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Modeling Solar Radiations Series in Nigeria using ARIMA-GARCH Models

A.O Adejumoab[1] & E.A Suleimana[2]

[1]Department of Mathematics,
College of Science and Technology,
Covenant University, Ota, Nigeria
[2]Department of Statistics,
Faculty of Physical Sciences,
University of Ilorin, Ilorin, Nigeria
adejumo.adebowale@covenantuniversity.edu.ng,
aodejumo@unilorin.edu.ng,
suleiman.esivue@stu.cu.edu.ng

Abstract: Modeling solar radiation is a necessity for the utilization of the benefits it brings to mankind. Time series analysis has proved to stand out amidst other statistical tools when estimating and forecasting solar radiations and their variations. In this paper, a mixture of the Autoregressive Moving Average (ARMA) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) time series models were implemented on the solar radiation series for three (3) representative meteorological stations in Nigeria namely; Ibadan, Sokoto and Port Harcourt to capture and model the conditional mean and volatility that may exist in the series. After subjecting the models to some evaluation metrics for model adequacy, the results gave appropriate ARMA models for the stations and indicated the presence of volatility in the radiations series. Furthermore, a-week-ahead forecasts were conducted for these stations using the ARMA-GARCH model combination which gave close convergence with the actual radiations for year 2016.

Keywords: Models, Solar radiation, ARMA, GARCH, Volatility

1. Introduction

In most developed countries, the use of solar energy as an alternative source for generating power is gaining an edge over other sources, despite its maintenance expenses. It is vital to understand the behaviour of solar energy for proper utilization [1]. Solar radiation is the radiant energy transferred from the sun to the surface of the earth. Solar energy warms our planet and gives us our everyday wind and weather. Without the sun's radiant energy, the earth will gradually cool and become encased in a layer of ice [2]. The sun is an unending source of natural energy that when compared with other forms of renewable energy, has the potential for a broad range of applications due to its accessibility. The closer the earth is to the sun, the more the intensity of solar energy it receives. Some factors that affect the amount of solar radiation the earth's surface receives are the geographic region, time of day, time of year, local landscape and local climate condition [3]. Solarimeters are the instruments used to measure solar radiation [2]. Nigeria has the potential for a wide range of application of solar energy due to the availability of sunshine throughout the year, which can in turn impact positively on her economy. This is true because, every hour the earth receives more energy from the sun than is consumed by mankind in a year [4. 5] found that there is an estimated 3,000 hours of sunshine annually and on an average per day, Nigeria receives as high as 20 Ms/m2 of solar radiation, depending on the time of the year and location [6]. In the western region of Africa, Nigeria is located between latitude 4oN and 13o N and longitude 3oE and 15oE. An insight as to how a particular geographical location encounter variations in solar energy distribution, would surely lead one to discovering that the solar energy received in the states makes up Nigeria, possesses different meteorological data which accounts for these variations [1]. Though the measurement of solar radiation is not having total coverage for all locations in most developing nations such as Nigeria, meteorological indicators like sunshine hours, temperature, relative humidity and rainfall to name but a few, are use to extrapolate the solar energy reaching the earth's surface [7]. Knowing that for various states in Nigeria, there are varying solar radiation intensities, for instance, there is higher intensity of sun-rays in the Northern part compared to the southern part of Nigeria, which are the differences that were considered to improve the accuracy of the models.

In the research community, Autoregressive Moving Average (ARMA) methods are widely used time series models when compared with other models like Artificial Neural Network Models, Markov Chains, Fuzzy networks, etc. [8]. The ARMA model is able to extract the useful statistical properties of many regions, and can easily take on the well-known Box-Jenkins methods [9]. In addition, these models are very flexible; therefore, they can be used in various types of time series modelling with different orders. Finally, it offers a regular pervasiveness at individual phases (identification, estimation and diagnostic checks) for a suitable model. In ARMA models, one of the greatest difficulties is the need for enormous amount of data [10]. Forecasts are essential in monitoring solar systems, energy systems sizing, and optimization and utility applications. Utilities and independent system operators utilize forecast information to manage generation and distribution. Hypothetically, there is no stochasticity in solar irradiance; hence, deterministic models are frequently incorporated to model this dataset. At ground level, the achievement of seasonal ARIMA models are ascribed to their abilities to capture the stochastic component of the irradiance series due to the effects of the unstable atmospheric conditions [11]. Relative to other electricity generating sources, solar powered systems produces electricity that are more prone to instability, which suggests the challenges present when integrating solar energy into traditional electricity systems [12]. In the utilization of solar radiation, one of the critical difficulties is modeling solar radiation [13].Although accurate prediction of the mean solar radiation can be provided from various techniques proposed by professionals, the turbulence (volatility or heteroscedasticity) of solar radiation is often missing [14].

