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Articles

Rural Off-grid Hybrid Wind/Hydro Renewable Power Generation for Rapid Development in Africa

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***Abstract*:** Many rural communities in Africa located far away from transmission corridors do not enjoy electricity supply and cannot hope to be connected to their national grids in the foreseeable future. However, many among such communities have small hydro potential and moderate wind resources. In this paper, an off-grid, hybrid, wind-hydro power generation model is proposed for such communities. Candidate sites for this model would possess small hydro capacity of up to 500 kW and annual mean wind speed of 7 – 10 m/s at 10 m hub height. The operation of an existing small, run-of-river hydro station in conjunction with a hypothetical, in-situ wind farm feeding loads typical of rural areas was simulated to demonstrate the feasibility of the scheme. Simulation results for the system operation were found to be satisfactory. Further, it was demonstrated that, in spite of being an off-grid scheme, the wind turbine generators, aided by the STATCOM are capable of Low Voltage Ride Through in the event of a severe three-phase fault on a major feeder in accordance with grid code requirements. It is anticipated that, with the provision of subsidy by government and/or development agencies, implementation of this model will translate into rapid economic and social transformation of numerous rural areas across Africa.

Keywords: Off-grid; hybrid; renewable power generation; rural development; STATCOM; sinusoidal pulse-width modulation; EMTP-ATP; low voltage ride through

Introduction

Electricity supply is an indispensable requirement for economic growth and development of all nations. Traditionally, electricity has been generated at large thermal and hydroelectric stations and transmitted on extra high voltage grids over very long distances before being distributed to consumers. The consumers are usually residents of big urban settlements and industrial centres.

Many rural communities have not benefitted from grid-based power supply because utilities have found it uneconomical to extend facilities to areas far removed from the transmission corridors. Meanwhile, great impetus has been given globally to power generation from renewable sources such as wind, solar, mini-hydro and biomass, with some countries targeting the attainment of up to 40 % of demand from renewables only (Renewable UK, 2012; Smith & Pasrsons, 2011; Ibrahim et al, 2010). Remarkably however, almost the entire power output from renewable sources are still being fed into the various national grids. That has led to the development of a number of national grid code requirements for wind turbines (The RSA Grid Code Secretariat; Research Institute for Danish Electric Utilities (DEFU), 1998; ELECTRA Transmission Planning, 2000). Where available, isolated renewable generating stations operate in single-modes only with the exception of a few, e.g. the wind power plant backed up by diesel generators described in (Carillo et al, 2004). Such single-mode, stand-alone power stations have been unable to meet local energy demands completely. On the other hand, for most of the studies on hybrid wind/hydro generation reported in the literature, pumped-storage hydro schemes are predominant (Taylor; Bakos, 2002; Papaefthimiou et al, 2009). One notable exception is (Jaramillo et al, 2004) in which the authors describe a theoretical framework in which two hypothetical facilities in Mexico - a wind farm located in the "LaVente" area and a hydro plant located in the "Presidente Benito Juarez" dam - would operate together to produce up to 20 MW of firm power in real time. That scheme was also intended for connection to the regional electricity distribution network. Thus, the needs of rural communities outside grid networks for reliable power supply still remain unmet.

The objective of this study is to look into the operation of off-grid *hybrid* renewable power generation with a view to establishing the means for a more reliable power supply for rural areas in Africa. In particular, the study will focus on the operation of an existing 150 kW, run-of-river hydro power plant (the Waya Dam Project in Nigeria) in conjunction with a hypothetical, in situ 150 kW wind farm in order to develop a model or template for the establishment of similar hybrid schemes at suitable sites around Africa. Data on the Waya Dam Project were obtained from (Mu'Azu) except for electrical data which were obtained from design of an equivalent generator. The electrical (machine) data are as shown below.

Section 1 contains the introduction and a statement of the problem. In section 2, the proposed system operation diagram is described and relevant equivalent circuits and the method of self-excitation derived.

Also in section 2, gross estimates of energy capture are determined. Network simulation results are presented in section 3. Low voltage ride through capability of the wind turbine generators with the aid of the STATCOM is dealt with in Section 4 and section 5 contains the conclusions.

- RA -- Armature resistance, pu = 0.0252
- XL --Armature leakage reactance , pu = 0.0675
- XD -- d-axis synchronous reactance, pu = 0.853
- XQ -- q-axis synchronous reactance, pu = 0.512
- XD' -- d-axis transient reactance, pu = 0.3
- XD'' -- d-axis sub-transient reactance pu = 0.0677
- T_d' - d-axis short-circuit transient time constant, s = 1.5
- T_d'' -- d-axis short-circuit sub-transient time constant, s =0.03
- X_0 -- Zero-sequence reactance, pu = 0.15
- Z_n -- Neutral grounding impedance, pu = 0. .
- H -- Inertia constant, s = 4

Excitation Data

- VF , Rated Field Voltage, volts = 40
- IF, Rated Field Current, amps = 62
- RF, Field Winding Resistance, ohms = 0.6

2. System Operation Diagram

Fig 1 shows the system operation diagram (circuit breakers not shown for simplicity). It consists of (a) the two existing 75 kW hydro units which feed into a single 200 kVA, 0.4/33 kV transformer; a 33 kV transmission line, and a 33/0.4 kV distribution transformer, the secondary of which is connected to the main 400 V busbars; and (b) the two 75 kW wind turbine generator units. Each wind turbine unit comprises the wind turbine generator and its step-up 100 kVA, 0.4/11 kV transformer the HV side of which is connected to 11 kV Collector busbars. From the Collector busbars, an 11 kV transmission line conveys the power to a 200 kVA, 11/0.4 kV station transformer, the secondary of which is connected to the main 400 V busbars. At least one hydro unit would be expected to be in service to supply a small base load and be available to provide the required ramping up or down of real power output as dictated by the wind turbine performance, and also to provide start-up (magnetizing) power (as back-up to self-excitation) at cut-in times of the wind turbine. Also shown in the system diagram are 400 V outgoing feeders for typical rural loads: two 51 kVA village feeders for domestic lighting and power; a 9 kVA feeder for farm and irrigation pumps; a 24 kVA sawmill feeder and a 15 kVA quarry feeder.

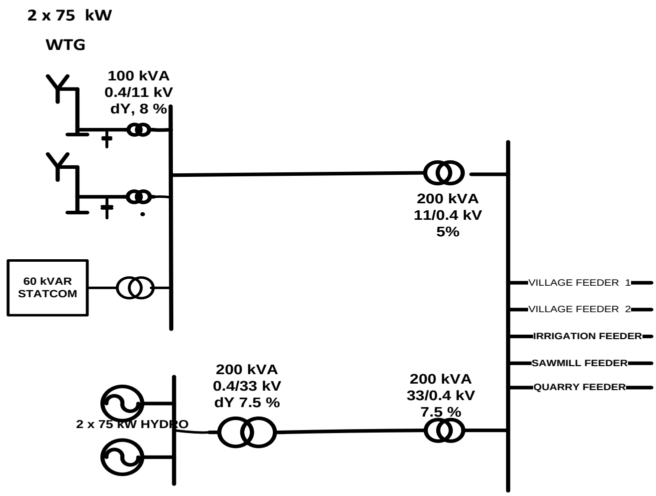

Figure 1 -- System Operation Diagram

A 60 kVAR Synchronous Staic Compensator (STATCOM) – which is a sinusoidal pulse-width modulated voltage source inverter - is connected to the collector busbars via a 100 kVA, 0.4/11 kV, 5 % power transformer for continuous wind turbines' terminal voltage regulation (by supplying reactive power to the induction generators when required) and to provide protection during system faults (low voltage ride-through) as demonstrated below in Section 4. In addition, by increasing the short circuit MVA, it enhances overall system stability (ABB Power Systems; Ronner et al, 2009; CIGRE, 2003; Singh et al, 2004; Mohan et al). The power transformer is also expected to act as a low pass filter for the output of the STATCOM, in addition to its normal role as the inductance between the STATCOM and the collector busbars. The Universal Machine Model was used to represent the wind turbine generator and all simulations of the power system were based on EMTP–ATP (Leuven EMTP Center, 1992).

The hydro plant and water resources

The essential data of the Waya Dam relevant to this study are (Mu'Azu) :

- Storage Capacity : 30 million cubic meters
- Dead Storage : 8 million cubic meters
- Live Storage : 22 million cubic meters
- Ave Potential Head : 12 meters
- Design Discharge : 2.4 m^3 / s
- Potential Power : 200 kW

The wind plant and wind resources

The wind resource at the WAYA dam site (Bauchi) is very poor. The annual mean wind speed is less than 3 m/s.. Such is generally the case throughout Nigeria except for very few sites (Adekoya & Adewale, 1992) . However, in Africa, locations exist with bountiful wind resources , e. g. the Klein Karoo, near Worcester in the Western Cape; the Great Karoo, near the town of Graaf Reinet in the Eastern Cape (Blakeway); and the Wind Atlas of South Africa (WASA) sites in South Africa (see www.wasa.csir.co.za) (CSIR). Candidate sites for the hybrid power plants proposed in this study should have wind resources, at least intermediate between the above two for satisfactory results. A mean wind speed of 7 m/s at 10 m hub height is assumed for the candidate site. The horizontal-axis wind turbine (HAWT) – 30 m in diameter – is employed. It consists of three blades attached to the hub at one end of a horizontal shaft atop a 40 m mast. The configuration of the blades is upwind. At the other end of the shaft, a 75 kW, 400 V, 3-phase, 50 Hz, 4-pole, 1475 RPM squirrel-cage induction motor is installed via a 1:60 gearbox.

