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Position and Trajectory Tracking Control for the Ball and Plate System using Mixed Sensitivity Problem

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Abstract: This paper presents the position and trajectory tracking control scheme for the ball and plate system (BPS) using the double feedback loop structure (a loop within a loop) for effective control of the system. The inner loop was designed using linear algebraic method by solving a set of Diophantine equations. The outer inner loop was designed using sensitivity approach. Simulation results showed that the plate was stabilized at 0.3546 seconds, and the ball was able to settle at 1.7087 seconds, when given a circular trajectory of radius 0.4 m with an angular frequency of 1.57 rad/sec, with a trajectory tracking error of 0.0095 m, which shows that the controllers have adaptability, strong robustness and control performance for the ball and plate system.

Keywords: Ball and plate system (BPS), linear algebraic method, Diophantine equation, controller, mixed sensitivity problem.

1. Introduction

The ball and plate system (BPS) problem is a benchmark for testing of control algorithms. The BPS is one of the most enduringly popular and important laboratory models for teaching of the control systems engineering (Oravec et al., 2015; Zeeshan et al., 2012). It is amongst the most well-known and challenging test platform for the control engineers. Example of such systems include the ball and beam system (BBS), traditional cart-pole system (inverted pendulum), double and multiple inverted pendulums (Cheng & Tsai, 2016; Mohajerin et al., 2010). The BBS is a well-known problem for nonlinear control where the system is under-actuated and has two degree of freedom (DOF), while the BPS can be considered as an extension of the BBS, consisting of a ball that can roll freely on a plate. Therefore, the BPS has four DOF, and it is more complicated due to the coupling between the variables (Kassem et al., 2015). This under-actuated system has only two actuators, which is stabilized by just two control inputs (Borah et al., 2017; Ghiasi & Jafari, 2012). The system consist of a plate pivoted at its center such that the slope of the plate can be manipulated in two perpendicular directions (Dong et al., 2011). However, since the movements of the ball can reach high speeds, the design of a suitable controller for the BPS is a major challenge (Galvan-Colmenares et al., 2014). The BPS is made of a small ball and plate with two mutually perpendicular axis of rotation (Fei et al., 2011).

A servo system consist of motor controller card and two servo motors to tilt the plate. Intelligent vision system is used for measurement of a ball position from a CCD camera. The problem of the motion control of this system is to control the position of a ball on a plate for both static positions and desired paths. The slope of the plate can be manipulated in two perpendicular directions, so that the tilting of the plate will make the ball move on the plate (Dong et al., 2009; Dong et al., 2011).

The system has demonstrated various controller design methods for positioning and trajectory tracking of the ball; proportional integral derivative (PID) control, fuzzy control, neural network control and model predictive control (Mochizuki & Ichihara, 2013).

The system finds application in areas like humanoid robot, satellite control and unmanned aerial vehicle (UAV) (Mukherjee et al., 2002) in the field of path planning, trajectory tracking and friction compensation (Oriolo & Vendittelli, 2005).

However, various control methods have been introduced in the recent years for the BPS. A controller design for two dimensional electro-mechanical ball and plate system that was based on the classical and modern control theory was proposed by (Knuplež et al., 2003). In the work of (Hongrui et al., 2008), the position of the ball was regulated with a double feedback loop system, in which recursive back-stepping design was employed for the external loop, while switching control scheme was employed for the inner loop. (Farooq et al., 2013) designed a simple interval type-2 fuzzy gain scheduling controller for the stabilization and reference tracking of the BPS. The developed controller was compared with pole placement and type-1 fuzzy logic controller, in which it outperformed the two other controllers with respect to settling time, percentage overshoot and disturbance rejection capability.

In the work of (Lin et al., 2014) , a controller was designed which ensured the BPS stability by employing a loop shaping method based on Normalized Right Coprime Factor (NRCF) perturbation model, in which the well-known lead-lag series compensation design methods were innovatively adopted to obtain appropriate pre and post compensators as the weighting functions to guarantee the BPS time domain performance requirements. A unique motion controller, based on the evolved lookup tables have been developed by (Beckerleg & Hogg, 2016) in order to move a ball on a set-point on a typical BPS. Also, in order to overcome the problem of under-actuation, instability and nonlinearity, which is attributed to the BPS.