In this paper, some time series statistical tools that have been extensively utilized in finance and financial decisions were applied to solar energy so as to better estimate the mean and volatility (variations) in solar radiation received in Nigeria. Although countless researchers in Nigeria who are more of physicists and engineers have developed some good models for estimating global radiation, there is little or no attention on modelling and forecasting solar radiation using time series tools especially S/ARMA, GARCH models for mean and volatility of solar radiation series. Time is an important factor in virtually every aspect of life and human endeavours, which have made researchers from various works of life, explore all areas ranging from economy, business, archaeology, engineering, academia etc. As a result of this, time series analysis has grown to be relevant in all of these fields. Among the most effective approaches for analysing time series data is the model introduced by Box and Jenkins, Autoregressive Integrated Moving Average (ARIMA). For instance, in a study by [15], an ARIMA model was developed in MATLAB environment for simulating and forecasting the rainfall data for the study area Krisnanaga, India using the Box-Jenkins methodology. The rainfall data covered the period of 1971 to 2010, where the first thirty (30) years i.e. from 1971 to 2000 of the data was used for the model development and the remaining ten (10) years i.e. from 2001 to 2010 of the data was used to verify the developed model. From the study, it was found that the ARIMA model is suitable for forecasting monthly rainfall over the study area and further suggested that the model could be used for forecasting the monthly rainfall for up-coming years. Suitable solar data modeling and reliable forecasting of solar radiation is vital for design, performance forecast and monitoring of solar energy conversion systems. One category of models used effectively to achieve this are the short-memory Box-Jenkins seasonal/non-seasonal Autoregressive Integrated Moving Average (S/ARIMA) stochastic models [16, 17, 18].

Also, [19] applied Box-Jenkins method to average solar radiation data that covered the period of 31st May to 14th October, 2007 for Bangi, Malaysia and discovered that the non-seasonal autoregressive model of order 1 i.e. is adequate after using Ljung-Box statistics for diagnostic checking. In the study, they reported that there were missing measurements in the data on 4th to 8th of July, 5th, 6th and 15th of August and these were replaced with the value derived from the average of the data in the same week. Meanwhile, an analysis of the international variability of solar radiation and sunshine hours for Brazil was done by [20] to generate statistical parameters for model checking which was to be used as an input data for synthetic time series generation. The AR-1 was the suggested approach for monthly solar radiation synthesis time series generation with auto-correlation coefficient varying from 0.30 to 0.40 for the localities in the north of Brazil and 0 for the other regions.

Generally, it is well-known in time series analysis that the ARMA-GARCH models are used in finance for modeling the mean and volatility [21, 22], yet these models have not received much attention in the energy community except for wind-speed forecasting [23, 24, 25]. Recently, [14] conducted an empirical investigation of solar radiation series using ARMA-GARCH models. Representative dataset from two china stations were incorporated into six different ARMA-GARCH models to model and predict the mean and volatility of monthly time series which out-performed the traditional point forecasting models like the simple Artificial Neural Network (ANN), because ANN was a poorer model in dealing with volatility of solar radiation data. In their work, the results reported that the ARMA-GARCH (-M) models are effective in radiation series estimation. The remaining part of this paper is organised as follows. Section 2 reviews the general ARIMA and GARCH methodologies. Section 3 describes in details the representative meteorological sites under investigation. Section 4 uses the daily solar radiation time series from the sites to describe the appropriate ARIMA-GARCH models for estimating the mean and volatility that exist in the series. Finally, in Section 5, the summaries of the results from the study were given with a brief remark to conclude the paper.

2. Method

2.1 Foundations for ARMA Models

A stationary time series is said to be an autoregressive moving average process of order p and q written as ARMA (p, q), if it satisfies the difference,

$$S_t - \emptyset_1 X_{t-1} - \cdots - \emptyset_p S_{t-p} = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} \quad (1)$$

$\{S_t\}$ are the solar radiation series, $\{\varepsilon_t\}$ are white noise (shocks) for the solar radiation process and the coefficients $\emptyset's\ and\ \theta's$ are such that the model is stationary and invertible. For stationarity, the roots of $\Phi(B)$ must lie outside the unit circle i.e. $|B| > 1$ while the invertibility condition is that the roots of $\theta(B)$ must lie outside the unit circle

A general non-seasonal $ARIMA(p, d, q)$ *model is* $\Phi(B)\nabla(B)^d X = \theta(B)\epsilon$

where $\nabla(B) = I - B$.

$$\Phi(B) = 1 - \emptyset_1 B - \emptyset_2 B^2 - \ldots - \emptyset_p B^p$$

And

$$\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \ldots + \theta_q B^q$$

$$\Phi(B) X_t = \theta(B)\varepsilon_t \quad (2)$$

For non-stationary series $\{S_t\}$, [26] proposed that differencing of sufficient order d could make the series stationary. If the d^{th} difference denoted by $\{\nabla^d S_t\}$ satisfies (2) then $\{S_t\}$ is said to follow an autoregressive integrated moving average model of order p, d and q, denoted by ARIMA (p,d,q).

The Box-Jenkins procedure is concerned with fitting an ARIMA model to a data, which are of three parts: Identification, Estimation and Verification.

A popular way to choose p is by minimizing Akaike Information Criterion (AIC), introduced by [27, 28] defined as,

$$AIC = -2logL + 2k \qquad (3)$$

where k is the number of parameters estimated, (in the above case p). The optimal model order is determined by the value of k for which $AIC_{(k)}$ is minimum.