No-load, locked-rotor, and stator DC resistance tests of the 3-phase motor were conducted. The no-load characteristics are plotted in Fig. 2. The locked–rotor and stator DC resistance test results are shown in Tables 1 and 2 respectively. No-load self-excitation capacitance was estimated from the plot of no-load terminal voltage versus magnetizing susceptance (Fig 2A) , similar to the methods used in (Say, 2002;Burton et al, 2001; Bassett & Potter, 1935). A capacitance of C = 500 μF was found to be satisfactory (the exact figure from the plot was 573.25 μF). For the full-load case, a capacitance of 3000 μF was found to be adequate.

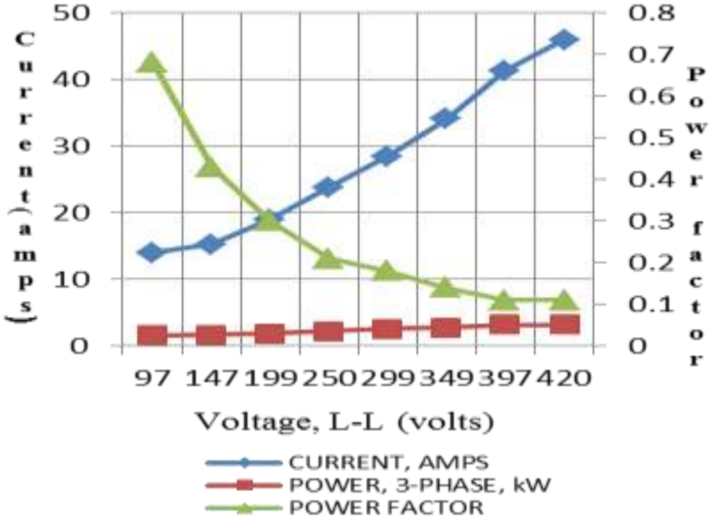

Figure 2 -- No-Load Characteristics of 75 KW Induction Motor

Gross estimates of energy capture

(a) Energy from the hydro plant

The total annual energy obtainable from the hydro plant was estimated by (Mu'Azu) as 1.08 GWh. This figure assumed 300 days of operation in a year.

(b) Energy from the wind -- The wind resource at a candidate site may be modeled by the Weibull probability density function (Burton et al, 2001; Hogg & Tanis, 2010);

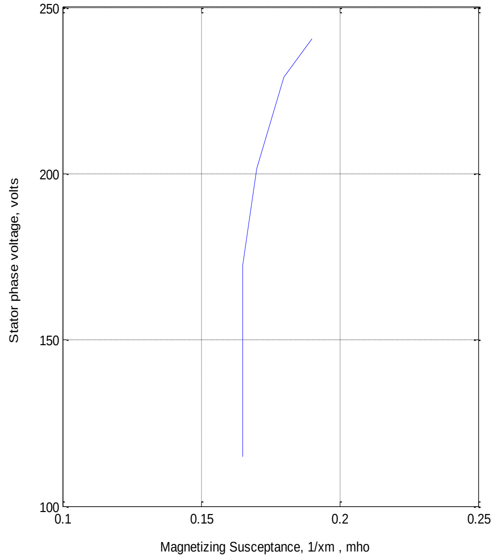

Fig 2a -- Phase Voltage versus Magnetizing Susceptance

$g(u) = \frac{\propto u^{\propto-1}}{\beta^{\alpha}} \exp\left(-\left(\frac{u}{\beta}\right)^{\alpha}\right)$, u > 0

where u = the wind speed. The mean wind speed is given by:

$$\mu = \beta \Gamma \left(1 + \frac{1}{\alpha}\right)$$

where Γ is the Gamma function. At typical sites, α = 2 is adequate. That corresponds to the Rayleigh distribution. For a mean wind speed of 7 m/s at 10 m hub height, the equivalent at 40 m hub height is (Heier, 1998):

$$u_{40} = u_{10} \left(\frac{h}{h_{10}}\right)^{a} ,$$

Where: u_h = wind speed at height , h m

a = Hellman Exponent = 0.14 – 0.17

Let a = 0.15, then

$$u_{40} = 7\left(\frac{40}{10}\right)^{0.15} = 8.62 \quad \text{m/s}$$

Thus 8.62 = β Γ(1.5)

β = 8.62 / 0.88623 = 9.73

thus: g (u) = 0.021 u exp (- 0.01 u^2) , u ≥ 0 (1)

$$P_{aero}(u) = C_p \cdot \frac{1}{2} \rho A u^3 \qquad (2)$$

Where $\rho = 1.225\ kg/m^3$ = density of air , and C_p is the power coefficient. The dimensionless $C_p - \lambda$ curve is approximated by two piecewise linear curves (see p. 174 of (Burton et al)) :

$$C_p = 0.18\lambda - 0.45 \ , \ 0 \le \lambda \le 7$$

$$C_p = -0.04375\lambda + 0.74375 \ , \ \lambda > 7$$

Now, $\lambda = \frac{\Omega R}{u}$

For a gearbox ratio of 1:60 and given a slip of -1.7 % , generator constant rotor speed of 1500 x 1.017 = 1525.5 RPM,

Wind Turbine shaft speed = $\frac{1525.5}{60}$ = 25.425 RPM

Therefore, $\Omega = 25.425 \times \frac{2\pi}{60}$ = 2.66 rad/s

The assumed rotor diameter = 30 m. Then R = 15 m implies that tip-speed ratio , $\lambda = \frac{2.66 \times 15}{u} = \frac{39.9}{u}$

Thus λ = 7 implies u = 5.7 .

$C_p = \frac{7.182}{u} - 0.45$, $\infty \ge u \ge 5.7$ (3a)

$C_p = -\frac{1.746}{u} + 0.74375$, $u < 5.7$ (3b)

The expected power generation, E (P_{aero}) is:

$$E(P_{aero}) = \int_{u_0}^{u_\infty} P_{aero}(u) g(u) du \quad (4)$$

Therefore, from eqs. (1) to (4), assuming a cut-in wind speed of 3 m/s at 10 m hub-height = 3.69 m at 40 m, and a cut-out wind speed of 15 m/s at 10 m hub height = 18.47 m/s at 40 m , energy captured over one year, T = 8760 hours will be:

$$E = 8760 \int_{3.69}^{18.47} C_p \frac{1.225 x \pi x 15^2}{2 x 1000} u^3 g(u) du$$

kW

$$E = 3793.08 \int_{3.69}^{18.47} C_p u^3 g(u) du \ \text{kWh}$$

Applying (3a) and (3b), $E = E_1 + E_2$ kWh

$$E_1 = 79.655 \int_{5.7}^{18.49} (7.182 u^3 - 0.45 u^4) \exp(-0.01 u^2) du$$

$$E_2 = 79.655 \int_{3.69}^{5.7} (-1.746 u^3 + 0.74375 u^4) \exp(-0.01 u^2) du$$

This gives $E_1 = 5.31 \times 10^5$ kWh and $E_2 = 2.56 \times 10^4$ kWh. The total energy per wind turbine is thus $E_1 + E_2$ = 5.57 x 10^5 kWh. The two wind turbines could generate up to 2 x 5.57 x 10^5 kWh = 1.114 GWh.

Table 1-- Locked - Rotor Test

VOLTAGE, L - L, VOLTS	CURRENT, AMPS	POWER, 3 - PHASE, kW	POWER FACTOR
60	109	3.3	0.3

Table 2 - DC Resistance Test (Stator Winding)

WINDING	**RESISTANCE (mΩ)**
A1 - A2	69.9
B1 - B2	70.1
C1 - C2	69.8
AVERAGE	**69.93**

3. Network Simulation

Figures 3 – 8 below show results of the simulation of the network. As can be observed, the results were very satisfactory except for the minimal distortion of the waveforms due to incomplete filtration by the STATCOM transformer.