However, (Roy et al., 2016a) presents a comparative study between a cascaded fractional order sliding mode controller (FOSMC) and sliding mode controller (SMC) for trajectory control of a ball in a BPS, the FOSMC was designed by choosing a fractional order sliding surface. The proposed control strategies was experimentally validated on a ball and plate laboratory setup (Feedback Instrument Model No. 033-240), simulation and experimental studies showed that the FOSMC outperformed the SMC in terms of tracking accuracy, speed of response, chattering effect and energy efficiency. Also, a cascaded SMC was proposed by (Roy et al., 2016b) for position control of a ball in a BPS. The effectiveness of the proposed controller was tested through simulation studies by making the ball follow a circular and a square shaped trajectories, and the effect of undesirable phenomenon of chattering associated with SMC was found within satisfactory limit. And (Das & Roy, 2017) dealt with a comparative analysis of the performance of SMC and FOSMC when applied to the problem of trajectory control of ball in a BPS. The two control algorithm were simulated on MATLAB-Simulink environment, and experimental validation was later carried out on a BP laboratory setup (Feedback Instrument Model No. 033-240). Simulation and experimental results conveyed that FOSMC performs better than SMC in terms of speed of response and tracking accuracy without increasing the level of chattering and control effort. Also, (Cheng & Tsai, 2016) presented a skillful robotic wrist system using a visual control technique to demonstrate dexterity of the mechanical wrist from the viewpoint of the table tennis. Intelligent control algorithm using Linear Quadratic Regulator (LQR) approach was developed to adjust the plate's altitude to guide the ball to a specific position to achieve given balancing tasks.

(Debono & Bugeja, 2015) proposed and examined the application of sliding mode control (SMC) to the control problem of the ball and plate. The linear full-state feedback controller was compared with the SMC, and the performance between the SMC and linear full-state feedback controller was tested experimentally on a designed and constructed physical test bed that was meant for the purpose of the research.

One of the most effective control scheme of the BPS is the double feedback loop structure, i.e. a loop within a loop. The inner loop serves as a dc motor servo position controller, while the outer loop controls the position of the ball (Liu & Liang, 2010).

In this paper, the ball and plate system is considered as a double feedback loop structure for position and trajectory tracking control. The outer feedback loop is used regulate the ball's position on the plate, and this is based on controller. The inner feedback loop, which is being controlled by a servo controller, drives the plate's slope to follow the reference position, which will be designed based on linear algebraic method by solving a set of Diophantine equations. The rest of the paper is organized as follows. Section 2 introduces the modelling of the BPS, also the design of the inner and outer loop controller is discussed in section 3. However, section 4 shows the simulation results and the trajectory tracking control of the ball on the plate. And finally, section 5 presents the conclusion.

2.Mathematical Modelling

Figure 1 shows a typical laboratory model of the BPS by HUMUSOFT

Figure 1. The ball and plate system (bps) by humusoft (humusoft ltd., 2012a)

Figure 2 shows the schematic model diagram of the BPS.

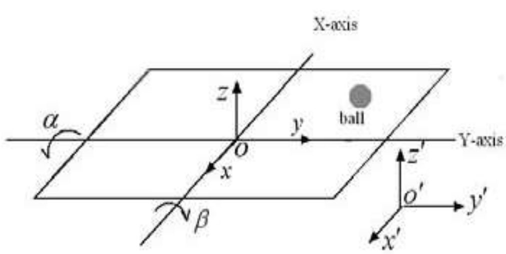

Figure 2. The schematic model for the ball and plate system

The plate rotates around the x and y-axis in two perpendicular directions. The kinematic differential equations of the BPS are obtained using the Euler-Lagrange equation given as (Morales *et al.*, 2017; Yıldız & Gören-Sümer, 2017):

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \tag{1}$$

Where q_i stands for the i^{th}-direction coordinate, T is the kinetic energy of the system, V is the potential energy of the system, and Q is the composite force.

