \ldots + \theta_q B^q$$

$$\Phi(B)X_t = \theta(B)\varepsilon_t \quad (2)$$

For non-stationary series $\{S_t\}$, [26] proposed that differencing of sufficient order d could make the series stationary. If the d^{th} difference denoted by $\{\nabla^d S_t\}$ satisfies (2) then $\{S_t\}$ is said to follow an autoregressive integrated moving average model of order p, d and q, denoted by ARIMA (p,d,q).

The Box-Jenkins procedure is concerned with fitting an ARIMA model to a data, which are of three parts: Identification, Estimation and Verification.

A popular way to choose p is by minimizing Akaike Information Criterion (AIC), introduced by [27, 28] defined as,

$$AIC = -2logL + 2k \quad (3)$$

where k is the number of parameters estimated, (in the above case p). The optimal model order is determined by the value of k for which $AIC_{(k)}$ is minimum.

[29, 30] developed Bayesian Information Criterion (BIC) which is an extension of minimum AIC procedure defined as

$$BIC = (n-p-q)\ln[n\hat{\sigma}^2/(n-p-q)] + n(1+\ln\sqrt{2\pi}) + (p+q)\ln\left[\left(\sum_{t=1}^{n} Y_t^2 - n\hat{\sigma}^2\right)/(p+q)\right] \quad (4)$$

where $\hat{\sigma}^2$ is the maximum likelihood estimate of the white noise variance. The BIC is a consistent order-selection criterion

1.2. Foundations for GARCH models

To model volatility in the series if it exists, Autoregressive Conditional Heteroscedasticity (ARCH) or Generalized ARCH models as suggested by [21] and [22] for univariate volatility can be used, having the following properties;

ARCH Model

$$r_t = \mu + \varepsilon_t \quad (5)$$

where r_t is the return series (transformed solar radiation series), μ is a constant and ε_t is the random shock (error term) which is distributed as $\varepsilon_t = \sigma_t \Omega_t$ and $\{\Omega_t\}$ is a sequence of identically and independently distributed random variable with mean zero and variance unity. Then for $\alpha_0 > 0$ and $\alpha_i \geq 0\ (i > 0)$, the innovation is derived,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 \quad (6)$$

The model in (6) is called $ARCH\ (p)$ model. Note, the distribution of Ω_t can be standard normal, standardizedstudent-t, generalized error distribution (GED) or skewed student-t distribution.

GARCH Model

The ARCH model of [21], conditional variance σ_t^2 is determined based on the dependencies among lags of the return series alone. In the GARCH model, lags of the conditional variance, $\sigma_{t-j}^2 (j > 0)$ are introduced to further remove the linear dependencies in the return series.

GARCH Specification

$GARCH\ (p,q)$ Model is then specified as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \quad (7)$$

Then

for $\alpha_0 > 0, \alpha_i, \beta_j \geq 0\ (i,j > 0)$,

and $\sum_{i,j=1}^{\max(p,q)} (\alpha_i + \beta_j) < 1$, the $GARCH\ (p,q)$ model in (7) can be parameterized by applying

$a_t = \varepsilon_t^2 - \sigma_t^2$. Then we have,

$$\varepsilon_t^2 = a_0 + \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i)\varepsilon_{t-i}^2 + a_t - \sum_{j=1}^{q} \beta_j a_{t-j} \quad (8)$$

which is an ARMA representation of the squared residuals, ε_t^2

2. Solar Radiation Data from the Sites

The solar radiation dataset for this study were obtained from the Nigerian Meteorological Agency (NIMET), Oshodi, Lagos State office, Nigeria. The parameter made available was the solar radiation series measured in millilitres (ml) using the Gunn-Bellani Radiation Integrator as the instrument for reading the radiation in those stations. The representative sites under investigation were Ibadan, Sokoto, and Port Harcourt. The investigation periods were from the 1st of January 2011 to 31st of December, 2015 which covered daily observations within those periods. In order to understand the data, some basic statistical summaries like means, standard deviation, etc. were conducted on the data as seen in Table 3. From the table, on an average, Sokoto obviously has the highest intensity of solar radiation relative to the other stations. However, on an unusual day, Port Harcourt received 394.58W/m^2, which outshined that of Sokoto. The data distribution for the sites are negatively skewed save Port Harcourt and exhibits positive kurtosis except for Ibadan. Before using the dataset from the stations, a standard conversion was made from ml to watts per sq. meters (1 ml to 13.153 W/m^2). And the reason for the use of Gunn-Bellani Radiation Integrator relative to a Solarimeter for taking solar radiation readings was because the former was inexpensive and easy to use compared to the later. For computational purposes, R statistical programming software was used incorporated.

Table 1: Daily solar radiation readings for each site

DATE	Ibadan	Sokoto	Port H
01/01/2011	177.56	205.18	226.23
01/02/2011	136.79	228.86	140.74
01/03/2011	152.57	226.23	193.35
01/04/2011	142.05	240.7	194.66

01/05/2011	115.74	252.53	111.8
01/06/2011	132.84	210.44	142.05

Table 2: Daily solar radiation readings for each site for the last six days

DATE	Ibadan	Sokoto	Port H
12/26/2015	148.63	164.41	217.02
12/27/2015	157.83	215.71	21965
12/28/2015	165.73	242.01	261.74
12/29/2015	184.14	238.07	247.27
12/30/2015	174.93	260.43	210.44
12/31/2015	207.81	219.65	236.75

Table 3: Summary statistics of the solar radiation series for the sites

Sites	Mean	Std. Dev.	Min	Max	Skewness	Kurtosis
Ibadan	142.10	47.52	2.63	281.47	-0.46	-0.26
Sokoto	234.74	53.57	11.84	373.54	-1.06	1.53
Port. H	153.23	52.58	3.95	394.58	0.09	0.27

4.0. Results and Discussion

Figure 1 shows the time series plots of the solar radiation measured at each station to their respective years of observation. From the plot, there seems to be no trends and seasonality in the solar radiation throughout these years and also that there appears to be some kind of non-stationarity in the daily solar radiation for Sokoto. It is

also worth noting, that the time plot does not adequately supply all needed information. After the solar radiation time series were converted and the time plots constructed, the next step was to perform a test for serial correlation within the series using autocorrelation function (ACF) and partial autocorrelation function (PACF) plots to have a visual display of its behaviour. All of these tests are the basic time series criteria that must be satisfied for a particular model to be appropriate for estimation and forecasting purposes. Augmented Dickey Fuller (ADF) Test of Table 4 reports a p-value that is less than 0.05 for Ibadan, Sokoto and Port Harcourt, therefore the null hypothesis for the presence of a unit root was rejected. This implies that the solar radiation for the three sites are stationary and need no differencing. However, a further test was conducted to validate the ADF's result due to the peculiarity of the radiation data. The p-values for the three sites after carrying out the test must be greater than 0.05. Kwatowski-Phillips-Schimdt-Shin (KPSS) test of Table 4 reports a p-value that is greater than 0.05, therefore the null hypothesis was rejected for Stationarity for only Ibadan and Port Harcourt which agrees with their respective ADF. The ADF test for Sokoto was in disagreement with the KPSS implying that the series must be differenced at least once to attain stationarity before it can be appropriate to fit the ARMA model for the series.