[29, 30] developed Bayesian Information Criterion (BIC) which is an extension of minimum AIC procedure defined as

$$BIC = (n-p-q)\ln[n\hat{\sigma}^2/(n-p-q)] + n\left(1+\ln\sqrt{2\pi}\right) + (p+q)\ln\left[\left(\sum_{t=1}^{n} X_t^2 - n\hat{\sigma}^2\right)\Big/(p+q)\right] \qquad (4)$$

where $\hat{\sigma}^2$ is the maximum likelihood estimate of the white noise variance. The BIC is a consistent order-selection criterion

1.2. Foundations for GARCH models

To model volatility in the series if it exists, Autoregressive Conditional Heteroscedasticity (ARCH) or Generalized ARCH models as suggested by [21] and [22] for univariate volatility can be used, having the following properties;

ARCH Model

$$r_t = \mu + \varepsilon_t \qquad (5)$$

where r_t is the return series (transformed solar radiation series), μ is a constant and ε_t is the random shock (error term) which is distributed as $\varepsilon_t = \sigma_t \Omega_t$ and $\{\Omega_t\}$ is a sequence of identically and independently distributed random variable with mean zero and variance unity. Then for $\alpha_0 > 0$ and $\alpha_i \geq 0\ (i > 0)$, the innovation is derived,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 \qquad (6)$$

The model in (6) is called $ARCH\ (p)$ model. Note, the distribution of Ω_t can be standard normal, standardizedstudent-t, generalized error distribution (GED) or skewed student-t distribution.

GARCH Model

The ARCH model of [21], conditional variance σ_t^2 is determined based on the dependencies among lags of the return series alone. In the GARCH model, lags of the conditional variance, $\sigma_{t-j}^2 (j > 0)$ are introduced to further remove the linear dependencies in the return series.

GARCH Specification

GARCH (p,q) Model is then specified as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \qquad (7)$$

Then for $\alpha_0 > 0, \alpha_i, \beta_j \geq 0\ (i,j > 0)$, and $\sum_{i,j=1}^{\max(p,q)} (\alpha_i + \beta_j) < 1$, the $GARCH\ (p,q)$ model in (7) can be parameterized by applying $a_t = \varepsilon_t^2 - \sigma_t^2$. Then we have,

$$\varepsilon_t^2 = \alpha_0 + \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i)\varepsilon_{t-i}^2 + a_t - \sum_{j=1}^{q} \beta_j a_{t-j} \qquad (8)$$

which is an ARMA representation of the squared residuals, ε_t^2

2. Solar Radiation Data from the Sites

The solar radiation dataset for this study were obtained from the Nigerian Meteorological Agency (NIMET), Oshodi, Lagos State office, Nigeria. The parameter made available was the solar radiation series measured in millilitres (ml) using the Gunn-Bellani Radiation Integrator as the instrument for reading the radiation in those stations. The representative sites under investigation were Ibadan, Sokoto, and Port Harcourt. The investigation periods were from the 1st of January 2011 to 31st of December, 2015 which covered daily observations within those periods. In order to understand the data, some basic statistical summaries like means, standard deviation, etc. were conducted on the data as seen in Table 3. From the table, on an average, Sokoto obviously has the highest intensity of solar radiation relative to the other stations. However, on an unusual day, Port Harcourt received 394.58W/m^2, which outshined that of Sokoto. The data distribution for the sites are negatively skewed save Port Harcourt and exhibits positive kurtosis except for Ibadan. Before using the dataset from the stations, a standard conversion was made from ml to watts per sq. meters (1 ml to 13.153 W/m^2). And the reason for the use of Gunn-Bellani Radiation Integrator relative to a Solarimeter for taking solar radiation readings was because the former was inexpensive and easy to use compared to the later. For computational purposes, R statistical programming software was used incorporated.

Table 1: Daily solar radiation readings for each site

DATE	Ibadan	Sokoto	Port H
01/01/2011	177.56	205.18	226.23
01/02/2011	136.79	228.86	140.74
01/03/2011	152.57	226.23	193.35
01/04/2011	142.05	240.7	194.66
01/05/2011	115.74	252.53	111.8

01/06/2011	132.84	210.44	142.05

Table 2: Daily solar radiation readings for each site for the last six days

DATE	Ibadan	Sokoto	Port H
12/26/2015	148.63	164.41	217.02
12/27/2015	157.83	215.71	21965
12/28/2015	165.73	242.01	261.74
12/29/2015	184.14	238.07	247.27
12/30/2015	174.93	260.43	210.44
12/31/2015	207.81	219.65	236.75

Table 3: Summary statistics of the solar radiation series for the sites

Sites	Mean	Std. Dev.	Min	Max	Skewness	Kurtosis
Ibadan	142.10	47.52	2.63	281.47	-0.46	-0.26
Sokoto	234.74	53.57	11.84	373.54	-1.06	1.53
Port. H	153.23	52.58	3.95	394.58	0.09	0.27

4.0. Results and Discussion

Figure 1 shows the time series plots of the solar radiation measured at each station to their respective years of observation. From the plot, there seems to be no trends and seasonality in the solar radiation throughout these years and also that there appears to be some kind of non-stationarity in the daily solar radiation for Sokoto. It is also worth noting, that the time plot does not adequately supply all needed information. After the solar radiation time series were converted and the time plots constructed, the next step was to perform a test for serial correlation within the series using autocorrelation function (ACF) and partial autocorrelation function (PACF) plots to have a visual display of its behaviour. All of these tests are the basic time series criteria that must be satisfied for a particular model to be appropriate for estimation and forecasting purposes. Augmented Dickey Fuller (ADF) Test of Table 4 reports a p-value that is less than 0.05 for Ibadan, Sokoto and Port Harcourt, therefore the null hypothesis for the presence of a unit root was rejected. This implies that the solar radiation for the three sites are stationary and need no differencing. However, a further test was conducted to validate the ADF's result due to the peculiarity of the radiation data. The p-values for the three sites after carrying out the test must be greater than 0.05. Kwatowski-Phillips-Schimdt-Shin (KPSS) test of Table 4 reports a p-value that is greater than 0.05, therefore the null hypothesis was rejected for Stationarity for only Ibadan and Port Harcourt which agrees with their respective ADF. The ADF test for Sokoto was in disagreement with the KPSS implying that the series must be differenced at least once to attain stationarity before it can be appropriate to fit the ARMA model for the series.