The BPS can be simplified into a system made up of two rigid bodies; the geometry of the plate has limits in translation along x, y and z-axis. It also has a geometry limit in rotation about its z-axis. The plate has two degree of freedom (DOF) in rotation about the x and y-axis. The ball's geometry has a limit in translation along the z-axis. It also has two DOF along the x and y-axis respectively. The BPS system model has four DOF, in which the generalized coordinates are (Hongrui *et al.*, 2008):

$q_1 = x$, $q_2 = y$, $q_3 = \alpha$, $q_4 = \beta$.

The total kinetic energy of the BPS is given as:

$$T = T_{ball} + T_{plate} \tag{2}$$

$$T_{ball} = \frac{1}{2}\left[\begin{array}{l}\left(m_b + \frac{J_b}{R_b^2}\right)\left(\dot{x}^2 + \dot{y}^2\right) + J_b\left(\dot{\alpha}^2 + \dot{\beta}^2\right) \\ \qquad + m_b\left(x\dot{\alpha} + y\dot{\beta}\right)^2\end{array}\right] \tag{3}$$

$$T_{plate} = \frac{1}{2}\left[\begin{array}{l}\left(J_{Px}\dot{\alpha}^2 + J_{py}\dot{\beta}^2\right) + \left(m_b + \frac{J_b}{R_b^2}\right)\left(\dot{x}^2 + \dot{y}^2\right) \\ \qquad + J_b\left(\dot{\alpha}^2 + \dot{\beta}^2\right) + m_b\left(x\dot{\alpha} + y\dot{\beta}\right)^2\end{array}\right] \tag{4}$$

The potential energies of the BPS along the x and y-axis is given as:

$$V_x = m_b g x \sin\alpha \tag{5}$$

$$V_y = m_b g y \sin\beta \tag{6}$$

And the mathematical equation of the BPS is given as:

$$\left(m_b + \frac{J_b}{R_b^2}\right)\ddot{x} - m_b x\left(\dot{\alpha}\right)^2 - m_b y\dot{\alpha}\dot{\beta} + m_b g\sin\alpha = 0 \tag{7}$$

$$\left(m_b + \frac{J_b}{R_b^2}\right)\ddot{y} - m_b y\left(\dot{\beta}\right)^2 - m_b x\dot{\alpha}\dot{\beta} + m_b g\sin\alpha = 0 \tag{8}$$

$$\left(m_b x^2 + J_b + J_{Px}\right)\ddot{\alpha} + 2m_b x\dot{x}\dot{\alpha} + m_b xy\ddot{\beta} + m_b\left(\dot{x}y + x\dot{y}\right)\dot{\beta} + m_b g x\cos\alpha = \tau_x \tag{9}$$

$$\left(m_b y^2 + J_b + J_{Py}\right)\ddot{\beta} + 2m_b y\dot{y}\dot{\beta} + m_b xy\ddot{\alpha} + m_b\left(\dot{x}y + x\dot{y}\right)\dot{\alpha} + m_b g y\cos\beta = \tau_y \tag{10}$$

$m_b\left(kg\right)$ represents the mass of the ball, $J_b\left(kgm^2\right)$ is the rotational moment of inertia of the ball, $J_{P_x}\left(kgm^2\right)$ and $R_b\left(m\right)$ are the rotational moment of inertia of the plate and the radius of the ball; $x\left(m\right)$ and $y\left(m\right)$ gives the position of the ball along the x and y-axis; $\dot{x}\left(m/s\right)$ and $\ddot{x}\left(m/s^2\right)$ are the velocity and acceleration along the x-axis; $\dot{y}\left(m/s\right)$ and $\ddot{y}\left(m/s^2\right)$ gives the velocity and acceleration along the y-axis; $\alpha\left(rad\right)$ and $\dot{\alpha}\left(rad/\text{sec}\right)$ gives the plate deflection angle, and angular velocity along x-axis; $\beta\left(rad\right)$ and $\dot{\beta}\left(rad/\text{sec}\right)$ gives the plate deflection angle and angular velocity along y-axis; and $\tau_x\left(Nm\right)$ and $\tau_y\left(Nm\right)$ gives the torques on the plate in the x and y-axis.