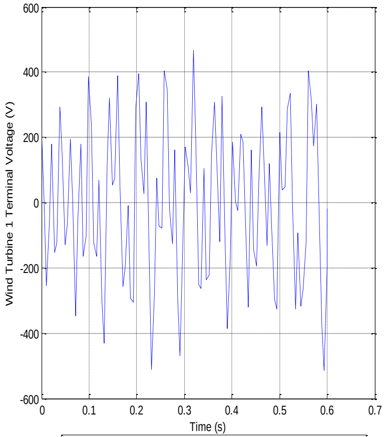

Fig 3 -- Wind Turbine Generator 1 Terminal -- A-phase Voltage, V

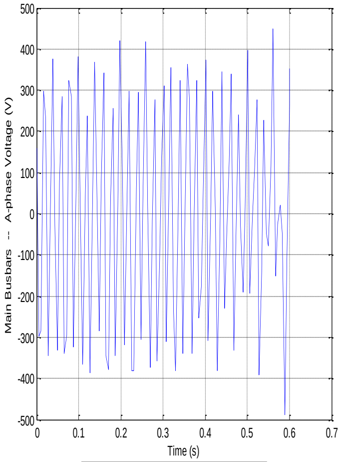

Fig 4 -- Main Busbars - A-phase Voltage, V

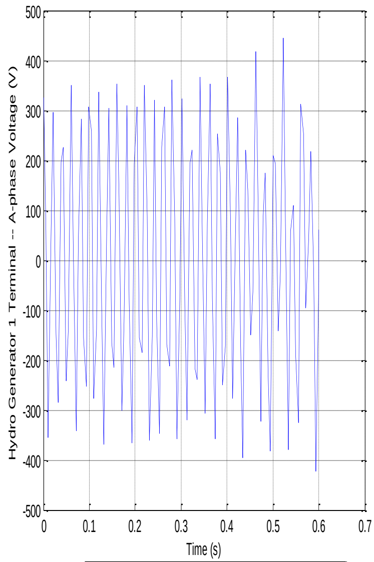

Fig 5 -- Hydro Generator 1 Terminal -- A-phase Voltage, V

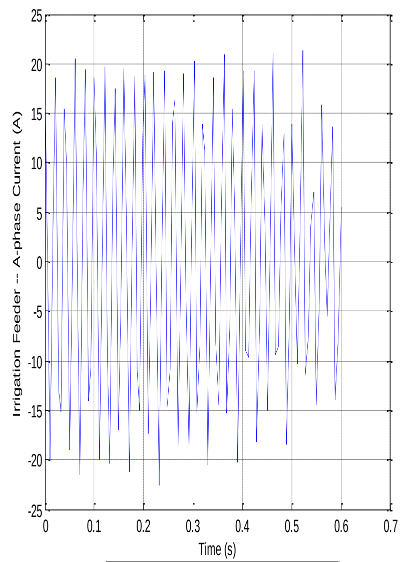

Fig 6 -- Irrigation Feeder - A-phase Current, A

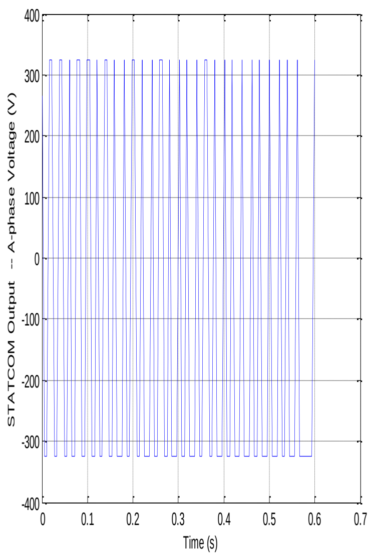

Fig 7 -- STATCOM Output -- A-phase Voltage, V

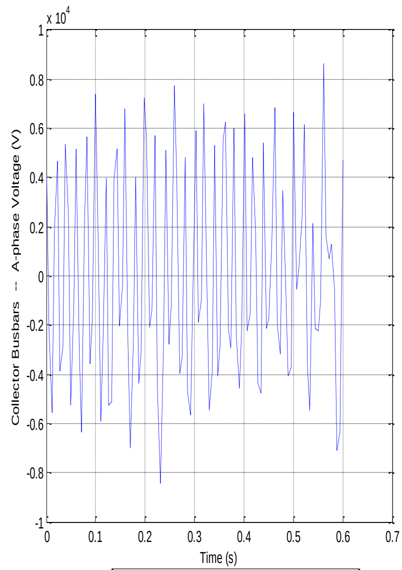

Fig 8 -- Collector Busbars -- A-phase Voltage, V

4. Low Voltage Ride Through

Even though the system in this study was off-grid, it was considered desirable to determine if the wind turbine generators were capable of low voltage ride through (LVRT) in the event of faults. The South African and the German Grid Codes, (The RSA Grid Code Secretariat; Pannell et al, 2010; Vidal et al, 2013; Pannell et al, 2013) require that wind turbine generators remain connected to the grid for 150 ms following all types of fault resulting in voltage dips down to zero volts. That is to aid system recovery following the clearance of the fault and to provide voltage support (reactive power). As pointed out earlier, the STATCOM enabled the wind turbine generators to successfully ride through the voltage dips in compliance with grid code requirements. Figs 9 to 16 below show simulation results for a three-phase fault – the severest – on one of the village feeders. The fault was applied at t = 1 sec and cleared at t = 1.15 secs. It is clear from the figures that the wind turbine generators remained connected to the network during the fault in compliance with grid-code requirements. However, shaft over speed occurred during the fault but normal speed was restored soon after the fault was cleared.

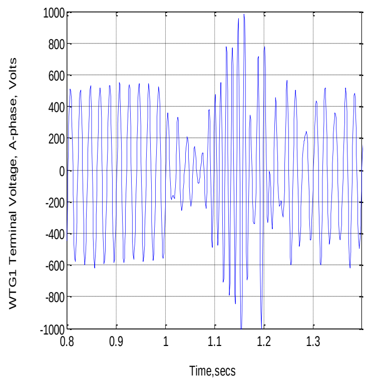

Fig 9 -- LVRT: Wind Turbine Generator 1 Terminal Voltage, Volts

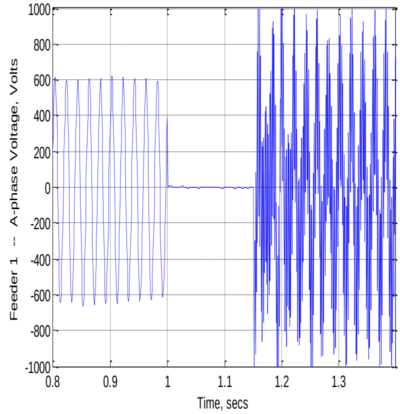

Fig 10 -- LVRT: Village Feeder 1 A-phase Voltage, Volts

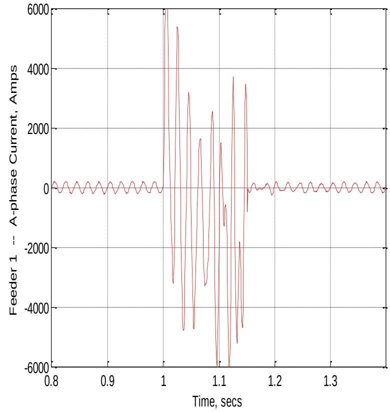

Fig 11 -- LVRT: Village Feeder 1 A-phase Current

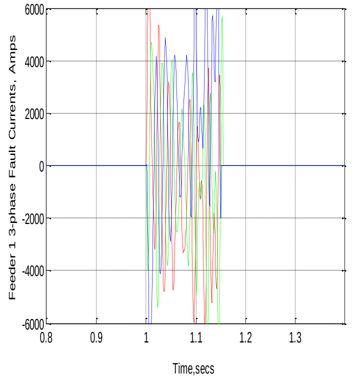

Fig 12 -- LVRT: Village Feeder 1 3-phase Fault Currents

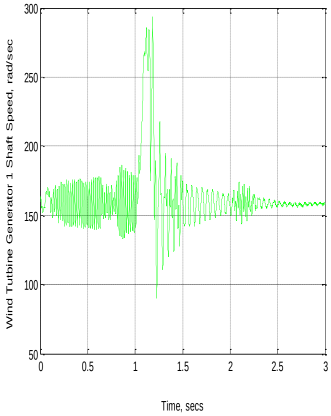

Fig 13 -- LVRT: Wind Turbine Generator 1 Shaft Speed

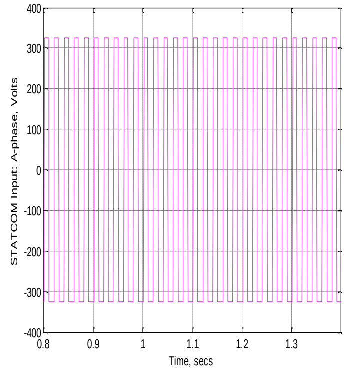

Fig 14 -- LVRT: STATCOM Input - A-phase

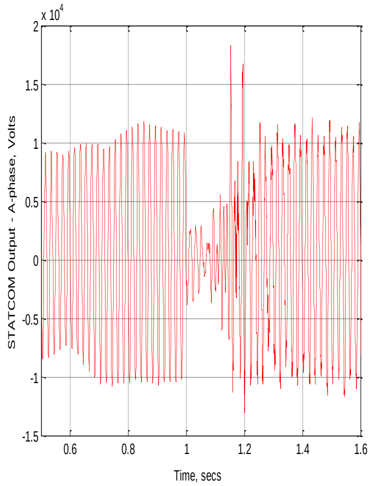

Fig 15 -- LVRT: STATCOM Output Voltage - A-phase

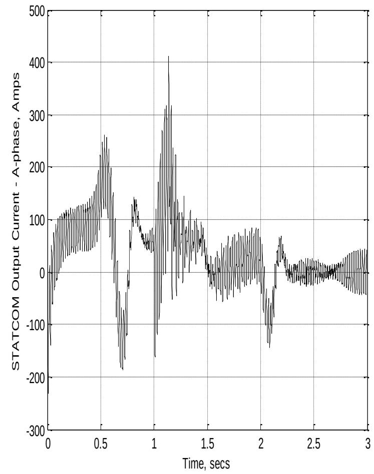

Fig 16 -- LVRT: STATCOM Output Current - A-phase

5. Conclusion

Results from this study demonstrate that rural communities in Africa that possess modest wind and hydro resources can benefit immensely from the off-grid hybrid electricity schemes long before they get connected to their national grids. In particular, in spite of the very moderate wind speeds considered, the potential energy production capability at the WAYA dam site can be seen to have doubled and thus more villages could be connected to the supply than hitherto. Furthermore, the wind turbine generators were capable of low voltage ride through as required by grid codes.

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Colored Petri Net: Its application to Sucrose Biosynthesis Pathway in *Plasmodium falciparum*

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Abstract: Sucrose plays major role as macromolecule used in organisms including *Plasmodium falciparum* (P.f.) to generate glucose for energy production in the glycolysis pathway. A metabolic pathway is a series of chemical reactions, which goes through various intermediate compounds to transform input compounds into a product. In this work, we modelled a metabolic pathway in *Plasmodium falciparum* using Colored Petri Net Markup Language (CPNML). The model was used to examine the dynamic behavior of the sucrose biosynthesis pathway which shows the interactions between the metabolites and the reactions in the sucrose biosynthesis pathway of *Plasmodium falciparum*. We further analyzed the model for its structural and quantitative properties using Petri Net theory. Our model gives more insight to the structure of the pathway and helps to improve our understanding of the biological processes within this pathway.