Table 4: Test for stationarity and normality of residuals for the sites

	Augmented Dickey Fuller			KPSS TEST			Residual Test (Box-Ljung Test)		
SITES	Lag	Value	p-value	Lag	Value	p-value	Value	d.f	p-value
IBADAN	12	-4.2782	0.01	9	0.22406	0.1	0.258	1	0.6111
SOKOTO	12	-5.4687	0.01	9	1.9286	0.01	13.68	20	0.8464
PORT HARCOURT	12	-4.5374	0.01	9	0.38057	0.08553	20.581	20	0.4222

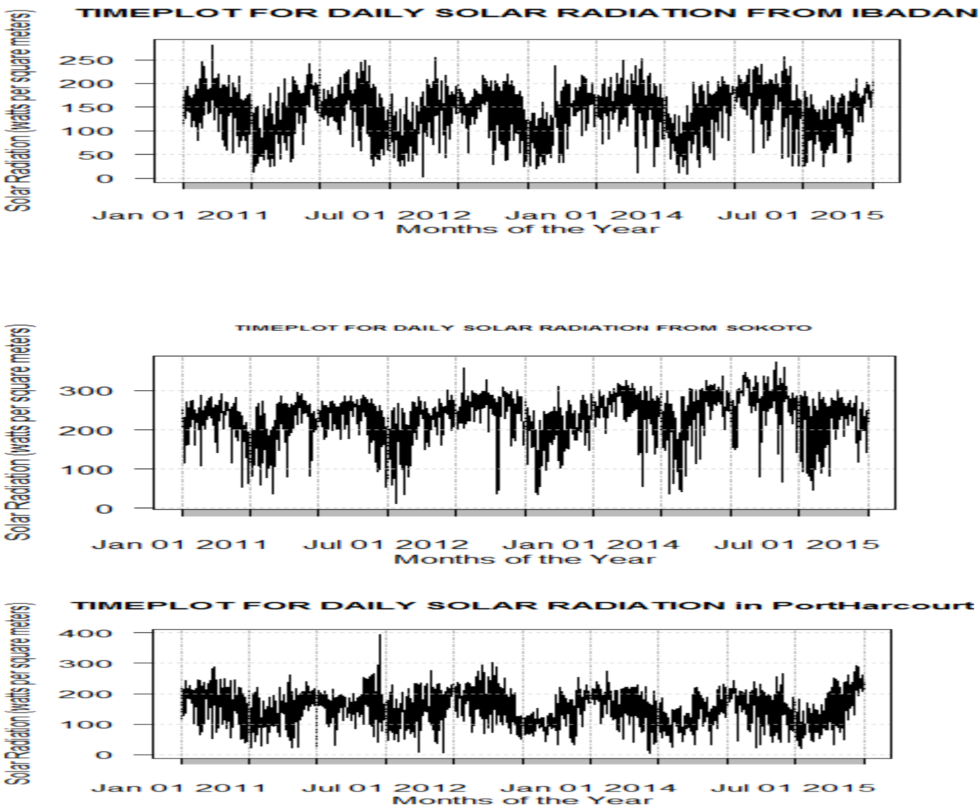


Figure 1: Time plots for Ibadan, Sokoto and Port Harcourt

Table 5: SARMA (2, 2) x (2, 2)$_7$ model for Solar Radiation at Ibadan

Ibadan Site Model: SARMA (2,2) x (2,2)$_7$

Coefficient	**AR1**	**AR2**	**MA1**	**MA2**	**SAR1**	**SAR2**	**SMA1**	**SMA2**	**Intercept**
	0.1001	0.8616	-0.0319	-0.8408	0.1001	0.8616	-0.0319	-0.8408	142.8656
Standard Error	0.0868	0.0848	0.0798	0.0739	0.0868	0.0848	0.0798	0.0739	9.4432

Sigma^2 estimated as 1437: Log likelihood = -9229.43 AIC =18478.87 BIC = 18533.97

The SARMA (2, 2) x (2, 2)$_7$ model for solar radiation from Ibadan site is;

$$S_{IBt} = 0.8616(S_{IBt-2} + S_{IBt-14}) - 0.7424S_{IBt-16} + \varepsilon_{IBt} + 0.8408(\varepsilon_{IBt-2} + \varepsilon_{IBt-14}) + 0.7069\varepsilon_{IBt-16}$$

where $\{S_{IBt}\}$ are the stationary time series for Ibadan Solar radiations, $\{\varepsilon_{IBt}\}$ are the white noise (or shocks) existing in the series.