Equation (7) and (8) describes the ball's movement on the plate; it also shows how the effect of the ball's acceleration relies on the plate's angular deflection and its angular velocity. However, equation (9) and (10) describes how the dynamics of the plate's deflection rely on the external forces driving it, and the ball's position (Duan *et al.*, 2009).

Considering the state variable assignment of the BPS (Fan *et al.*, 2004):

$$X = \left(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\right)^T = \left(x, \dot{x}, \alpha, \dot{\alpha}, y, \dot{y}, \beta, \dot{\beta}\right)^T \tag{11}$$

And the state space equation of the BPS is as follows (Fan *et al.*, 2004):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} x_2 \\ B\left(x_1 x_4{}^2 + x_4 x_5 x_8 - g\sin x_3\right) \\ x_4 \\ 0 \\ x_6 \\ B\left(x_5 x_8{}^2 + x_1 x_4 x_8 - g\sin x_7\right) \\ x_8 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix} \quad (12)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} [X] \quad (13)$$

$$B = \frac{m_b}{\left(m_b + \frac{J_b}{R_b{}^2}\right)} \quad (14)$$

In the steady state, the plate should be at a position that is horizontal, where both the inclination angles of the x and y-axis are equal to zero, if the inclination angle of the plate does not have much change, i.e. $\pm 5^0$, then, the sine function can be substituted by its argument (Fabregas *et al.*, 2017). The mathematical model of the BPS can be simplified and decomposed into x and y-axis as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ B\left(x_1 x_4{}^2 - g\sin x_3\right) \\ x_4 \\ 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [u_x] \quad (15)$$

$$\begin{bmatrix} \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} x_6 \\ B\left(x_5 x_8{}^2 - g\sin x_7\right) \\ x_8 \\ 0 \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [u_y] \quad (16)$$

3.Controller Design

Determination of the Actuator Parameters

The actuator with a permanent DC motor is considered in the inner loop design. Also, the relationship between θ_L and e_a is given as (Golnaraghi & Kuo, 2010):

$$\frac{\theta_L}{e_a} = \frac{K_t \frac{N_1}{N_2}}{\left[J_{eq} L_a s^3 + \left(J_{eq} R_a + D_{eq} L_a\right)s^2 + \left(D_{eq} R_a + K_t K_e\right)s\right]} = T_L \quad (17)$$

The parameters of the DC motors are derived based on the requirements of the load torque, the speed of the motor, and the moment of inertia. This is given in Table I.

Table 1. BPS System Parameters (Humusoft Ltd., 2012b)

S/N	Description	Symbol	Unit	Value
1	Mass of the ball	m	kg	0.11
2	Radius of the ball	R	m	0.02
3	Dimension of the plate (square)	lxb	m^2	0.16
4	Mass moment of inertia of the plate	$J_{Px,y}$	kgm^2	0.5
5	Mass moment of inertia of the ball	J_b	kgm^2	1.76e-5

6	Maximum velocity of the ball	V	m/s	0.04

Equation (17) is given as (Umar, 2017):

$$\frac{\theta_L}{e_a} = \frac{0.105}{\left[0.47005s^2 + 421.113s\right]}$$
(18)

$$= \frac{0.2234}{s(s+895.89)} \approx \frac{2.49x10^{-4}}{s}$$
(19)

3.2 *Two-Port Parameter Configuration*

The two-port parameter configuration was used in the design of the inner loop. To find the value of ω_o, a step response is required, which could settle in 0.4 seconds. From the Humusoft ball and plate system manual, through simulation, $\omega_o = 20\,rad/\text{secs}$ was found to give the steady state response. The ITAE optimal overall transfer function with zero position error of the system, which is $G_0(s)$ is (Chen, 1995):

$$G_0(s) = \frac{\omega_0^{\;2}}{s^2 + 1.4\omega_0 s + \omega_0^{\;2}} \qquad (20)$$

An additional gain of 494 was provided to limit the step response using a preamplifier. $G_0(s)$ is implemented as shown in Figure 3 using the two-port configuration