Table 4: Test for stationarity and normality of residuals for the sites

	Augmented Dickey Fuller			KPSS TEST			Residual Test (Box-Ljung Test)		
SITES	Lag	Value	p-value	Lag	Value	p-value	Value	d.f	p-value
IBADAN	12	-4.2782	0.01	9	0.22406	0.1	0.258	1	0.6111
SOKOTO	12	-5.4687	0.01	9	1.9286	0.01	13.68	20	0.8464
PORT HARCOURT	12	-4.5374	0.01	9	0.38057	0.08553	20.581	20	0.4222

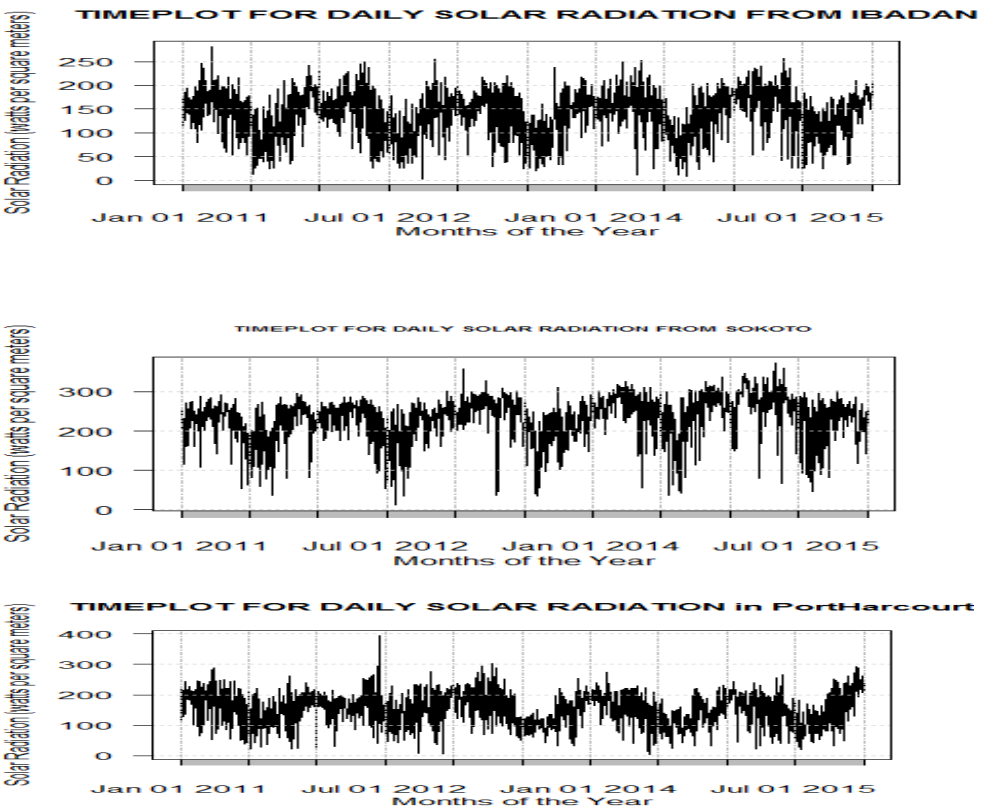


Figure 1: Time plots for Ibadan, Sokoto and Port Harcourt

Table 5: SARMA (2, 2) x (2, 2)$_7$ model for Solar Radiation at Ibadan

Ibadan Site Model: SARMA (2,2) x (2,2)$_7$

Coefficient	**AR1**	**AR2**	**MA1**	**MA2**	**SAR1**	**SAR2**	**SMA1**	**SMA2**	**Intercept**
	0.1001	0.8616	-0.0319	-0.8408	0.1001	0.8616	-0.0319	-0.8408	142.8656
Standard Error	0.0868	0.0848	0.0798	0.0739	0.0868	0.0848	0.0798	0.0739	9.4432

Sigma^2 estimated as 1437: Log likelihood = -9229.43 AIC =18478.87 BIC = 18533.97

The SARMA (2, 2) x (2, 2)$_7$ model for solar radiation from Ibadan site is;

$$S_{IBt} = 0.8616(S_{IBt-2} + S_{IBt-14}) - 0.7424S_{IBt-16} + \varepsilon_{IBt} + 0.8408(\varepsilon_{IBt-2} + \varepsilon_{IBt-14}) + 0.7069\varepsilon_{IBt-16}$$

where $\{S_{IBt}\}$ are the stationary time series for Ibadan Solar radiations, $\{\varepsilon_{IBt}\}$ are the white noise (or shocks) existing in the series.

Table 6: ARIMA (3, 1, 2) model for Solar Radiation at Sokoto

Port Harcourt Site	**ARIMA (1,0,2) with non-zero mean**			
	Ar1	**Ma1**	**Ma2**	**Mean**
Coefficient	0.9867	-0.9389	0.0562	156.1667
Standard Error	0.0049	0.0245	0.0243	9.0270
Sigma^2 estimated **as 2022:** log likelihood =**-9539.07** AIC=**19088.14** AICc =**19088.17** BIC = **19115.69**				
Sokoto Site	**ARIMA (3,1,2) with non-zero mean**			
	AR1	**AR2**	**AR3**	**MA1**	**MA2**
Coefficient	-0.7362	0.2486	0.1233	-0.0012	-0.8207
Standard Error	0.0549	0.0399	0.0270	0.0502	0.0435
Sigma^2 estimated **as 1744:** log likelihood = **- 9413.94** AIC=**18839.88** AICc= **18839.93** BIC = **18872.94**					

The ARIMA (3, 1, 2) model for solar radiation from Sokoto site is;

$$(S_{SKt} - S_{SKt-1}) = -0.7362(S_{SKt-1} - S_{SKt-2}) + 0.2486(S_{SKt-2} - S_{SKt-3}) + 0.1233(S_{SKt-3} - S_{SKt-4}) + \varepsilon_{SKt} - 0.8207\varepsilon_{SKt-2}$$

where $\{S_{SKt}\}$ are the stationary time series for Sokoto Solar radiations, $\{\varepsilon_{SKt}\}$ are the white noise (or shocks) existing in the series.