Table 6: ARIMA (3, 1, 2) model for Solar Radiation at Sokoto

Port Harcourt Site	**ARIMA (1,0,2) with non-zero mean**			
	Ar1	**Ma1**	**Ma2**	**Mean**
Coefficient	0.9867	-0.9389	0.0562	156.1667
Standard Error	0.0049	0.0245	0.0243	9.0270

Sigma^2 estimated **as 2022:** log likelihood **=-9539.07** AIC**=19088.14** AICc **=19088.17** BIC = **19115.69**

Sokoto Site	**ARIMA (3,1,2) with non-zero mean**				
	AR1	**AR2**	**AR3**	**MA1**	**MA2**
Coefficient	-0.7362	0.2486	0.1233	-0.0012	-0.8207
Standard Error	0.0549	0.0399	0.0270	0.0502	0.0435

Sigma^2 estimated **as 1744:** log likelihood = **- 9413.94** AIC**=18839.88** AICc**= 18839.93** BIC **= 18872.94**

The ARIMA (3, 1, 2) model for solar radiation from Sokoto site is;

$$(S_{SKt} - S_{SKt-1}) = -0.7362(S_{SKt-1} - S_{SKt-2}) + 0.2486(S_{SKt-2} - S_{SKt-3}) + 0.1233(S_{SKt-3} - S_{SKt-4}) + \varepsilon_{SKt} - 0.8207\varepsilon_{SKt-2}$$

where $\{S_{SKt}\}$ are the stationary time series for Sokoto Solar radiations, $\{\varepsilon_{SKt}\}$ are the white noise (or shocks) existing in the series.

The ARMA (1, 2) or ARIMA (1, 0, 2) model for solar radiation from Port Harcourt site is;

$$S_{PHt} = 156.17 + 0.98675S_{PHt-1} + \varepsilon_{PHt} - 0.9389\varepsilon_{PHt-1} + 0.0562\varepsilon_{PHt-2}$$

where $\{S_{PHt}\}$ are the stationary time series for Port Harcourt solar radiation, $\{\varepsilon_{PHt}\}$ are the white noise (or shocks) existing in the series

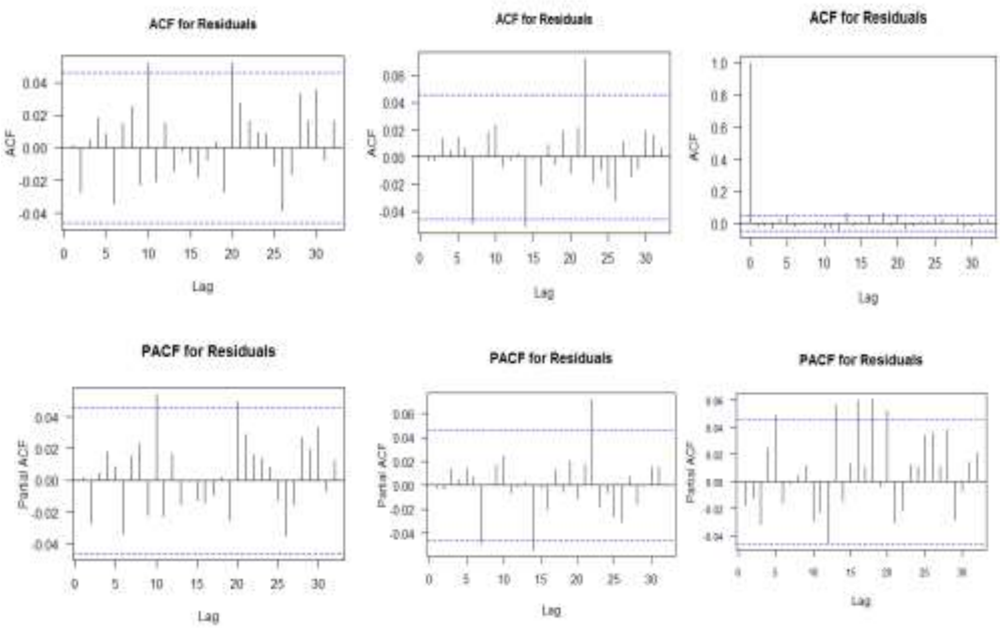
Having confirmed that the solar radiation for Ibadan, Port Harcourt and Sokoto (after 1st differencing) are stationary, the next step was to fit an appropriate Auto Regressive Moving Average (ARMA) model to the series for Ibadan and Port Harcourt, while for Sokoto, an Auto Regressive Integrated Moving Average Model (ARIMA) model was fitted, which gives the result as seen from Tables 5, 7 and 6 respectively.

Table 7: ARIMA (1, 0, 2) model for solar radiation at Port Harcourt

Table 8: GARCH (1, 1) Results for the Three Sites

	Coefficients			Jacque-Bera			Box-Ljung Test		
Sites	A_0	A_1	B_1	Chi-Squared	d.f	p-value	Chi-Squared	d.f	p-value
Ibadan	41.3538	0.0425	0.9293	158.9	2	<2.2e-16	0.004688	1	0.9454
Sokoto	71.1581	0.0770	0.8837	1768.1	2	<2.2e-16	0.1617	1	0.6876
Port Harcourt	74.2488	0.0845	0.8799	78.197	2	< 2.2e-16	0.1876	1	0.6649

Figure 2: ACF and PACF plots for the residuals of the sites Port Harcourt, Sokoto and Ibadan respectively