Keywords: Sucrose, Colored Petri Net, *Plasmodium falciparum*

Introduction

A model is a representation of the build-up and working process of a particular system. A model is a similar but simpler version of the system it represents. Modeling is simply the process of creating a model (Anu, 1997). There are different types of models, qualitative or quantitative, deterministic or stochastic, static or dynamic, continuous or discrete. While a qualitative or structural model e.g. a network graph indicates the relationship among model elements, a quantitative model attaches weights to those relationships. (Riesig, 1982).

Petri Net (PN) was suggested by (Reddy *et al*., 1993) for modeling metabolic networks to overcome some limitations like applying ordinary differential equations (ODE) to a very large complex system which can be a very tediuos task. Lots of further conceptual works, technical tool implementations and applications into biological problems have been reported since then and have demonstrated the importance and usefulness of this concept referred today as systems biology.

Colored Petri Nets (CPN) was first proposed by (Jensen, 1981). They combine PN alongside the advantages of computer programming to narrate data type and operations, which gives an easier approach to produce a parameterized model. Colors are used to distinguish the tokens within a colored Petri Net. An arc expression (extension of arc weights from the basic PN) indicates the flow of the tokens either above the arcs or guards (Boolean expressions) which can be used to define additional constraints on enabling of transitions (Jensen *et al*. 2007).

CPN is an extension of an ordinary PN which provides a framework for the construction and analysis of distributed and concurrent systems (Kristensen *et al*., 1998). Its representation in a system shows the status of the system and the possible changes between the states involved. CPNs have been applied to a vast range of areas, such as biological systems (Liu & Heiner, 2012) communication protocols (Huber & Pinci, 1991, Floreani *et al*., 1996), audio/video systems (Christensen & Jørgensen, 1997), operating systems (Cherkasova *et al*., 1993a & Cherkasova *et al*., 1993b), hardware design, embedded systems (Rasmussen & Singh, 1996), software system designs (McLendon & Vidale, 1992, Scheschonk & Timpe, 1994) and business process re-engineering (Pinci & Shapiro, 1991 & Mortensen & Pinci, 1994).

Definition1: Colored Petri Net (CPN) can be formally defined as a 9-tuple (Jensen *et a*l. 2007):

$$CPN = (Pl, TR, Ar, \sum, VR, Cl, Gd, E, I)$$

, where:

Pl denotes a bounded set places

TR denotes a bounded set transitions $TR \ni Pl \cap TR = \phi$

$Ar \subseteq PlXTR \cup TRXPl$ denotes directed arcs

$\sum$ denotes bounded non-empty color sets

VR is a bounded typed variables $\ni$ Type[VR] $\in \sum$ $\forall$ variables $v \in VR$.

$Cl : Pl \rightarrow \sum$ denotes colored set function attaching color set to each place.

$Gd : TR \rightarrow PR_v$ denotes a guard function attaching guard to each transition $tr \ni Type[Gd(tr)] = bool$.

$E : Ar \rightarrow PR_v$ denotes arc expression function attaching arc expression to each arc $ar \ni Type[E(ar)] = Cl(p_l)$, where p_l denoted the place connected to the arc ar.

$I : Pl \rightarrow PR_\theta$ denotes initialization function connecting initialization expression to each place

$p_l \ni Type[I(p_l)] = C(p_l)$.

Definition2: A Colored Petri Net module contains 4-tuples Jensen *et al*. 2007):

$CPN_m = (CPN, T_s, P_{pt}, PT)$, where

CPN is a non-hierachical colored petri Net defined in definition 1 above.

$T_s \subseteq TR$ denoted substitution transitions

$P_{pt} \subseteq Pl$ denoted port places

$P_T : Pl \rightarrow \{I_{IN}, O_{OUT}; I/O\}$ denoted port type function connecting port type to each port places.

Metabolic Networks

A Metabolic pathway is a series of connected enzymatic reactions that occur within a cell. It consists of consecutive chemical reactions, which changes input compound(s) (substrates) into an output compound (product) with the aid of enzymes (Reddy *et al*., 1993). It is defined as a subsystem that deals with some specific function, generate the most essential components that are vital for the synthesis, usage for life and energy (Baldan *et al*., 2011). It can also be defined as a combination of reactions that are chemical in nature that is combined with enzymes in which some of the reactants are converted to other products. These products can in-turn serve as the substrate for the next reaction.

There are various advantages in the use of mathematical representations in the modeling of metabolic networks such as: organization of disparate information into coherent and self-consistent whole, to think logically about the essential components and interactions in a complex system, simulation, prediction, and optimization of procedures, experiments and therapies, in order to validate and to define improved hypotheses and to understand the important features of a given system (van Riel, 2006).

Consequently, the task of any metabolic pathway is to modify a principal chemical compound to form another chemical compound which can be used up, passed on to start another pathway or stored up by the cell.

Sucrose consists of monosaccharide glucose and fructose with the molecular formula $C_{12}H_{22}O_{11}$. The sucrose biosynthesis is obtained through UDP-D-Glucose and fructo-6-phosphate reaction in the presence of an enzyme called sucrose-6-phopshate synthase. This reaction obtained energy from the cleavage of Uridine diphosphate (UDP-$C_9H_{11}N_2O_{12}P_2$).

This paper focuses on the use of a modeling tool called Petri Net to construct an *in-silico* metabolic network that shows the interactions between the metabolites and the reactions in the sucrose biosynthesis pathway of *Plasmodium falciparum*

(P.f.). Reddy et al., 1993 were the first to introduce the use of Petri Nets to qualitative modeling of biochemical Networks. In their work, simple case condition systems were used for simulation of simple biochemical processes. Since then lot of deeper papers have been published using this method of simulation of metabolic, regulatory, genetic and signal transduction nnetworks.

Materials and Methods

The data for the Sucrose Biosynthesis pathway for *P.f.* was obtained from the **BioCyc** database - (www.biocyc.org) which consists of 3530 Pathway/Genome Databases (PGDBs). Sucrose synthesis is performed by first generating the phosphorylated form, sucrose 6^F-phosphate with chemical formula: $C_{12}H_{21}O_{14}P$ (the "F" indicates that the phosphate group is attached to the furanose functional group), then this is followed by dephosphorylation. The first step is catalyzed by sucrose-phosphate synthase (E.C 2.4.1.14), which condenses β-D-fructofuranose 6-phosphate ($C_6H_{11}O_9P$) with UDP-α-D-glucose ($C_{15}H_{22}N_2O_{17}P_2$). The second step is catalyzed by sucrose-phosphate phosphatase (EC 3.1.3.24), which hydrolyzes, sucrose 6^F-phosphate to sucrose ($C_{12}H_{22}O_{11}$). In photosynthetic organisms both precursors originate from photosynthetic-derived carbon, via β-D-fructofuranose 6-phosphate ($C_6H_{11}O_9P$) (Hasunuma et al., 2010). The image in figure 2 below is the representation of the Sucrose Biosynthesis pathway for *P.f.* from BioCyc, which was then converted by building a Colored Petri Net model.

The Petri Net model was built and analyzed using CPN Tool version 4.0.0. (http://cpntools.org/). Colored Petri Net tool helps to gain better understanding of a colored Petri Net by providing room for making changes, implementing a prototype and carrying out analysis. For more details on this tool see (Ratzer *et al*, 2003, Jensen *et al*. 2007, Westergaard & Kristensen, 2009).

The use of colored Petri Net model for pathway modelling requires to appoint the elements of the colored Petri Net to the corresponding metabolic pathway. Places (P_l) would be equivalent to byproduct of metabolism (i.e Metabolites (M_b), Enzymes (E_z) and Compound(C_p)). The reactants or substrates represent input places (I_p) and the products(R_p) represent output places (O_p). In simple terms, an arc is directed outwards from an input place, while an arc is directed inwards into an output place. Transitions (T_R) represent chemical reactions (such as Reaction (R_t) and Interaction

(Int). The pathway's stoichiometric matrix is equivalent to the incidence matrix of the petri Net and the arc weights are obtained by the given stoichiometric coefficients. In each place, the number of tokens suggests the amount of substance integrated with that place, the flux's modes and the conservation relations of metabolites correlate with the specific properties of PNs. For example, the minimal (semi-positive) T-invariants correlate with the flux modes of a metabolic pathway, this form a basis for the set of semi-positive T-invariant (Hilbert basis) which is unique and characteristic of PN (Hofestadt, 1994).

According to (Baldan *et al.*, 2013), Table 1 shows the relationship between Petri Net elements and metabolic pathway elements. For the Colored Petri Net model, the only change to this table is the inclusion of the colored tokens refered to as Colored Sets (metabolites, enzymes, and compounds quantities). Figure 1 is the colored Petri Net representation of the well-known chemical reaction $2H_2 + O_2 \rightarrow 2H_2O$. The first Petri Net represents the state before the reaction occurs (i.e. before the transition fires), while the second represents the state after the reaction has occurred.

Table 1. Relationship Between Petri Nets and Pathway Elements

PN Elements	**Biological Pathway Elements**
Places(P_l)	Metabolism(M_b, E_z, C_p)
Transitions	Chemical reaction(R_t, Int)
Input Places	Substrates, reagents
Output places	Products(R_p)
Incident matrix (Arc Weights)	Stoichiometric coefficients
Number of tokens on places	Metabolism(M_b, E_z, C_p)
Transition rates	Kinetic reactions

For this Petri Net to be constructed, an enumerated type color set IN with members H and O had to be defined using the following syntax *closet IN = with H | O;* and two variables of type *closet IN,* in1 and in2 were declared, using this syntax *var in1, in2: IN;* Then a compound type color set OUT was declared using this syntax *closet OUT = product IN * IN;*. This was defined as a product in order to contain hold the different tokens of the various input places (i.e. H and O) The variables in 1 and 2 were bound to the various members of the color set

IN so that the arc expression for each of the input places contained the condition in which the transition would be enabled to fire. In this example the transition required 2 moles of H^+ and 1 mole of O_2 to be enabled to fire and result in the formation of 2 moles of H_2O.