The ARMA (1, 2) or ARIMA (1, 0, 2) model for solar radiation from Port Harcourt site is;

$$S_{PHt} = 156.17 + 0.9867S_{PHt-1} + \varepsilon_{PHt} - 0.9389\varepsilon_{PHt-1} + 0.0562\varepsilon_{PHt-2}$$

where $\{S_{PHt}\}$ are the stationary time series for Port Harcourt solar radiation, $\{\varepsilon_{PHt}\}$ are the white noise (or shocks) existing in the series

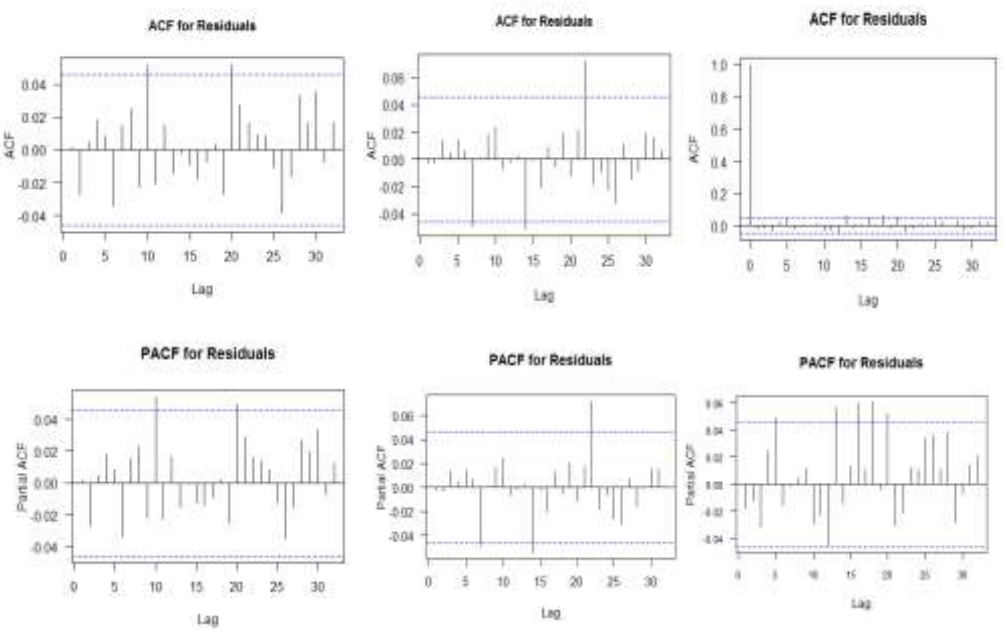
Having confirmed that the solar radiation for Ibadan, Port Harcourt and Sokoto (after 1st differencing) are stationary, the next step was to fit an appropriate Auto Regressive Moving Average (ARMA) model to the series for Ibadan and Port Harcourt, while for Sokoto, an Auto Regressive Integrated Moving Average Model (ARIMA) model was fitted, which gives the result as seen from Tables 5, 7 and 6 respectively.

Table 7: ARIMA (1, 0, 2) model for solar radiation at Port Harcourt

Table 8: GARCH (1, 1) Results for the Three Sites

Sites	Coefficients			Jacque-Bera			Box-Ljung Test		
	A_0	A_1	B_1	Chi-Squared	d.f	p-value	Chi-Squared	d.f	p-value
Ibadan	41.3538	0.0425	0.9293	158.9	2	<2.2e-16	0.004688	1	0.9454
Sokoto	71.1581	0.0770	0.8837	1768.1	2	<2.2e-16	0.1617	1	0.6876
Port Harcourt	74.2488	0.0845	0.8799	78.197	2	< 2.2e-16	0.1876	1	0.6649

Figure 2: ACF and PACF plots for the residuals of the sites Port Harcourt, Sokoto and Ibadan respectively



The ACF plot for the residuals of Ibadan displayed above (Figure 2) suggests that there is no significant autocorrelation which implies that the model is a good fit, meanwhile the ACF plots for Port Harcourt and Sokoto shows some significant lags. Further confirmation was carried out via Box-Ljung test (Table 4). The null hypothesis states that the autocorrelation is not different from 0. The Box-Ljung test with a reported p-value greater than 0.05 for Ibadan, Port Harcourt and Sokoto implies that the null hypothesis of insignificant autocorrelations will not be rejected. Also, the model must follow Normal distribution with mean zero and a constant variance. Squared residuals plot in Figure 3 shows volatility clustering at some points in time. Since the ACF and PACF of the squared residuals for all sites displays some significant lags, it implies that volatility can be modeled for average solar radiation in these sites because there exists a strict white noise which are independent with zero mean and

normally distributed. The residuals show some patterns that might be modeled. To implement this, the GARCH method was used to model the conditional variance of the series. The p-values (Table 8) for all the parameters are less than 0.05, indicating statistical significance. In addition, the p-value of Box-Ljung test is greater than 0.05, and so the null hypothesis that the autocorrelation of the residuals is different from 0, will not be rejected. The model therefore adequately represents the residuals. These conclusions are suitable for all the sites (Ibadan, Port Harcourt and Sokoto) under investigation.

The GARCH (1, 1) models for Ibadan, Port Harcourt and Sokoto respectively are as follows;

$$\sigma^2_{IBt} = 41.354 + 0.043\varepsilon^2_{IBt-1} + 0.929\sigma^2_{IBt-1} \quad (9)$$

$$\sigma^2_{PHt} = 74.249 + 0.0845\varepsilon^2_{PHt-1} + 0.8798\sigma^2_{PHt-1} \quad (10)$$

$$\sigma^2_{SKt} = 71.158 + 0.077\varepsilon^2_{SKt-1} + 0.8837\sigma^2_{SKt-1} \quad (11)$$

The Mixed ARIMA-GARCH Models are:

For Ibadan: - SARMA (2, 2) x $(2, 2)_7$ + GARCH (1, 1)

$$S_{IBt} + \sigma^2_{IBt} = 0.8616(S_{IBt-2} + S_{IBt-14}) - 0.7424S_{IBt-16} + \varepsilon_{IBt} + 0.0319(\varepsilon_{IBt-2} + \varepsilon_{IBt-14}) + 0.001\varepsilon_{IBt-16} + 41.354 + 0.043\varepsilon^2_{IBt-1} + 0.929\sigma^2_{IBt-1} \quad (12)$$

For Sokoto: - ARIMA (3, 1, 2) + GARCH (1, 1)

$$(S_{SKt} - S_{SKt-1}) + \sigma^2_{SKt} = -0.7362(S_{SKt-1} - S_{SKt-2}) + 0.2486(S_{SKt-2} - S_{SKt-3}) + 0.1233(S_{SKt-3} - S_{SKt-4}) + \varepsilon_{SKt} - 0.0012\varepsilon_{SKt-1} - 0.8207\varepsilon_{SKt-2} + 71.158 + 0.077\varepsilon^2_{SKt-1} + 0.8837\sigma^2_{SKt-1} \quad (13)$$