The ACF plot for the residuals of Ibadan displayed above (Figure 2) suggests that there is no significant autocorrelation which implies that the model is a good fit, meanwhile the ACF plots for Port Harcourt and Sokoto shows some significant lags. Further confirmation was carried out via Box-Ljung test (Table 4). The null hypothesis states that the autocorrelation is not different from 0. The Box-Ljung test with a reported p-value greater than 0.05 for Ibadan, Port Harcourt and Sokoto implies that the null hypothesis of insignificant autocorrelations will not be rejected. Also, the model must follow Normal distribution with mean zero and a constant variance. Squared residuals plot in Figure 3 shows volatility clustering at some points in time. Since the ACF and PACF of the squared residuals for all sites displays some significant lags, it implies that volatility can be modeled for average solar radiation in these sites because there exists a strict white noise which are independent with zero mean and normally distributed. The residuals show some patterns that might be modeled. To implement this, the GARCH method was used to model the conditional variance of the series. The p-values (Table 8) for all the parameters are less than 0.05, indicating statistical significance. In addition, the p-value of Box-Ljung test is greater than 0.05, and so the null hypothesis that the autocorrelation of the residuals is different from 0, will not be rejected. The model therefore adequately represents the residuals. These conclusions are suitable for all the sites (Ibadan, Port Harcourt and Sokoto) under investigation.

The GARCH (1, 1) models for Ibadan, Port Harcourt and Sokoto respectively are as follows;

$$\sigma_{IBt}^2 = 41.354 + 0.043\varepsilon_{IBt-1}^2 + 0.929\sigma_{IBt-1}^2 \quad (9)$$

$$\sigma_{PHt}^2 = 74.249 + 0.0845\varepsilon_{PHt-1}^2 + 0.8798\sigma_{PHt-1}^2 \quad (10)$$

$$\sigma_{SKt}^2 = 71.158 + 0.077\varepsilon_{SKt-1}^2 + 0.8837\sigma_{SKt-1}^2 \quad (11)$$

The Mixed ARIMA-GARCH Models are:

For Ibadan: - SARMA (2, 2) x $(2, 2)_7$ + GARCH (1, 1)

$$S_{IBt} + \sigma_{IBt}^2 = 0.8616(S_{IBt-2} + S_{IBt-14}) - 0.7424S_{IBt-16} + \varepsilon_{IBt} + 0.0319(\varepsilon_{IBt-2} + \varepsilon_{IBt-14}) + 0.001\varepsilon_{IBt-16} + 41.354 + 0.043\varepsilon_{IBt-1}^2 + 0.929\sigma_{IBt-1}^2 \quad (12)$$

For Sokoto: - ARIMA (3, 1, 2) + GARCH (1, 1)

$$(S_{SKt} - S_{SKt-1}) + \sigma_{SKt}^2 = -0.7362(S_{SKt-1} - S_{SKt-2}) + 0.2486(S_{SKt-2} - S_{SKt-3}) + 0.1233(S_{SKt-3} - S_{SKt-4}) + \varepsilon_{SKt} - 0.0012\varepsilon_{SKt-1} - 0.8207\varepsilon_{SKt-2} + 71.158 + 0.077\varepsilon_{SKt-1}^2 + 0.8837\sigma_{SKt-1}^2 \quad (13)$$

For Port Harcourt: - ARMA (1, 2) + GARCH (1, 1)

$$S_{PHt} + \sigma_{PHt}^2 = 0.9867S_{PHt-1} + \varepsilon_{PHt} - 0.9389\varepsilon_{PHt-1} + 0.0562\varepsilon_{PHt-2} + 74.249 + 0.0845\varepsilon_{PHt-1}^2 + 0.8798\sigma_{PHt-1}^2 \quad (14)$$

Tables 9, 10 and 11 are the forecast for solar radiation in Port Harcourt, Sokoto and Ibadan respectively, for first week of the new year 2016 using only their single AR(*I*)MA models which neither considers volatility or reflect changes as new information are available but focuses only on analysing time series data linearly. In other words, the mixed model will consider modeling the noise existing in the ARMA based on the conditional variances as seen in last column of the tables. Looking at the tables, it is no doubt that both models have a conflicting or overlapping forecasts and variations relative to the actual radiations and there is little or no significant variations among the models. Furthermore from figures 4, 5, and 6, the 95% confidence intervals effectively captures the actual radiations for the first week of the year, which has the potential to capture the remaining part of the year considerably. Though, the forecast for the first week looks linear, however, when the length of the forecast is increased, the fluctuation surfaces. The Figures 4, 5, 6 help to visualize the pattern of the forecast for the solar radiations received at the sites.