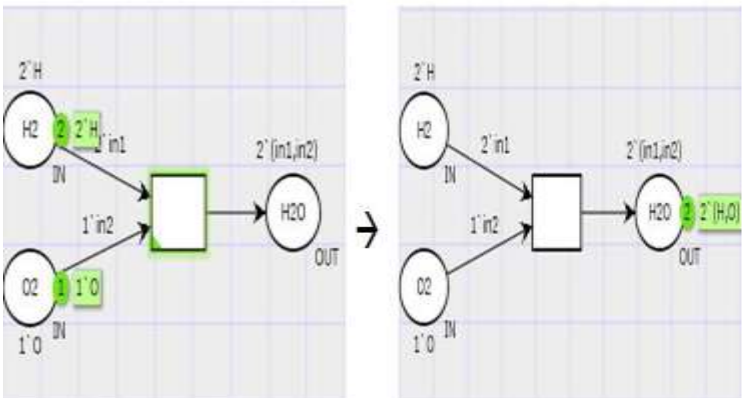

Figure 1. The Petri Net Representation of the Formation OF H_2O

Rx1	β -D-glucose-6-phosphate ⇆ α-D-glucose-6-phosphate	Reversible	5.1.3.15 (spontaneous)
Rx2	α-D-glucose-6-phosphate ⇆ α-D-glucose-1-phosphate	Reversible	5.4.2.2
Rx3	α-D-glucose-1-phosphate + UTP + H ⇆ UDP-D-Glucose + Diphosphate	Reversible	2.7.7.9
Rx4	UDP-D-Glucose+β-D-Fructofuranose ⇆ Sucrose	Reversible	2.4.1.13

	+ UDP		
Rx5	UDP-D-Glucose + D-Fructose-6-phosphate →Sucrose-6-Phosphate+UDP+H^+	Irreversible	2.4.1.14
Rx6	Sucrose-6-phosphate + H_2O → Sucrose + Phosphate	Irreversible	3.1.3.24

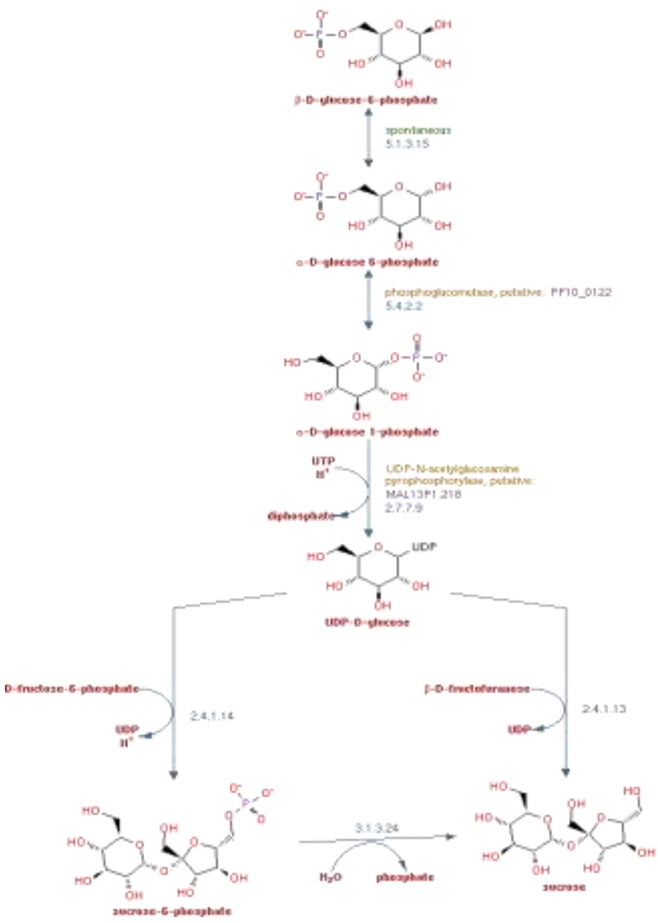

Figure 2. The Sucrose Biosynthesis Pathway in *Plasmodium Falciparum* from BioCyc

Results And Discussion

The Constructed Model consists of 6 reactions which have a total of 9 reactants and 11 products. 4 of the reactions are reversible and the other two are irreversible. The reactions were catalyzed by 5 enzymes, one of them (5.1.3.15) being spontaneous (meaning it's not compulsory that enzyme is present for the corresponding reaction to occur). Table 2 shows the overall reaction layout for the Sucrose biosynthesis pathway in *P.f.*

From the reaction equations, a corresponding stoichiometric matrix (shown in figure 3) was built using the stoichiometric coefficients. The substrates are multiplied by -1 and the products by +1. The zero value entries signify the given metabolite did not participate in the given reaction. This was used in assigning the values of the color sets in the construction of the Colored Petri Net model. The CPN model is shown in figure 4 (before it fires) and figure 5 (after it fires).

Abbreviations	**Full Meanings**
a-D-g6p	α-D-glucose-6-phosphate
b-D-g6p	β -D-glucose-6-phosphate
B-D-FF	β-D-Fructofuranose
Df6p	D-Fructose-6-phosphate
d-p	Diphosphate
Dg1p	α-D-glucose-1-phosphate
s-6-p	Sucrose-6-Phosphate
UDP	Uridine diphosphate
UDP-g	Uridine diphosphate- Glucose
UTP	Uridine triphosphate

Table 3. Abbreviations of Compounds and their full meaning

	*Rx*1*a*	*Rx*1*b*	*Rx*2*a*	*Rx*2*b*	*Rx*3*a*	*Rx*3*b*	*Rx*4*a*	*Rx*4*b*	*Rx*5	*Rx*6
b - D - g6p	1	-1	0	0	0	0	0	0	0	0
a - D - g6p	-1	1	-1	1	0	0	0	0	0	0
Dg1p	0	0	1	-1	-1	1	0	0	0	0
UTP	0	0	0	0	-1	1	0	0	0	0
H +	0	0	0	0	-1	1	0	0	1	0
UDP - G	0	0	0	0	2	−1	−1	1	−1	0
dp	0	0	0	0	1	−1	0	0	0	0
b − *DFF*	0	0	0	0	0	0	−1	1	0	0
sucrose	0	0	0	0	0	0	1	−1	0	1
UDP	0	0	0	0	0	0	1	−1	1	0
*F*6*P*	0	0	0	0	0	0	0	0	−1	0
*S*6*P*	0	0	0	0	0	0	0	0	1	−1
H_2O	0	0	0	0	0	0	0	0	0	−1
Phosphate	0	0	0	0	0	0	0	0	0	1

Figure 3. The Stoichiometric Matrix for Sucrose Biosynthesis Pathway

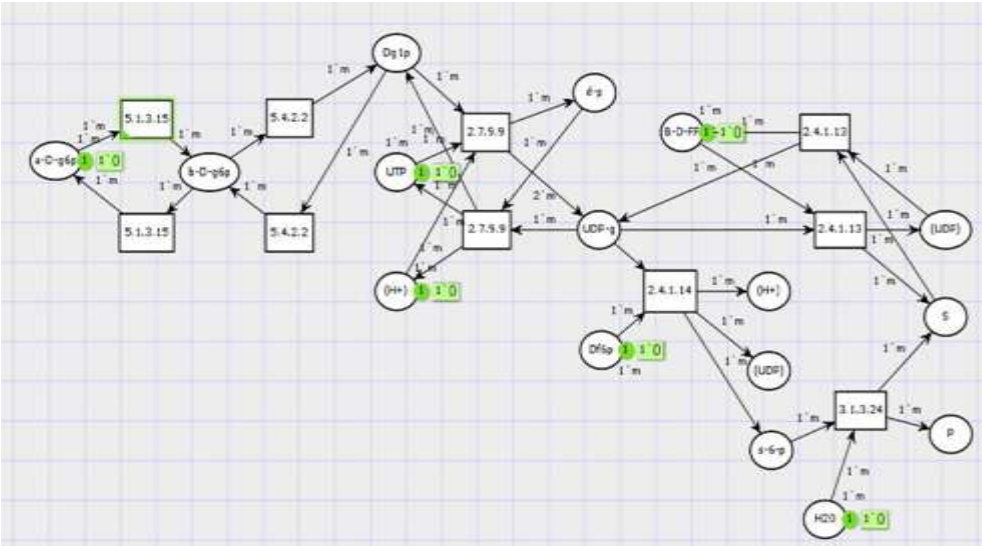

Figure 4: Petri Net Construction of the Sucrose Biosynthesis Pathway (before Firing)

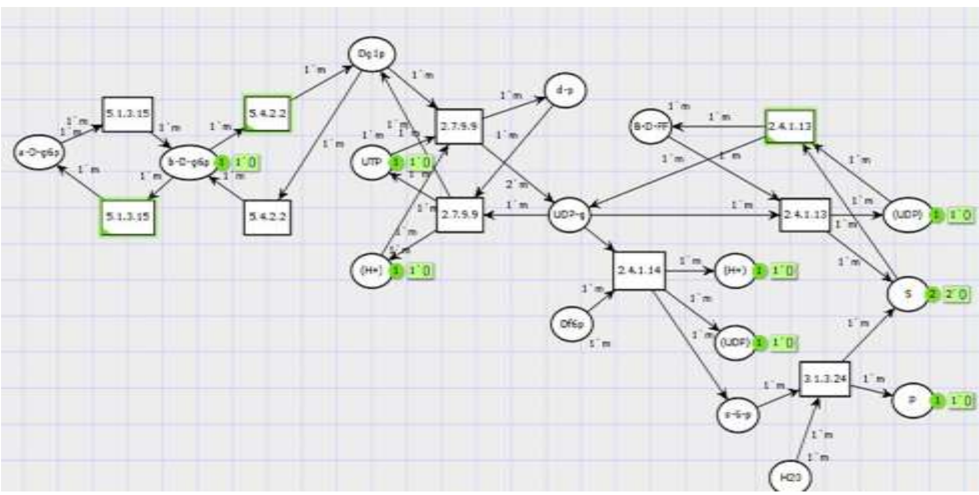

Figure 5: Petri Net Construction of the Sucrose Biosynthesis Pathway (after Firing)

Model Validation & Analysis

The aim of model validation of a constructed model is to check for inconsistencies in that given system, hence deriving statements on the structural and dynamic properties that reflect the activities of that system in real life (Koch et al, 2005). The CPN model of the sucrose biosynthesis pathway in P.f. after firing shows one token of UDP (Uridine diphosphate), one token of Phosphate and two tokens of Sucrose. The two tokens of Sucrose produced is because there are two distinguishing "routes" in which Sucrose is produced from UDP-D-Glucose (UDP being an activator).