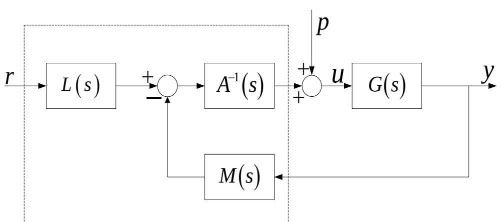

Figure 3. Two-port parameter configuration

Where $L(s)$, $M(s)$ and $A(s)$ are the polynomials that defines the compensator, p which is the input disturbance. Solving the Diophantine equation, the compensator and the DC motor actuator has the following values (Umar, 2017):

$$A(s) = 28 + s$$
(21)

$$M(s) = 3252 + s$$
(22)

$$L(s) = 3252$$
(23)

3.3 H_∞ Controller

The augmented plant model for H_∞ controller can be constructed as (Dingyu *et al.*, 2007):

$$P(s) = \left[\begin{array}{c:cc} A & B_1 & B_2 \\ \hdashline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$
(24)

With the augmented state space description as follows:

$$\dot{x} = Ax + \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (25)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (26)$$

Straightforward manipulations gives the following closed loop transfer function:

$$Ty_1u_1(s) = P_{11}(s) + P_{12}(s)\left[I - F(s)P_{22}(s)\right]^{-1} F(s)P_{21}(s) \qquad (27)$$

The above expression is also known as the linear fractional transformation (LFT) of the interconnected system. The objective of robust control is to find a stabilizing controller.

$$u_2(s) = F(s)\,y_2(s) \qquad (28)$$

Such that

$$\|Ty_1u_1\|_\infty < 1 \qquad (29)$$

The design objective is to find a robust controller $F_c(s)$ guaranteeing the closed-loop system with an H_∞-norm bounded by a given positive number γ, i.e. (Dingyu *et al.*, 2007):

$$\| Ty_1u_1 \|_\infty < \gamma \qquad (30)$$

Then the controller can be represented by (Dingyu *et al.*, 2007)

$$F_c(s) = \left[\begin{array}{c|c} A_f & -ZL \\ \hline F & 0 \end{array}\right] \qquad (31)$$

Where:

$$A_f = A + \gamma^{-2} B_1 B_1^T X + B_2 F + ZLC_2$$

$$F = -B_2^T X \qquad (33)$$

$$L = -YC_2^T \qquad (34)$$

$$Z = \left(I - \gamma^{-2} YX\right)^{-1} \qquad (35)$$

X and Y are, respectively the solutions of the following two Algebraic Riccati Equations (AREs) (Dingyu *et al.*, 2007):

$$A^T X + XA + X\left(\gamma^{-2} B_1 B_1^T - B_2 B_2^T\right) X + C_1 C_1^T = 0 \qquad (36)$$

$$AY + YA^T + Y\left(\gamma^{-2} C_1^T C_1 - C_2^T C_2\right) Y + B_1^T B_1 = 0 \qquad (37)$$

The conditions for the existence of an H_∞ controller are as follows (Dingyu *et al.*, 2007):

1) D_{11} is small enough such that $D_{11} < \gamma$
2) The solution X of the controller ARE is positive-definite;
3) The solution Y of the observer ARE is positive-definite;
4) $\lambda_{\max}(XY) < \gamma^2$, which indicate that the eigenvalues of the product of the two Riccati equation solution matrices are all less than γ^2.

In the design of the optimal H_∞ controller, the optimal criterion is defined as (Dingyu *et al.*, 2007):

$$\max_{\gamma} \| Ty_1u_1 \|_\infty < \frac{1}{\gamma} \qquad (38)$$

3.4 H_∞ Mixed Sensitivity Problem

In the design of the H_∞ optimal control, using the mixed sensitivity problem, the weighting functions which are $\mathrm{W}_1(\mathrm{s})$, $\mathrm{W}_2(\mathrm{s})$ and $\mathrm{W}_3(\mathrm{s})$ are used for shaping the plant model $G(s)$. The weighting function $\mathrm{W}_1(\mathrm{s})$, penalizes the error signal, $\mathrm{W}_2(\mathrm{s})$ penalizes the input signal and $\mathrm{W}_3(\mathrm{s})$ penalizes the output signal (Hossain, 2007). This is shown in Figure 4.