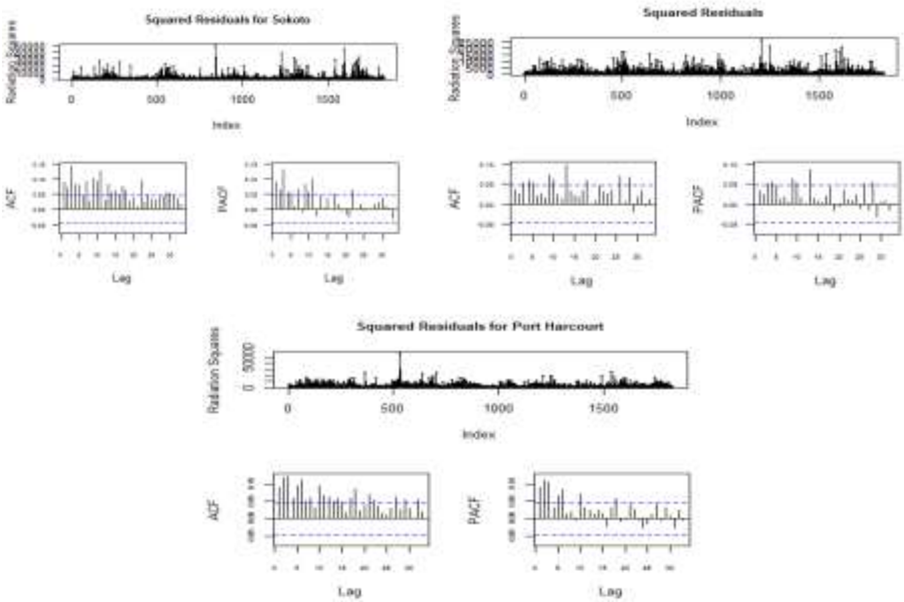


Figure 3: Shows the Squared Residual Plots with their ACF and PACF for

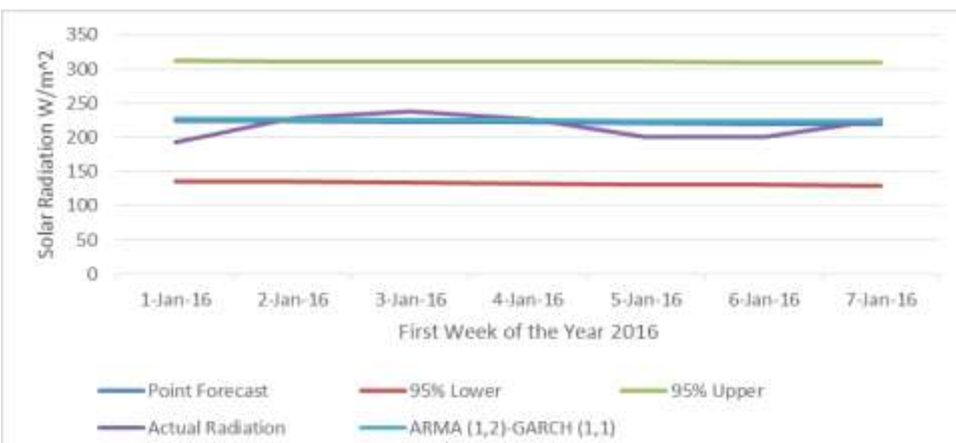


Figure 4: One-Week-Ahead Forecast for Port Harcourt

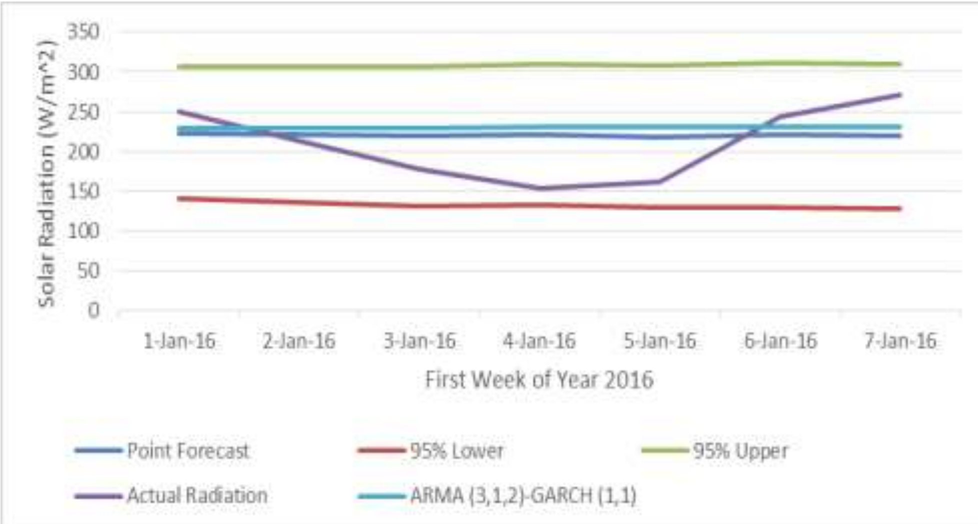


Figure 5: One-Week-Ahead Forecast for Sokoto

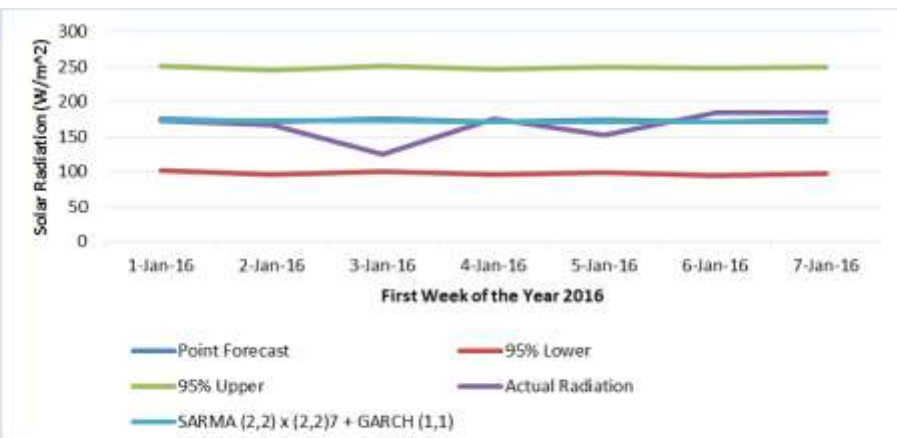


Figure 6: One-Week-Ahead Forecast for Ibadan

Table 9: One-Week ahead forecast for 2016 in Port Harcourt

Day	Point Forecast	95% Lower	95% Upper	Actual Radiation	ARMA (1,2)-GARCH (1,1)	Absolute Error (Single)	Absolute Error (Mixed)
01-Jan-16	223.788	135.658	311.918	193.35	227.2	30.438	33.85
02-Jan-16	223.554	135.324	311.784	228.36	226.6	4.806	1.76
03-Jan-16	222.661	133.961	311.361	238.07	226.0	15.409	12.07
04-Jan-16	222.779	132.625	310.934	227.54	225.3	4.761	2.24
05-Jan-16	220.909	131.314	310.505	201.24	224.7	19.669	23.46
06-Jan-16	220.051	130.029	310.073	199.92	224.0	20.131	24.08
07-Jan-16	219.204	128.768	309.64	226.23	223.4	7.026	2.83

Table 10: One-Week ahead forecast for 2016 in Sokoto

Day	Point Forecast	95% Lower	95% Upper	Actual Radiation	ARMA (3,1,2)-GARCH (1,1)	Absolute Error (Single)	Absolute Error (Mixed)
01-Jan-16	223.462	140.918	306.007	249.90	229.2	26.438	20.7
02-Jan-16	221.763	136.42	307.106	213.08	229.5	8.683	16.42
03-Jan-16	218.935	131.445	306.425	177.56	229.8	41.375	52.24
04-Jan-16	221.065	132.109	310.021	153.89	230.1	67.175	76.21
05-Jan-16	218.585	129.032	308.137	161.78	230.4	56.805	68.62
06-Jan-16	220.591	130.03	311.152	243.33	230.7	22.739	12.63
07-Jan-16	218.76	127.72	309.8	270.95	230.9	52.19	40.05

Table 11: One-Week ahead forecast for 2016 in Ibadan

Day	Point Forecast	95% Lower	95% Upper	Actual Radiation	SARMA (2,2) x $(2,2)_7$ + GARCH (1,1)	Absolute Error (Single)	Absolute Error (Mixed)
01-Jan-16	176.458	102.162	250.753	172.3	172.9	4.158	0.6
02-Jan-16	171.104	96.120	246.088	167.04	172.6	4.064	5.56
03-Jan-16	175.477	100.361	250.592	124.95	172.2	50.527	47.25
04-Jan-16	171.393	95.689	247.097	176.25	171.9	4.857	4.35
05-Jan-16	174.637	98.76	250.515	152.57	171.6	22.067	19.03
06-Jan-16	171.511	95.116	247.906	184.14	171.3	12.629	12.84