For Port Harcourt: - ARMA (1, 2) + GARCH (1, 1)

$$S_{PHt} + \sigma^2_{PHt} = 0.9867S_{PHt-1} + \varepsilon_{PHt} - 0.9389\varepsilon_{PHt-1} + 0.0562\varepsilon_{PHt-2} + 74.249 + 0.0845\varepsilon^2_{PHt-1} + 0.8798\sigma^2_{PHt-1} \quad (14)$$

Tables 9, 10 and 11 are the forecast for solar radiation in Port Harcourt, Sokoto and Ibadan respectively, for first week of the new year 2016 using only their single AR(*I*)MA models which neither considers volatility or reflect changes as new information are available but focuses only on analysing time series data linearly. In other words, the mixed model will consider modeling the noise existing in the ARMA based on the conditional variances as seen in last column of the tables. Looking at the tables, it is no doubt that both models have a conflicting or overlapping forecasts and variations relative to the actual radiations and there is little or no significant variations among the models. Furthermore from figures 4, 5, and 6, the 95% confidence intervals effectively captures the actual radiations for the first week of the year, which has the potential to capture the remaining part of the year considerably. Though, the forecast for the first week looks linear, however, when the length of the forecast is increased, the fluctuation surfaces. The Figures 4, 5, 6 help to visualize the pattern of the forecast for the solar radiations received at the sites.