The first being the hydrolysis of Sucrose-6-phoshpate to form sucrose and one mole of Phosphate and the second is the condensation of β-D-fructofuranose with D-Glucose to form another mole of sucrose and a mole of UDP.

This verifies that the constructed model conforms to the reaction layout. The constructed model consists of 6 reactions which have a total of 9 reactants and 11 products. Four of the reactions are reversible and the other two are irreversible, and the reversible reactions are modeled as forward and backward reactions. The reactions were catalyzed by 5 enzymes, one of them (5.1.3.15) being spontaneous (meaning it's not compulsory that enzyme is present for the corresponding reaction to occur).

Structural Analysis

The aim of structural analysis is to discover underlying structures that allows conclusions on dynamic properties. It involves the calculation of elementary properties. Structural analysis checks if the constructed net is ordinary, homogenous, conservative, pure, static, conflict free & connected or strongly connected in the graph theoretical sense. The net constructed is unbounded and it is not ordinary because the arc weights are not all equal to one. The net is also not homogenous (the pathway modeled in a metabolic pathway). It is also not conservative because not all transitions have the sum of the input arc weights equal to the sum of the output arc weights.

For example: Reaction 1: 1 mole of α-D-glucose-1-phosphate and 1 mole of UTP and 1 mole of H^+would result in one mole UDP-D-Glucose and one mole of Diphosphate. For this example we have a total of 2 moles of reactants and 3 moles of products.

Since the net is not conservative then it's not bounded. The net is not pure because there's no transition in which the pre-place is also a post-place (except in reversible reactions). This net is not static conflict free because there are transitions sharing the same input places. A metabolic petri net would not be free of static conflicts because compounds may be used by several reactions. Since the model is not a bounded model, we cannot compute the reachability graph. Also the cover ability graph which is a subset of the reachability graph cannot be computed because there is a huge amount of possible states.

Invariant Analysis

The net contains but is not covered by the following P invariants;

Invariant B – These are the set of all the compounds that provide a phosphate group either directly or indirectly. If the phosphate group is transferred from one compound to another the sum of the phosphorylated metabolite remains unchanged. If no phosphate is consumed or secreted by a cell, the sum of phosphate groups in all metabolites, including inorganic phosphate would also remain unchanged.

The following compounds are the compounds that contain a phosphate group.

UTP (Uridine triphosphate), UDP (Uridine diphosphate) and Phosphate.

5. Conclusion

Various Petri Net representations have been successfully used for biological networks such as gene regulation, signal transduction and metabolic systems. In this work, we used Colored Petri Net model which is more compact and readable to build the sucrose biosynthesis pathway and further analyze the model for its structural and quantitative properties using Petri Net theory. Our model gives more insight to the structure of the pathway and helps to improve our understanding of the biological processes within this pathway.

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Optimization of Production Plan of Hebron Drinks using Operational Research Technique

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Abstract: Manufacturing companies frequently face challenging operational problems. In such business environment, operations that compete for the same resources must be planned in a way that deadlines are met. Certain expertise in optimization is often required for successful solution of these problems. In this paper, we attempted to optimize the production plan of a manufacturing company - Hebron Drinks, by minimizing the Labour hours, Marching hours and Materials used in producing six different types of products. Linear programming technique was use to model the production plan of Hebron Drinks. The resulted model was solved using simplex method with the aid of computer software (LIP Solver 1.11.1 and 1.11.0). The optimal value obtained shows a reduction in the total cost of production for the period considered.

Keywords: Linear programming, Simplex method, Optimization, Operational research, Production plan.

1. Introduction

Production systems are area of operations research that concentrates on real-word operational problems. Production planning is concerned primarily with the adaptation of the industrial limited resources of the firm in order to satisfy demand for its product. (Bruce R. Feiring, 1991; Shapiro, J.F., 1993). The following settings, among others, usually produce production systems problems: manufacturing, telecommunications, health-care delivery, facility location and layout, and staffing (Hillier, F., & Lieberman, G., 2001). Operations research fundamentals are required to solve production problems since they are operations research problems. Additionally, the solution of production systems problems frequently draws on expertise in more than one of the primary areas of operations research, implying that the successful production researcher cannot be one-dimensional (Banks, J., Carson II J., & Nelson, B., 1995: Taha, H. A., 2002).. In order to solve production problems, an in-depth understanding of the real problem is required, since invoking assumptions that simplify the mathematical structure of the problem may lead to an elegant solution for the wrong problem. Common attributes of successful production planners are common sense and practical insight (Hillier, F., & Lieberman, G., 2001; Epply, T., 2004). Design objective could be simply to minimize the cost of production or to maximize the efficiency of production (Agarana, M.C. Anake T.A., and Adeleke O.J., 2014).

An optimization algorithm is a procedure which is executed iteratively by comparing various solutions till an optimum or a satisfactory solution is found. Optimization or mathematical programming is the selection of a best element (with regard to some criteria) from some set of available alternatives (Ofori S., 2013). In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function (Voss, C., Tsikriktsis, N. & Frohlich, M. 2002). The generalization of optimization theory and techniques to other formulations comprises a large area of applied mathematics. More generally, optimization includes finding "best available" values of some objective functions given a defined domain (or a set of constraints), including a variety of different types of objective functions and different types of domains (Fagotinbo, I. S., Akinbo, R. Y., Ajibode, I. A., Olaniran Y. O. A., 2011) A production plan is the administrative process that takes place within a manufacturing business and which involves making sure that sufficient raw materials, staff and other necessary items are procured the schedule specified (Graves, C. S., 1999)..

The Hebron drinks is a unit under Strategic Business Unit (SBU) of Covenant University Consultancy Services. It was established in December 2005, to cater, primarily, for worshippers at Canaan land and Covenant University students. The unit adequately provides Sachet water and Bottle water to Canaan land community. Hebron drinks is driving towards having distributors nationwide and maintaining high quality products at all times. Products range from Sachet water, Bottle water, Black currants flavour drink, Orange flavour drink. In March 2014, new products were added to the production line, these are: Hebron yoghurt and apple drink. In the near future, more products are expected to be added to the production, this will include pineapple drink and kola drink which are already being experimented to determine its demand and acceptance. (Covenant

University Student Handbook, 2013-2017). A typical manufacturing business engaging in production planning would usually aim at maximizing profit while maintaining a satisfied customer base (Voss, C., Tsikriktsis, N. & Frohlich, M., 2002). A planning problem exists because there are limited production resources that cannot be stored from period to period. Choices must be made as to which resources to include and how to model their capacity, behavior, and their costs (Unti, J. G., 1968). Also, there may be uncertainty associated with the production function, such as uncertain yields or lead times. One might only include the most critical or limiting resource in the planning problem, e. g., a bottleneck (Unti, J. G., 1968). Alternatively, when there is not a dominant resource, then one must model the resources that could limit production. We describe two types of production functions can be described as follows: The first assumes a linear relationship between the production quantity and the resource consumption. The second assumes that there is a required fixed charge or setup to initiate production and then a linear relationship between the production quantity and resource usage (Voss, C., Tsikriktsis, N. & Frohlich, M. 2002; Unti, J. G., 1968). In this paper attempt is made to maximize the production plan of Hebron drinks using Linear programming model.

2. Model Formulation

Production plan of Hebron Drinks is modeled using linear programming technique. The model include the objective function, the constraints; including the non-negativity constraints. Decision variables, constraints, per unit usage of resources. The available resources are labour hours, machine hours and materials. They are represented by the variables and parameters involved in the model formulation.