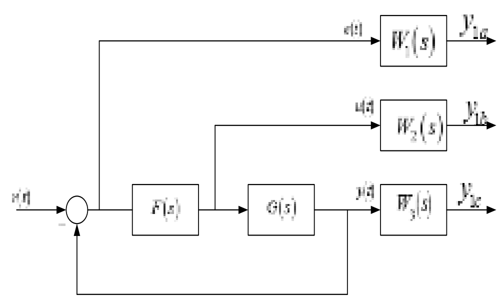

Figure 4. Mixed sensitivity problem

The augmented plant model $P(s)$ is written as:

$$P(s) = \left[\begin{array}{c|c} W_1 & -W_1 G \\ 0 & W_2 \\ 0 & W_3 G \\ \hline I & -G \end{array}\right] \qquad (39)$$

The linear fractional transformation (LFT) of the mixed sensitivity problem Ty_1u_1, the sensitivity transfer function

$S(s)$ and the complementary sensitivity transfer function $T(s)$ are given as:

$$Ty_1u_1 = \begin{bmatrix} W_1S \\ W_2FS \\ W_3T \end{bmatrix} \quad (40)$$

$$S(s) = \left[1 + F(s)G(s)\right]^{-1} \quad (41)$$

$$T(s) = 1 - S(s) = F(s)G(s)\left[1 + F(s)G(s)\right]^{-1} \quad (42)$$

$F(s)$ is the controller, $S(s)$ is the sensitivity transfer function and $T(s)$ is the complementary transfer function.

3.5 Determination of the H_∞ Controller

The weighting functions $W_1(s)$, $W_2(s)$ and $W_3(s)$ for the control of the system were selected after extensive simulation and fine tuning as:

$$W_1(s) = \frac{100\left(1.5s^2 + 12.64s + 18.49\right)}{100\left(1.5s^2 + 103.2s + 18.49\right)} \quad (43)$$

$$W_2(s) = 1 \quad (44)$$

$$W_3(s) = \frac{s^2}{100} \quad (45)$$

4. Simulation Results

The following simulations results were obtained from MATLAB 2017a software.

4.1 Inner Loop Design

The step response of the actuator is shown in Figure 5.

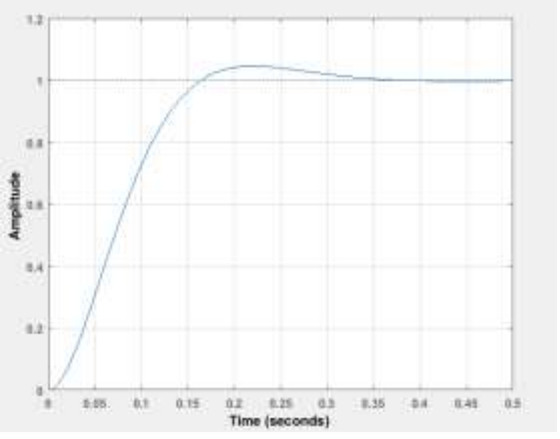

Figure 5. Step response of the actuator

Table II: shows the Properties of the Actuator.

System Response	Value
Settling Time (sec)	0.2989
Overshoot (%)	4.5989

From Table II, the settling time of the actuator is 0.2989 seconds, which shows that the plate will settle before 0.4 seconds that was set for it. However, the actuator $U(s)$ due to a step input should not exceed the rated DC motor voltage of the actuator, which is 75V. Figure 6 shows the step response of the actuator with open parameters (rated power and voltage) of the DC motor.

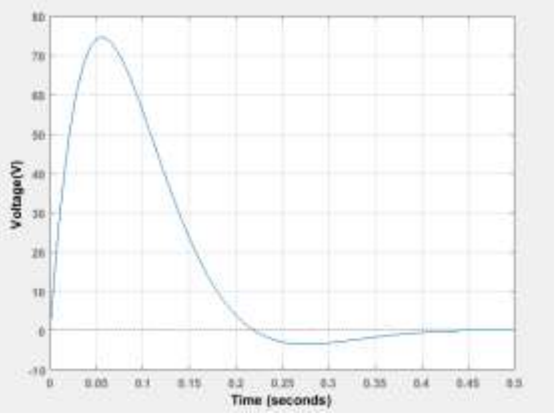

Figure 6. Step response of the Actautor with open Parameters

From Figure 6, the plate stabilized at 0.3546 seconds. Also, the peak voltage is 74.56V, which is closer to the rated voltage of 75V. From this, it shows that a proper inner loop design of the DC motor actuator has the following properties, which is given in Table III.