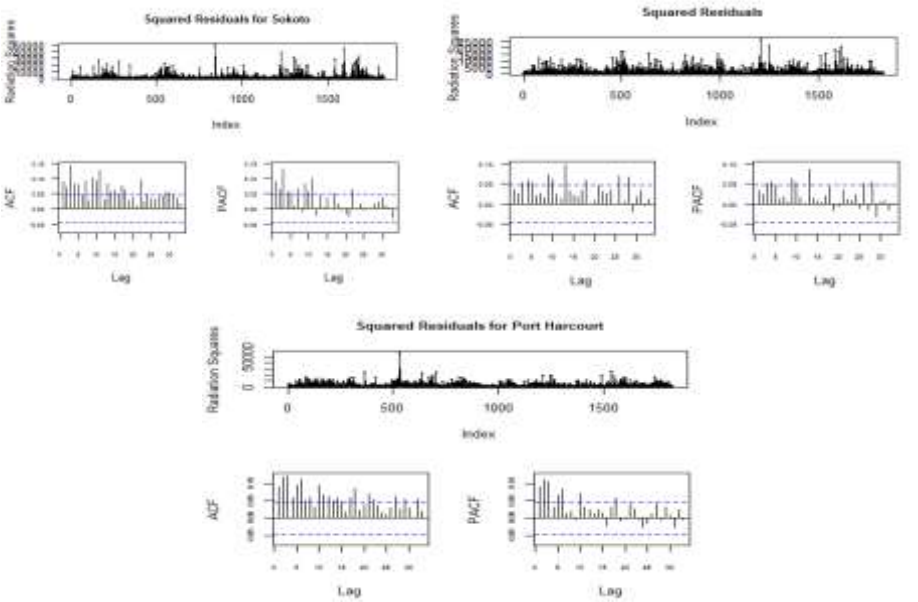


Figure 3: Shows the Squared Residual Plots with their ACF and PACF for

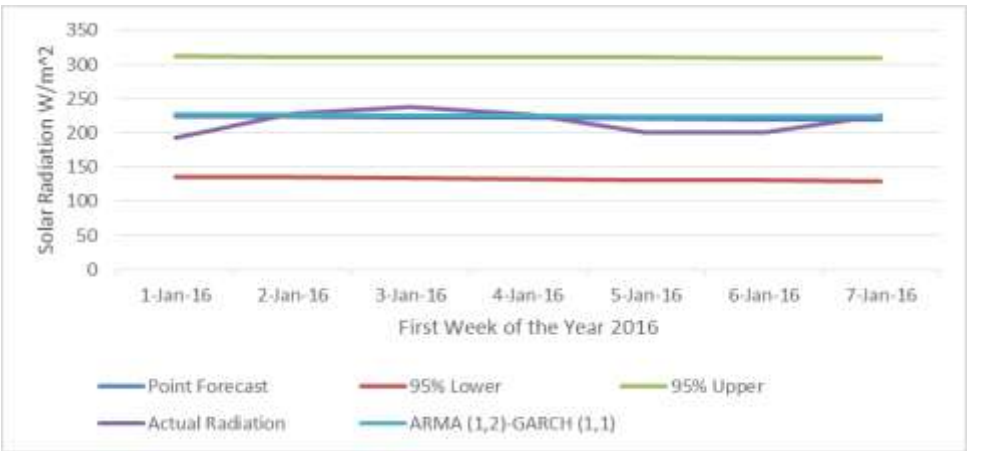


Figure 4: One-Week-Ahead Forecast for Port Harcourt

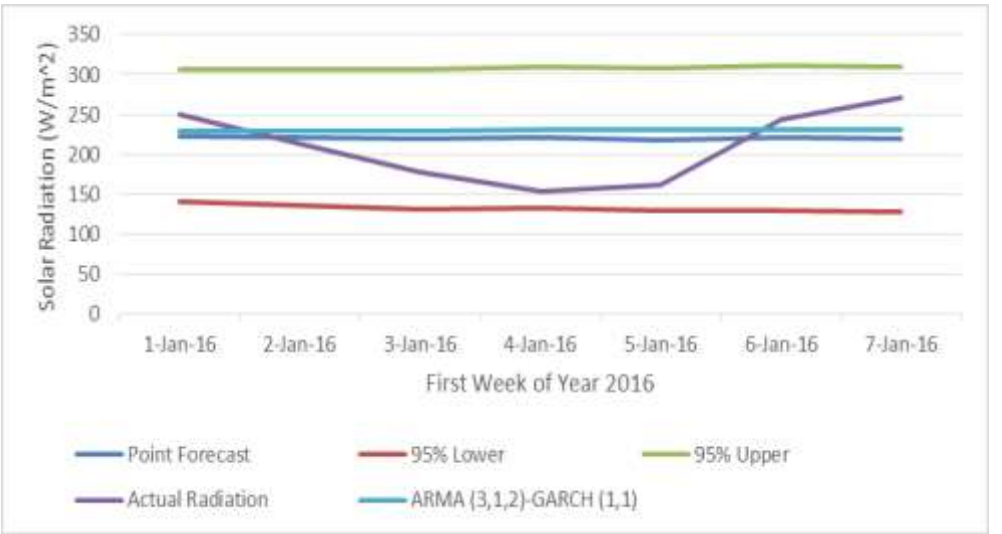


Figure 5: One-Week-Ahead Forecast for Sokoto

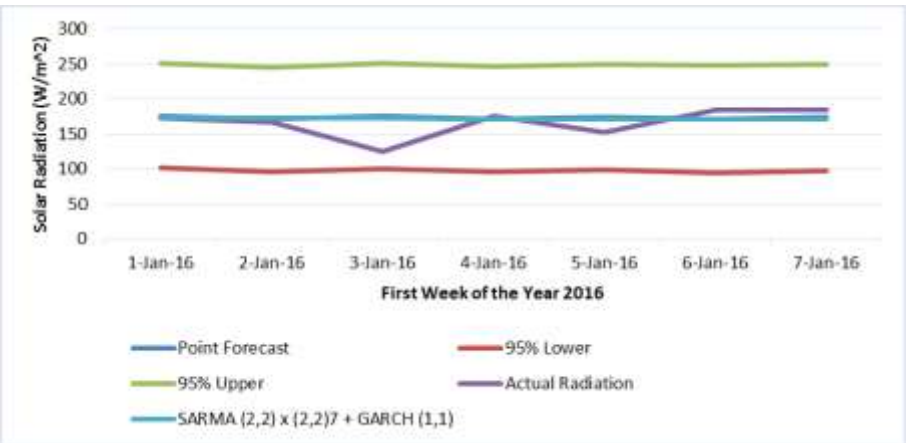


Figure 6: One-Week-Ahead Forecast for Ibadan

Table 9: One-Week ahead forecast for 2016 in Port Harcourt

Day	Point Forecast	95% Lower	95% Upper	Actual Radiation	ARMA (1,2)-GARCH (1,1)	Absolute Error (Single)	Absolute Error (Mixed)
01-Jan-16	223.788	135.658	311.918	193.35	227.2	30.438	33.85
02-Jan-16	223.554	135.324	311.784	228.36	226.6	4.806	1.76
03-Jan-16	222.661	133.961	311.361	238.07	226.0	15.409	12.07
04-Jan-16	222.779	132.625	310.934	227.54	225.3	4.761	2.24
05-Jan-16	220.909	131.314	310.505	201.24	224.7	19.669	23.46
06-Jan-16	220.051	130.029	310.073	199.92	224.0	20.131	24.08
07-Jan-16	219.204	128.768	309.64	226.23	223.4	7.026	2.83

Table 10: One-Week ahead forecast for 2016 in Sokoto

Day	Point Forecast	95% Lower	95% Upper	Actual Radiation	ARMA (3,1,2)-GARCH (1,1)	Absolute Error (Single)	Absolute Error (Mixed)
01-Jan-16	223.462	140.918	306.007	249.90	229.2	26.438	20.7
02-Jan-16	221.763	136.42	307.106	213.08	229.5	8.683	16.42
03-Jan-16	218.935	131.445	306.425	177.56	229.8	41.375	52.24
04-Jan-16	221.065	132.109	310.021	153.89	230.1	67.175	76.21
05-Jan-16	218.585	129.032	308.137	161.78	230.4	56.805	68.62
06-Jan-16	220.591	130.03	311.152	243.33	230.7	22.739	12.63
07-Jan-16	218.76	127.72	309.8	270.95	230.9	52.19	40.05

Table 11: One-Week ahead forecast for 2016 in Ibadan

Day	Point Forecast	95% Lower	95% Upper	Actual Radiation	SARMA (2,2) $(2,2)_7$ GARCH (1,1)	x +	Absolute Error (Single)	Absolute Error (Mixed)
01-Jan-16	176.458	102.162	250.753	172.3	172.9		4.158	0.6
02-Jan-16	171.104	96.120	246.088	167.04	172.6		4.064	5.56
03-Jan-16	175.477	100.361	250.592	124.95	172.2		50.527	47.25
04-Jan-16	171.393	95.689	247.097	176.25	171.9		4.857	4.35
05-Jan-16	174.637	98.76	250.515	152.57	171.6		22.067	19.03
06-Jan-16	171.511	95.116	247.906	184.14	171.3		12.629	12.84
07-Jan-16	173.901	97.299	250.504	184.14	171.0		10.239	13.14

5. Conclusion

It was observed that, the proposed model which closely mimics the solar radiation received in Ibadan, Sokoto and Port Harcourt are, the seasonal ARMA $(2,2)(2,2)_7$, ARIMA (3,1,2), and the combined ARMA (1, 2)-GARCH (1, 1) models respectively. The single models for Ibadan and Sokoto have no significant differences relative to their GARCH combinations when used for forecasting one-week ahead, implying that they are more suitable models due to the fluctuating patterns they exhibit. Meanwhile, the model for Port-Harcourt made provision for variations (or volatility) that exist in the surface radiation compared to the single ARMA (1, 2) model which only focuses on the linearity of the radiation time series. From the one-week ahead forecast, it was observed that as the day increases, both models follow a consistent decreasing pattern relative to the actual values. It is important to recall the mathematical expression of the suggested models as follows;

The seasonal ARMA (2, 2) x $(2, 2)_7$ model for solar radiation from Ibadan site is;

$$S_{IBt} = 0.8616(S_{IBt-2} + S_{IBt-14}) - 0.7424S_{IBt-16} + \varepsilon_{IBt} + 0.8408(\varepsilon_{IBt-2} + \varepsilon_{IBt-14}) + 0.7069\varepsilon_{IBt-16}$$

The ARIMA (3, 1, 2) model for solar radiation from Sokoto site is;

$$(S_{SKt} - S_{SKt-1}) = -0.7362(S_{SKt-1} - S_{SKt-2}) + 0.2486(S_{SKt-2} - S_{SKt-3}) + 0.1233(S_{SKt-3} - S_{SKt-4}) + \varepsilon_{SKt} - 0.8207\varepsilon_{SKt-2}$$

The Mixed ARMA-GARCH Model: - ARMA (1, 2) + GARCH (1, 1)

$$S_{PHt} + \sigma^2_{PHt} = 0.9867S_{PHt-1} + \varepsilon_{PHt} - 0.9389\varepsilon_{PHt-1} + 0.0562\varepsilon_{PHt-2} + 74.249 + 0.0845\varepsilon^2_{PHt-1} + 0.8798\sigma^2_{PHt-1}$$

It can therefore, be safely recommended that, the above models are adequate enough to forecast the solar radiation for Ibadan, Sokoto and Port Harcourt, which is an integral part in the application of solar energy and systems in the energy sector of the economy.

Acknowledgment

We cease this opportunity to appreciate God Almighty for this work, for the enablement in putting this paper together. This work is sponsored by the Centre for Research, Innovation and Discovery, Covenant University, Ota, Nigeria.