2.1 Decision Variables

Decision variables are a set of quantities that need to be determined in order to solve the linear programming problem. They are so called because the problem is to decide what value each variable should take. Typically, the variables represent the amount of a resource to use or the level of some activity. (Agarana, M. C., Anake T.A., and Adeleke O.J.,2014)

For this paper, let the decision variables be represented as follows:

x_{ij} : Number of product j produced by employing resource i

x_{1j} : Number of product j produced by employing resource 1

x_{2j} : Number of product j produced by employing resource 2

x_{3j} : Number of product j produced by employing resource 3

Specifically, we have the following representations:

x_{11}: Number of sachet water produced by employing machine hour.

x_{12}: Number of bottle water produced by employing machine hour

x_{13}: Number of apple juice produced by employing machine hour

x_{14}: Number of orange juice produced by employing machine hour

x_{15}: Number of yoghurt produced by employing machine hour

x_{16}: Number of communion drink produced by employing machine hour.

x_{21}: Number of sachet water produced by employing labour hour.

x_{22}: Number of bottle water produced by employing labour hour.

x_{23}: Number of apple juice produced by employing labour hour.

x_{24}: Number of orange juice produced by employing labour hour.

x_{25}: Number of yoghurt produced by employing labour hour.

x_{26}: Number of communion drink produced by employing labour hour.

x_{31}: Number of sachet water produced by employing material.

x_{32}: Number of bottle water produced by employing material

x_{33}: Number of apple juice produced by employing material

x_{34}: Number of orange juice produced by employing material

x_{35}: Number of yoghurt produced by employing material

x_{36}: Number of communion drink produced by employing material

x_{32}: Number of bottle water produced by employing material

x_{33}: Number of apple juice produced by employing material

x_{34}: Number of orange juice produced by employing material

x_{35}: Number of yoghurt produced by employing material

x_{36}: Number of communion drink produced by employing material

2.2 Resources Utilization

The resources used for production are represented as follows:

a_{ij} : Amount/Number of resource i used to produce a unit of product j

a_{1j} : Number of labour hour used to produce one unit of product j

a_{2j} : Number of machine hour used to produce one unit of product j

a_{3j} : Amount, in kilogram, of material used to produce one unit of product j

2.2.1 Labour Hours used:

a_{11}: Number of labour hour used to produce one unit of sachet water = 1.5seconds

a_{12}: Number of labour hour used to produce one unit of bottle water = 7.083seconds

a_{13}: Number of labour hour used to produce one unit of apple juice = 8.5714seconds

a_{14}: Number of labour hour used to produce one unit of orange juice = 8.5714seconds

a_{15}: Number of labour hour used to produce one unit of yoghurt = 75seconds

a_{16}: Number of labour hour used to produce one unit of apple juice = 14.094 seconds

2.2.2 Machine Hours used:

a_{21}: Number of machine hour used to produce one unit of sachet water = 1 second

a_{22}: Number of machine hours used to produce one unit of bottle water = 5 seconds

a_{23}: Number of machine hours used to produce one unit of apple juice = 8.75 seconds

a_{24}: Number of machine hours used to produce one unit of orange juice = 8.75 seconds

a_{25}: Number of machine hours used to produce one unit of yoghurt = 50 seconds

a_{26}: Number of machine hours used to produce one unit of apple juice = 12.5 seconds

2.2.3 Amount of Materials used:

a_{31}: Amount of material used to produce one unit of sachet water = 0.12

a_{32}: Amount of material used to produce one unit of bottle water = 0.34kg

a_{33}: Amount of material used to produce one unit of apple juice = 3.49kg

a_{34}: Amount of material used to produce one unit of orange juice = 2.01kg

a_{35}: Amount of material used to produce one unit of yoghurt = 1.24kg

a_{36}: Amount of material used to produce one unit of communion drink = 3.40kg

Table 1: Hebron Drinks' Daily Production Data

Days	x_1 (bag)	x_2 (tray)	x_3 (tray)	x_4 (tray)	x_5 (tray)	x_6 (tray)	Available resources
Day I	700	700	41	80	240	260	2400
Day II	600	600	79	84	240	149	2000
Day III	500	300	39	99	-	220	1700

Day IV	700	700	26	64	215	190	2125
Day V	750	600	60	100	200	149	2318
Cost per week	#85427	#664.630	#185,220	#240,828	#676,620	#789,880	#2,629,565.40

Where $x_1, x_2, x_3, x_4, x_5, x_6$ are defined as follows:

x_1: Number of sachet water produced per day using labour hour, machine hours and materials

x_2: Number of bottle water produced per day using labour hour, machine hours and materials

x_3: Number of apple juice produced per day using labour hour, machine hours and materials

x_4: Number of orange juice produced per day using labour hour, machine hours and materials

x_5 : Number of yoghurt produced per day using labour hour, machine hours and materials

x_6: Number of communion produced per day using labour hour, machine hours and materials

Table 2: Cost of Producing One Unit of Product at Hebron Drinks

c_j	cost per unit of producing product j	Cost in naira(#)
c_1	cost per unit of producing sachet water	1.314
c_2	cost per unit of producing bottle water	19.1
c_3	cost per unit of producing apple juice	51
c_4	cost per unit of producing orange juice	47
c_5	cost per unit of producing yoghurt	63
c_6	cost per unit of production communion drink	68

From the information gathered at Hebron Drinks, the total cost of producing all the products, on a weekly basis, is 2.63 million naira. Also the minimum required capacity, in terms of, labour hour, machine hour and material are 12600 minutes, 2160 minutes and 149 kilograms respectively.

2.1 The Model

The resulting linear programming model from the above representations and formulation is as follows:

$$\text{Minimize } Z = \sum_{j=1}^{6} c_j x_j$$

Subject to

$$\sum_{j=1}^{6} a_{1j}x_j \geq 12600$$

$$\sum_{j=1}^{6} a_{2j}x_j \geq 2160$$

$$\sum_{j=1}^{6} a_{3j}x_j \geq 149$$

$$\sum_{i=1}^{3} a_{i1}x_1 \geq 65000$$

$$\sum_{i=1}^{3} a_{i2}x_2 \geq 34800$$

$$\sum_{i=1}^{3} a_{i3}x_3 \geq 4165$$

$$\sum_{i=1}^{3} a_{i4}x_4 \geq 5124$$

$$\sum_{i=1}^{3} a_{i5}x_5 \geq 10740$$

$$\sum_{i=1}^{3} a_{i6}x_6 \geq 11580$$

$$x_j \geq 0,\ j = 1, 2, ..., 6$$

Substituting the values of Cj's and a_{ij}'s , the model becomes;

$$Minimize\ Z = 1.314x_1 + 19.1x_2 + 51x_3 + 47x_4 + 63x_5 + 68x_6$$

$$0.025x_1 + 0.118x_2 + 0.143x_3 + 0.143x_4 + 1.25x_5 + 0.235x_6 \geq 12600$$

$$0.016x_1 + 0.083x_2 + 0.146x_3 + 0.146x_4 + 0.83x_5 + 0.21x_6 \geq 2160$$

$$0.12x_1 + 0.39x_2 + 3.49x_3 + 2.01x_4 + 1.24x_5 + 3.4x_6 \geq 149$$

$$0.1617x_1 \geq 65000$$

$$0.540x_2 \geq 34800$$

$$5.114x_3 \geq 4165$$

$$3.634x_4 \geq 5124$$

$$3.32x_5 \geq 10740$$

$$3.843x_6 \geq 11580$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

2.3 The Dual

In this section we find the dual of the above minimization problem, resulting in the following maximization problem, with A,B,C,D,E,F,G,H,I,J,K,L,M,N,O as the new decision variables.

$$\text{Max}\,imize\ P = 12600A + 2160B + 149C + 65000D + 34800E + 4165F + 5124G + 10740H + 11580I$$

Subject to :

$0.025A + 0.016B + 0.12C + 0.1617D$ 34

$$0.118A + 0.083B + 0.39C + 0.54E \leq 19.1$$

$$0.143A + 0.146B + 3.49C + 5.114F \leq 51$$

$$0.143A + 0.146B + 2.01C + 3.634G \leq 47$$

$$1.25A + 0.83B + 1.24C + 3.32H \leq 63$$

$$0.235A + 0.21B + 3.4C + 3.843I \leq 68$$

$$A, B, C, D, E, F, G, H, I \geq 0$$

2.4 The Standardized Form of the Dual Problem

In order to form the initial tableau, we standardized the above dual problem by introducing slack variables as follows (M.C. Agarana and T.O. Olokunde, 2015)

$$\text{Max}\,imize\ P = 12600A + 2160B + 149C + 65000D + 34800E + 4165F + 5124G + 10740H + 11580I$$

$Subject\ to:$

$$0.025A + 0.016B + 0.12C + 0.1617D + J = 1.314$$

$$0.118A + 0.083B + 0.39C + 0.54E + K = 19.1$$

$$0.143A + 0.146B + 3.49C + 5.114F + L = 51$$

$$0.143A + 0.146B + 2.01C + 3.634G + M = 47$$

$$1.25A + 0.83B + 1.24C + 3.32H + N = 63$$

$$0.235A + 0.21B + 3.4C + 3.843I + O = 68$$

$$A, B, C, D, E, F, G, H, I, J, K, L, M, N, O \geq 0$$

Table 3: Initial Simplex Tableau

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	RHS
J	0.025	0.016	0.12	0.1617	0	0	0	0	0	1	0	0	0	0	0	1.314
K	0.118	0.083	0.39	0	0.54	0	0	0	0	0	1	0	0	0	0	19.1
L	0.143	0.146	3.49	0	0	5.114	0	0	0	0	0	1	0	0	0	51
M	0.143	0.146	2.01	0	0	0	3.634	0	0	0	0	0	1	0	0	47
N	1.25	0.83	1.24	0	0	0	0	3.32	0	0	0	0	0	1	0	63
O	0.235	0.21	3.4	0	0	0	0	0	3.843	0	0	0	0	0	1	68
P	12600	2160	149	65000	34800	4165	5124	10740	11580	0	0	0	0	0	0	0

This initial simplex tableau was solved using computer application software (LIP SOLVER), and the following results obtained. It was observed that six (6) iterations were involved. The tables resulting from the iterations give better results as the iterations progresses. The sixth iteration therefore gives the best result shown in table 9.