Table III: Properties of the Actuator with open Parameters

Actuator System Response	**Value**
Settling Time (sec)	0.3546
Peak Voltage (V)	74.5631

4.2 Outer Loop Design

The designed H_∞ controller is:

$$K = \frac{23622(s+1.631)(s+0.2213)(s+0.1794)}{(s+1109)(s+25.82)(s+0.9012)(s+0.3308)} \qquad (46)$$

The step response of the H_∞ controller is given in Figure 7

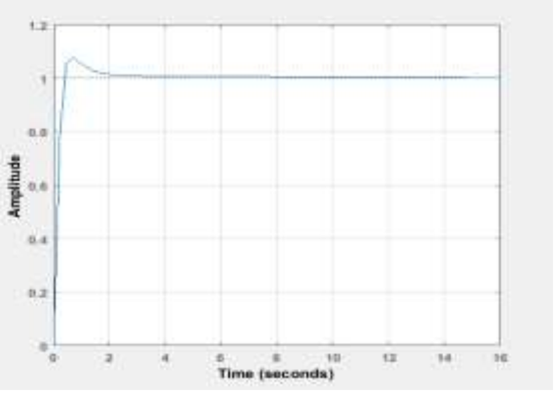

Figure 7. Step response of the H_∞ controller

From Figure 7, the properties of the H_∞ controller is given in Table IV.

Table IV. Properties of the H-Infinity Controller

H-infinity Controller System Response	Value
Settling Time (sec)	1.7087
Overshoot (%)	7.7246

From Table IV, it shows that the H_∞ controller stabilized the ball at 1.7087 seconds, with an overshoot of 7.7246 %. This shows a good indication of tracking the ball on the desired path on the plate.

A circular trajectory of radius 0.4 m, and a sinusoidal reference signal with a reference input of $x = 0.4(1-\cos\omega t)$ and $y = 0.4(\sin\omega t)$ was taken into consideration, and was used to demonstrate the trajectory tracking performance of the ball. The angular frequency of the sinusoidal reference signal used was 1.57 rad/sec. This is shown in Figure 8.

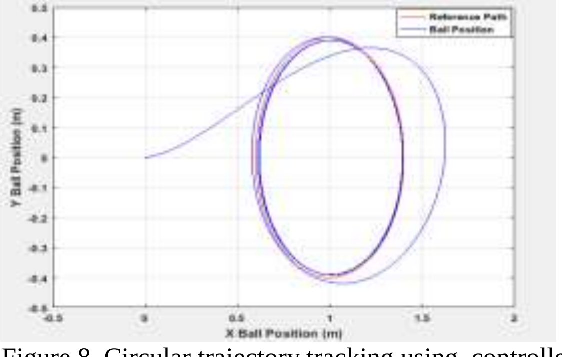

Figure 8. Circular trajectory tracking using controller

The ball was allowed to track a circular trajectory at a frequency of 0.52 rad/sec at a complete revolution of 12 seconds. When the speed was increased to 0.9 rad/sec, at a complete revolution of 7 seconds, the trajectory tracking error increased.
However, it was observed that using the controller, the steady state tracking error of the ball is 0.0095 m. Which shows that the ball was able to track the reference signal with a trajectory tracking error of 0.0095 m.

5. Conclusion

This paper presents the position and trajectory tracking control of the ball and plate system using the double feedback loop structure. For the effective control of the ball and plate system, the double feedback loop structure is considered. The inner loop was designed using linear algebraic method, while the outer loop was designed using function. Simulation results shows that the controllers has adaptability, strong robustness and control performance for the ball and plate system. The ball was able to settle with a good settling time and overshoot. However future research work will be to consider other controller techniques for the design of the outer and inner loop, and incorporating artificial intelligent techniques like grasshopper optimization algorithm (GOA) and weighted artificial fish swarm algorithm (wAFSA) with the controllers for the ball tracking a desired trajectory optimally.

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