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On A Truncated Accelerated Plan for Two Component Parallel Systems under Ramp-Stress Testing Using Masked Data for Weibull Distribution

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***Abstract*:** Several studies on design of Acceptance Life Test (ALT) focused on a subsystem (single system) totally ignoring its internal design. In most cases, it is not always possible to identify the components that cause the system failure or the cause can only be identified by a subset of its component resulting in a masked observation. This paper therefore investigates into the development of ramp-stress accelerated life testing for a high reliability parallel system that consist of two dependent components using masked failure data. This type of testing may be very useful in a twin-engine plane or jet. A ramp-stress results when stress applied on the system increases linearly with time. A parallel system with two dependent components is taken with dependency modeled by G umbel-Hougaard copula. The stress-life relationship is modeled using inverse power law and cumulative exposure model is assumed to model the effect of changing stress. The method of maximum likelihood is thereafter used for estimating design parameters. This optimal plan consists in finding the optimal stress rate using D-optimality criterion by minimizing the reciprocal of the determinant of Fisher information matrix. The projected plan is also explained using a real life example and sensitivity analysis carried out. This formulated model can help guide and assist engineers to obtain reliability estimates quickly with high reliability products that are sustainable.

***Keywords*:** Accelerate, Life test, Ramp-stress, Gumbel-Hougaard copula, Masked data, Fisher information matrix, D-optimality criterion, Dependent components.

Acronyms

ALT- accelerated life test

Avar- asymptotic variance
ML- maximum likelihood
cdf- cumulative distribution function
pdf- probability distribution function

1.0 Introduction

After production process has been carefully controlled up till the finished products, high reliability products of modern times have to be subjected to accelerated life test to detect early failures. This also helps the manufacturer to obtain timely reliability estimates about his products and live on in today's competitive market. Such products may be subject to different stress loading schemes. Such stress schemes include: constant-stress, step-stress, progressive-stress and their various combinations depending upon how they are to be used in service and other limitations both theoretical and practical [1, 2]. A ramp-stress results when stress applied linearly increases with time. A stress can be applied under fully accelerated environmental conditions in which all the test specimens are tested under accelerated condition or partially accelerated environmental conditions where they are tested both under normal and accelerated conditions [3, 4].

Several accelerated life test plans under different stress loading schemes have been devised in some literatures [5, 6]. Nevertheless, both plans are meant for a single system (i.e, a sub-system) with its internal configuration totally ignored. In many cases, it is not always probable to identify the component that caused the system failure or the cause of failure can only be identified by a subset of its component [7]. An observation is said to be masked when event cause of the system failure is not known except that it is as a result of some subset of the component of the system have used the exact maximum likelihood estimation of life time distribution of the component in the series system using masked data [8, 9]. [10] have used the Bayes estimation of component reliability from masked system-life data. [8, 9] have extended the results of [11] to a three component series system of exponential distribution. [12] has used the masked interval data in the series system of exponential components. Formulation of a ramp-stress ALT plan for a parallel system with two dependent components but without masking has been studied by [13]. This paper centered on formulation of a ramp-stress ALT plan for a system with parallel configuration in the presence of masked failure data. Such a testing may prove to be useful in a twin-engine plane or jet. A parallel system with two dependent components is taken with dependency modeled by Gumbel-Hougaard copula. The optimal stress rate is obtained using D-optimality criterion. A numerical example was used to demonstrate application of the developed projected plan and sensitivity analysis was also carried out to examine its robustness.

2.0 The Model

In this section, the model for formulation of a ramp-stress ALT plan for a system with parallel pattern in the presence of masked failure data is developed and its life distribution function with (and) likelihood functions are obtained.

Assumptions

i. The dependency between the two components of the parallel system

is modeled by Gumbel-Hougaard copula evaluated at two Weibull survival (reliability) marginals, viz., $\bar{G}_1(.)$ and $\bar{G}_2(.)$ with shape parameter β_1 and β_2 , and common scale parameter $\theta.\eta$ is the measure of association between the two components.

ii. The censoring time τ is pre specified.
iii. The two components of the system cannot fail simultaneously.
iv. Failed parallel systems are not replaced during the test.
v. The occurrence of masking is independent of the failure cause and time.
vi. The effect of changing stress is modeled by the linear cumulative exposure model.
vii. The stress applied to test units is continuously increased at a constant ramp rate ***k*** from zero.
viii. The inverse power law holds for stress-life relationship, i.e,

$$\eta(s(t)) = e^{\mu}\left(\frac{S_0}{S(t)}\right)^{\alpha} \qquad \textbf{(i)}$$

where μ is the characteristics of the product and α is the shape parameter, $s_{(0)}$ is the stress level under normal operating conditions or design stress and $s_{(t)}$ is a linear function of time in ramp-stress at time t.

2.1 Test

The reliability testing procedure is as follows:

i. If *n* independent and identical parallel systems are put to test and their failure times along with the cause of failure are recorded. An observation is said to be masked if its corresponding cause of failure cannot be recorded.
ii. The test is terminated when all the systems fail.

2.2 Parallel System

A parallel system fails if all the components fail. The configuration of a parallel system with two components is shown in Figure 1.

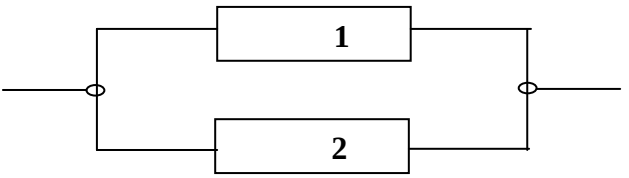

Figure 1: Parallel System

2.3 Copula Function

The dependency existing between the marginal random variables in bivariate and multivariate distributions are described by a copula [1]. The copula describes the way in which the marginals are linked together on the basis of their association.

Suppose X_1 and X_2 are two random variables and let $G_1(x_1)$ and $G_2(x_2)$ be their respective marginal reliability functions. If $H(x_1,x_2)$ are their joint reliability function, thus, according (Therefore according) to Sklar's theorem, there exists a copula reliability function C (x_1,x_2) such that for all that (x_1, x_2) in the defined array:

$$\overline{H}(x_1, x_2) = C\left(\hat{G}_1(x_1), \hat{G}_2(x_2)\right) \quad \text{(ii)}$$

Amongst the Gumbel-Hougaard copula is defined as:

$$C_{\mu}(a,b) = e^{-\left(-\log_e[a]\right)^{\mu}+\left(-\log_e[b]\right)^{\mu}\Big)^{\frac{1}{\mu}}}$$

(iii)

where $1 \leq \mu \leq \infty$ characterizes the relationship between the two variables. Gumbel-Hougaard copula is uni-parametric and symmetrical.