07-Jan-16	173.901	97.299	250.504	184.14	171.0	10.239	13.14

5. Conclusion

It was observed that, the proposed model which closely mimics the solar radiation received in Ibadan, Sokoto and Port Harcourt are, the seasonal ARMA $(2,2)(2,2)_7$, ARIMA (3,1,2), and the combined ARMA (1, 2)-GARCH (1, 1) models respectively. The single models for Ibadan and Sokoto have no significant differences relative to their GARCH combinations when used for forecasting one-week ahead, implying that they are more suitable models due to the fluctuating patterns they exhibit. Meanwhile, the model for Port-Harcourt made provision for variations (or volatility) that exist in the surface radiation compared to the single ARMA (1, 2) model which only focuses on the linearity of the radiation time series. From the one-week ahead forecast, it was observed that as the day increases, both models follow a consistent decreasing pattern relative to the actual values. It is important to recall the mathematical expression of the suggested models as follows;

The seasonal ARMA (2, 2) x $(2, 2)_7$ model for solar radiation from Ibadan site is;

$$S_{IBt} = 0.8616(S_{IBt-2} + S_{IBt-14}) - 0.7424S_{IBt-16} + \varepsilon_{IBt} + 0.8408(\varepsilon_{IBt-2} + \varepsilon_{IBt-14}) + 0.7069\varepsilon_{IBt-16}$$

The ARIMA (3, 1, 2) model for solar radiation from Sokoto site is;

$$(S_{SKt} - S_{SKt-1}) = -0.7362(S_{SKt-1} - S_{SKt-2}) + 0.2486(S_{SKt-2} - S_{SKt-3}) + 0.1233(S_{SKt-3} - S_{SKt-4}) + \varepsilon_{SKt} - 0.8207\varepsilon_{SKt-2}$$

The Mixed ARMA-GARCH Model: - ARMA (1, 2) + GARCH (1, 1)

$$S_{PHt} + \sigma^2_{PHt} = 0.9867S_{PHt-1} + \varepsilon_{PHt} - 0.9389\varepsilon_{PHt-1} + 0.0562\varepsilon_{PHt-2} + 74.249 + 0.0845\varepsilon^2_{PHt-1} + 0.8798\sigma^2_{PHt-1}$$

It can therefore, be safely recommended that, the above models are adequate enough to forecast the solar radiation for Ibadan, Sokoto and Port Harcourt, which is an integral part in the application of solar energy and systems in the energy sector of the economy.

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