2.4 Reliability Function for Bivariate-Weibull Distribution

The reliability function for Bivariate Weibull distribution is obtained by using Weibull reliability marginals in Gumbel-Hougaard reliability function. Using equation (iii) and assumption (i), according to [16], equation (iv) is arrived at:

$$C\left(\overline{G}_1(t_{i1}),\overline{G}_2(t_{i2})\right)=e^{-\left(\left(\frac{t_1}{\mu}\right)\beta_1\alpha+\left(\frac{t_2}{\mu}\right)\beta_2\alpha\right)^{\frac{1}{\alpha}}} \quad \text{(iv)}$$

Where t = testing time, μ = quality parameter, β = risk and α = shape parameter.

2.5 The Bivariate Weibull Reliability Function for Ramp-Stressed Data

The pdf of the bivariate Weibull distribution is given as:

$$f(t,\beta,\alpha)=\frac{\beta+1}{\beta}\alpha e^{-\alpha t\left(1-e^{-\beta\alpha t}\right)} \quad \text{(v)}$$

The Bivariate Weibull reliability function of a parallel system using Gumbel-Hougaard copula as given by [16] is

$$\hat{G}\left(t_1,t_2\right)=e^{-\left(\left(\frac{t_1}{\mu}\right)\beta_1\alpha+\left(\frac{t_2}{\mu}\right)\beta_2\alpha\right)^{\frac{1}{\alpha}}} \quad \text{(vi)}$$

The bivariate joint probability density function is given as:

(vii)

where $t_1 \geq 0, t_2 \geq 0, \mu_1 > 0, \beta_i > 0, i = 1,2 \text{ and } \alpha \geq 1$, μ_i are scale parameters, β_i are shape parameters and α is the association between the two variables. From the linear cumulative model, the joint reliability function of the parallel system under ramp-stress scheme is given as:

$$\hat{F}\left(t_1,t_2\right)=\hat{G}\left(E(t_1),E(t_2)\right) \quad \text{(viii)}$$

where $\hat{G}(.,.)$ is the underlying bivariate Weibull reliability function with assumed scale parameter taken to be one (1).

$$E(t)=\int_0^t \frac{1}{\mu(S(a))}da \quad \text{(ix)}$$

Equation above is the cumulative harm (damage) model at t. Therefore, the joint cumulative distribution (reliability) function and joint probability (failure) density function respectively of the system under ramp-stress loading are given as:

$$F\left(t_1,t_2\right)=e^{-\left(\left(\int_0^{t_1}\frac{1}{\mu(S(a))}da\right)\beta_1\alpha+\left(\int_0^{t_2}\frac{1}{\mu(S(a))}da\right)\beta_2\alpha\right)^{\frac{1}{\alpha}}} \quad \text{(x)}$$

Therefore,

$$F\left(t_1,t_2\right)=e^{-\left(\left(\left(\frac{t_1}{\varphi_1}\right)\beta_1\alpha+\left(\frac{t_2}{\varphi_2}\right)\beta_2\alpha\right)^{\frac{1}{\alpha}}\right)} \quad \text{(xi)}$$

$$F\left(t_1,t_2\right)=e^{-\left(\left(\left(\frac{t_1}{\varphi_1}\right)\beta_1\alpha+\left(\frac{t_2}{\varphi_2}\right)\beta_2\alpha\right)^{\frac{1}{\alpha}}\right)}\beta_1\beta_2\left(\frac{t_1}{\varphi_1}\right)\beta_{11}\alpha\left(\frac{t_2}{\varphi_2}\right)\beta_{22}\alpha \quad \text{(xii)}$$

$$\times\frac{\left(\left(\frac{t}{\varphi_1}\right)\beta_1\alpha+\left(\frac{t}{\varphi_2}\right)\beta_2\alpha\right)^{-2+\frac{1}{\alpha}\left(\left(\frac{t_1}{\varphi_1}\right)\beta_1\alpha+\left(\frac{t_2}{\varphi_2}\right)\beta_{22}\alpha\right)^{\frac{1}{\alpha}}+\alpha-1}}{t_1t_2}$$

where

$\varphi_i=\left(e^{\gamma_0}\left(\frac{S_0}{k}\right)\gamma_{i(1-\gamma_i)}\right)^{\frac{1}{1+\gamma_i}}$ is the scale parameter, (xiii)

$$\beta_{ii}=\beta_i\left(1+\gamma_i\right) \quad \text{(xiv)}$$

2.6 The D-Optimality

The D-optimality criterion is used in minimizing the reciprocal of the determinant of Fisher information matrix, the Fishers smaller value of the

determinant corresponds to a higher (joint) precision of the estimators of α, β [14].

2.7 Likelihood Function

This section deals with the case of the complete system but masked data. Likelihood for a parallel system is developed for two dependent components. Suppose we consider a sample of n-systems each consisting of two dependent components in parallel. Suppose T_i is the life time of system I and T_{ij} is the life time of component j in system i, i=1,2.....n and j=1,2, then

$$T_i = \max(T_{i1}, T_{i2}) \qquad \text{(xv)}$$

The probability that the system fails due to component 1, when $0 \le t_1 < \infty$ is obtained as:

$$P[T_{i2} \le t_i, t_i < T_{i1} \le t_i + \Delta t_i] = F_{T1}(t_i + \Delta t_i) - F_{T1,T2}(t_i, t_i)$$

$$= F_{T1}(t_i) - F_{T1,T2}(t_i, t_i)$$

As $\Delta \to 0$ and since F_{T1} is absolutely differentiable,

$$= 1 - F_{T2}(t_i) - \bar{F}_{T1,T2}(t_i, t_i)$$

$$\bar{F}_{T2}(t_i) - \bar{F}_{T1,T2}(t_i, t_i)$$

$$\text{Therefore, } L\infty - \frac{\partial}{\partial t_1}\left(\bar{F}_{T1,T2}(t_1, t_2)\right) I_{t_1=t_i, t_2=t_i} \qquad \text{(xvi)}$$

Also, the probability that the system fails due to component 2, when $0 \le t_i \le \infty$ is obtained as: $P[T_{i1} \le t_i, t_i < T_{i2} \le t_i + \Delta t_i] = F_{T2}(t_i + \Delta t_i) - F_{T1,T2}(t_i, t_i)$

$$= F_{T2}(t_i) - F_{T_1,T_2}(t_i, t_i)$$

As $\Delta t_i \to 0$ and since F_{T2} is absolutely differentiable,

$$= 1 - F_{T1}(t_i) - \bar{F}_{T1,T2}(t_i, t_i)$$

$$\bar{F}_{T1}(t_i) - \bar{F}_{T1,T2}(t_i, t_i)$$

Therefore, $L\infty - \frac{\partial}{\partial t_2}\left(\bar{F}_{T_1,T_2}(t_1, t_2)\right) \mathrm{I}_{t_1=t_i, t_2=t_i}$

(xvii)

2.8 The log-likelihood (L)

The log-likelihood of an n parallel system is as given below:

$$L = \prod_{S_i=1}^{n_1}\left(-\frac{\partial}{\partial t_1}\left(\bar{F}_{T1,T2}(t_1, t_2)\right)\right) \times \prod_{S_i=2}^{n_2}\left(-\frac{\partial}{\partial t_2}\left(\bar{F}_{T1,T2}(t_1, t_2)\right)\right) \times \prod_{S_i=1}^{n_1+n_2}\left(-\frac{\partial}{\partial t_1}\left(\bar{F}_{T1,T2}(t_1, t_2)\right) - \frac{\partial}{\partial t_2}\left(\bar{F}_{T1,T2}(t_1, t_2)\right)\right) \qquad \text{(xviii)}$$

where n is specified by the control engineer (experimenter).

$$L = \sum_{S_i=1}^{n_1}\log\left(-\frac{\partial}{\partial t_1}\left(\bar{F}_{T1,T2}(t_1, t_2)\right)\right) \times \sum_{S_i=2}^{n_2}\log\left(-\frac{\partial}{\partial t_2}\left(\bar{F}_{T1,T2}(t_1, t_2)\right)\right) \times \sum_{S_i=1}^{n_{12}}\log\left(-\frac{\partial}{\partial t_1}\left(\bar{F}_{T1,T2}(t_1, t_2)\right) - \frac{\partial}{\partial t_2}\left(\bar{F}_{T1,T2}(t_1, t_2)\right)\right) \qquad \text{(xix)}$$

3.0 Simulated of Parameter Estimation

The Maximum Likelihood Estimates of $\rho_1, \rho_2, \beta_1 \text{ and } \beta_2$ are obtained using R statistical software. The simulation is carried out following [15].

The algorithm is given below:

i. Select n units and put them to test.
ii. Specify the masking level (ρ).
iii. Calculate n_{12} such that $n_{12} \approx \left(\frac{\rho}{n} \times 100\right)$.
iv. Arbitrarily select a random sample of size n from the system life time, and the set of component causing the system failure $(t_1,s_1),\ldots,(t_n,s_n)$.

These random samples are generated following the steps below:

i. Generate n_{12} observations using the system cumulative (i.e, product's lifetime) distribution, which is known as time to failure.
ii. Generate n - n_{12} observations using the system cumulative distributi and determine Si for each i, (i 5 2,…,n-n_{12}), which gives the set of observations where the cause of system failure is known.

In table 1, the time to failure in minutes and the component that fails during the experiment is as shows below;

Table 1: Simulated data estimates

System No.	Time to Failure (t_i)	Component Failure-cause (S_i)
1.	0.0516	(2)
2.	0.1504	(1,2)
3.	0.1944	(1,2)
4.	1.2604	(1)
5.	3.1649	(1,2)
6.	5.437	(2)
7.	5.5425	(1)
8.	8.5725	(2)
9.	10.0166	(1)
10.	10.9509	(2)

System n umber 1, 2 and 2 has the least time to failure with component (2), (1, 2) and (1, 2) causing the failure respectively while system number 9 and 10 with system number 1 and 2 causing the failure respectively.

Maximum Likelihood Estimates (MLE) of the Design Parameters

The ML estimates of the design parameters obtained using simulated data estimates in table 1 are:

$$\rho_1 = -2.2,\ \rho_2 = 0.5, \beta_1 = 0.35 \text{ and } \beta_2 = 0.24$$

In selecting an optimum test plan, there is a need to estimate the design parameters

ρ_{01}, ρ_{02}, β_{11} *and* β_{12}.These estimates at times may affect the values of the resulting decision variables significantly. Therefore, their incorrect choice may result in poor estimate of the design constant stress. Therefore, it is significant to carry out a sensitivity analysis to evaluate the robustness of the resulting Acceptance Life Test plan. Sensitivity analysis helps to identify the design parameters ρ_0, ρ_1, β_1 *and* $\beta_2$which need to be estimated with care to avoid the risk of obtaining wrong solutions. An Acceptance Life Test plan is said to be robust if a small departure in any has no effect in relative change in the optimal plan. The percentage deviations (PD) of the optimal settings are obtained as $PD = \left(\frac{|T^{**} - T^{*}|}{T^{*}} \right) \times 100$, where T^{*} is obtained with the given design parameters and T^{**} is obtained when the parameter is miss-specified.

Table 2 illustrates the optimal test plans for various deviations from the design parameter estimates. The results explain that the optimal setting of T is robust to the small variance from baseline parameter estimates.

Table 2: Sensitivity Analysis for changes in design parameters
$\rho_1 = -3.45, \rho_2 = 0.65, S_0 = 20, \beta_1 = 0.35$ *and* $\beta_2 = 0.25, n = 10, n_{12} = 3$ *and* $\alpha = 0.8$

Parameter	%	K	T^{**}	Percent Deviation (%)
β_1	-5%	1.75	0.000574	3.6526
β_1	+5%	1.74	0.000596	7.6891
β_2	-5%	1.78	0.000583	5.3500
β_2	+5%	1.67	0.000587	5.9500
ρ_0	-5%	1.57	0.000585	5.5979
ρ_0	+5%	2.024	0.000585	5.5872
ρ_1	-5%	1.59	0.000589	4.9225
ρ_1	+5%	1.81	0.000581	6.2877

4.0 Discussion

This study deals with optimal planning of accelerated life test of a parallel system with two dependent components under ramp-stress loading for a Weibull distribution. The dependency is modeled by Gumbel-Hougaard copula evaluated at Weibull reliability marginals. The optimal plan consists in finding optimal stress rate using D-optimality criterion. A hypothetical ramp-stress ALT experiment for a parallel system with two dependent components is considered to illustrate the methods described in this paper. From the simulated dataset, system n umber 1, 2 and 2 were found to has the least time to failure with component (2), (1, 2) and (1, 2) causing the failure respectively while system number 9 and 10 with system number 1 and 2 causing the failure respectively.

5.0 Conclusion

This study has carefully developed a ramp-stress Acceptance Life Test for accelerated environmental conditions for a high reliability parallel system consisting of two dependent mechanisms using masked failure data. Such an experiment may be very useful in a two-engine plane or jet. The relationship between the two components is modeled using inverse power law and cumulative exposure. The method of maximum likelihood was used for estimating design parameters. D-optimality criterion was used to find the optimal stress rate using by minimizing the reciprocal of the determinant of Fisher information matrix. Conclusively, a simulation study (using R) is used to illustrate the method developed. The sensitivity analysis results proved that the proposed plan is better for a small departure from baseline parameters.

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