Table 4: Simplex Tableau 2 (First iteration)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	RHS
D	0.155	16/1617	0.742	1	0	0	0	0	0	6.18429	0	0	0	0	0	8.1262
K	0.219	0.083	0.39	0	0.54	0	0	0	0	0	1	0	0	0	0	19.1
L	0.143	73/500	3.49	0	0	5.114	0	0	0	0	0	1	0	0	0	51
M	0.143	73/500	2.01	0	0	0	3.634	0	0	0	0	0	1	0	0	47
N	1.25	0.83	1.24	0	0	0	0	3.32	0	0	0	0	0	1	0	63
O	0.235	0.21	3.4	0	0	0	0	0	3.843	0	0	0	0	0	1	68
P	2550.53	-4271.66	-48088.5	0	34800	4165	5124	10740	11580	-401979	0	0	0	0	0	528200

Table 5: Simplex tableau 3 (Second Iteration)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	RHS
D	0.155	0.099	0.742	1	0	0	0	0	0	6.18429	0	0	0	0	0	8.1262
E	0.219	0.083	0.722	0	1	0	0	0	0	0	1.85	0	0	0	0	35.37
L	0.143	0.146	3.49	0	0	5.114	0	0	0	0	0	1	0	0	0	51
M	0.143	0.146	2.01	0	0	0	3.634	0	0	0	0	0	1	0	0	47
N	1.25	0.83	1.24	0	0	0	0	3.32	0	0	0	0	0	1	0	63
O	0.235	0.21	3.4	0	0	0	0	0	3.843	0	0	0	0	0	1	68
P	-5053.92	-9620.55	-73221.8	0	0	4165	5124	10740	11580	-401979	-64444.4	0	0	0	0	1759090

Table 6: Simplex tableau 4 (Third Iteration)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	RHS
D	0.155	0.099	0.742	1	0	0	0	0	0	6.18429	0	0	0	0	0	8.1262
E	0.219	0.154	0.722	0	1	0	0	0	0	0	1.85	0	0	0	0	35.37
L	0.143	0.146	3.49	0	0	5.114	0	0	0	0	0	1	0	0	0	51
M	0.143	0.146	2.01	0	0	0	3.634	0	0	0	0	0	1	0	0	47
N	1.25	0.83	1.24	0	0	0	0	3.32	0	0	0	0	0	1	0	63
O	0.235	0.055	0.88473	0	0	0	0	0	1	0	0	0	0	0	0.26	17.6945
P	-5762.04	-10253.3	-83466.9	0	0	4165	5124	10740	0	-401979	-64444.4	0	0	0	-3013.27	1963990

Table 7: Simplex tableau 5 (Fourth Iteration)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	RHS
D	0.155	0.099	0.742	1	0	0	0	0	0	6.18429	0	0	0	0	0	8.1262
E	0.219	0.154	0.722	0	1	0	0	0	0	0	1.85	0	0	0	0	35.37
L	0.143	0.146	3.49	0	0	5.114	0	0	0	0	0	1	0	0	0	51
M	0.143	0.146	2.01	0	0	0	3.634	0	0	0	0	0	1	0	0	47
N	0.377	0.25	0.373	0	0	0	0	0	0	0	0	0	0	0.3012	0	18.9759
I	0.061	0.055	0.88473	0	0	0	0	0	1	0	0	0	0	0	0.26	17.6945
P	-9805.71	-12938.3	7478.3	0	0	4165	5124	0	0	-401979	-64444.4	0	0	-3234.94	-3013.27	2167790

Table 8: Simplex tableau 6 (Fifth iteration)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	RHS
D	0.155	0.099	0.742	1	0	0	0	0	0	6.18429	0	0	0	0	0	8.1262
E	0.219	0.154	0.722	0	1	0	0	0	0	0	1.85	0	0	0	0	35.37
L	0.143	0.146	3.49	0	0	5.114	0	0	0	0	0	1	0	0	0	51
G	0.143	0.0402	0.55311	0	0	0	1	0	0	0	0	0	0.2752	0	0	12.9334
H	0.377	0.25	0.373	0	0	0	0	0	0	0	0	0	0	0.3012	0	18.9759
I	0.061	0.055	0.88473	0	0	0	0	0	1	0	0	0	0	0	0.26	17.6945
P	-10007.3	-13144.2	-90312.4	0	0	4165	0	0	0	-401979	-64444.4	0	-1410.02	-3234.94	-3013.27	2167790

Table 9: Simplex tableau 7 (Sixth iteration)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	RHS
D	0.155	0.099	0.742	1	0	0	0	0	0	6.18429	0	0	0	0	0	8.1262
E	0.219	0.154	0.722	0	1	0	0	0	0	0	1.85	0	0	0	0	35.37
F	0.027	0.02855	0.68244	0	0	0	0	0	0	0	0	0.195	0	0	0	51
G	0.039	0.0402	0.55311	0	0	0	1	0	0	0	0	0	0.2752	0	0	12.9334
H	0.377	0.25	0.373	0	0	0	0	0	0	0	0	0	0	0.3012	0	18.9759
I	0.061	0.055	0.88473	0	0	0	0	0	1	0	0	0	0	0	0.26	17.6945
P	-10123.8	-13263.1	-93154.8	0	0	0	0	0	0	-401979	-64444.4	-814.423	-1410.02	-3234.94	-3013.27	2,275600

4. Interpretation of Result and Discussion

From the final tableau, the decision variables values are as follows:

$x_1 = 401979$

$x_2 = 64444.4$

$x_3 = 814.431$

$x_4 = 1410.02$

$x_5 = 3234.94$

$x_6 = 3013.27$

Substituting these values into the objective function of the primal problem, we have:

$$1.314(401979) + 19.1(64444.4) + 51(814.431) + 47(1410.02) + 63(3234.94) + 68(3031.27)$$
$$= 2,276,822.947 naira$$

This value is approximately the same as the optimal feasible solution shown in the final tableau (the sixth iteration). This means that the cost of production of the six (6) products in HEBRON DRINKS can be reduced to 2.276822.947 million naira as against the prevailing total cost of production of 2.63million naira per week. This implies that if the management of Hebron Drinks actually wants to maximize their profit, by efficiently putting the scarce available resources into use, the following must be put into consideration: Number of sachet water produced per day using labour hour, machine hours and materials must be 401979. Number of bottle water produced per day using labour hour, machine hours and materials must be 64444. Number of apple juice produced per day using labour hour, machine hours and materials must be 814. Number of orange juice produced per day using labour hour, machine hours and materials must be 1410.

Number of yoghurt produced per day using labour hour, machine hours and materials must be 3235.

Number of communion produced per day using labour hour, machine hours and materials must be 3013.

Looking at the values of the decision variables closely, we realized that less of apple juice should be produced while more of sachet water and bottled water should be produced. Doing this will minimize the usage of the available resources and maximize the company's profit.

4.1 Sensitivity Analysis

Sensitivity analyses were performed to identify key factors affecting the behaviour of the model. The sensitized parameters include:

- C_J 's and
- a_{IJ} 's

We had a new set of decision variables, as a result of varying the sensitized parameters, which when substituted into the objective function gave 2,037,604 and 2,168,249 respectively.

We can see that both the C_J 's and a_{IJ} 's are sensitive to the result of the model. The management of Hebron Drinks can manipulate these parameters to their benefit without necessarily altering their organizational policy or set goals in terms of expected income.

5. Conclusion

The data collected from HEBRON DRINKS was modeled into a Linear Programming Problem.

A primal problem was formed, and being a minimization case that would be solved using simplex method, it was imperative to find the Dual of the primal problem; hence a dual problem was developed out of the Linear Programming Problem. Upon obtaining the initial simplex tableau, the data was then run on a lip solver (linear integer problem solver) software application. The analysis done, using operations research technique (the linear programming problem, solved by using the simplex method) showed that average weekly cost of production at Hebron Drinks was 2,629,565.40 naira. Based on this research work, the company can reduce its weekly cost of production to 2,227,822.95 naira, which represents 13.41% decrease in the weekly production cost. The results also revealed that, for the decrease to be achieved there has to be an adherence to a new production routine. We can see that both the C_J's and a_{IJ}'s are sensitive to the result of the model. The management of Hebron Drinks can manipulate these parameters to their benefit without necessarily altering their organizational policy or set goals especially as it affects the contributions and amount of resources used in producing one unit of each of the products in order to maximize their expected income.

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Evaluation of Manufactured Product Performance Using Neural Networks

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Abstract: This paper discusses some of the several successful applications of neural networks which have made them a useful simulation tool. After several years of neglect, confidence in the accuracy of neural networks began to grow from the 1980s with applications in power, control and instrumentation and robotics to mention a few. Several successful industrial implementations of neural networks in the field of electrical engineering will be reviewed and results of the authors' research in the areas of food security and health will also be presented. The research results will show that successful neural simulation results using *Neurosolutions* software also translated to successful real-time implementation of cost-effective products with reliable overall performance of up to 90%.

Keywords: Neural Network, Ammonia, Back-propagation, NeuroSolutions, Supervised learning.

1. Introduction

The parallel processing capability of the human brain is the most powerful characteristic which has been applied to the design of neural networks. Parallelism is defined as the brain's ability to respond to an external stimulus in several different ways. Neural networks are designed to find the best-fitting solution to a problem given a number of specific 'objectives'. The objectives can be selected to minimize some cost, for example, or enhance utilization of limited resources. Neural networks are highly parallel, dynamic systems capable of performing data manipulations by means of their state response to input information (Lampinen and Heikkonen). Neural networks have been utilized in order to implement pattern recognition and data classification for decades. These networks mimic the way the human brain processes information through learning from past experience and parallel processing of information. Neural networks are particularly suited to tasks for which transformations of certain inputs to certain outputs have been established, but the transformations cannot be analytically determined due to certain constraints. Despite the fact that neural networks have been around since the 1940s, it was in the late 1980s that they began to be applied in industry. This is because the 'black box' approach to neural processing caused a lot of scepticism among researchers for nearly 50 years. However, since then, there has been a steady rise both in the frequency and areas of application of artificial neural networks from prediction and system modelling to pattern recognition systems and robust classifiers.

There are two common ways of classifying neural networks. The first is by the network structure and the second is by the training technique. In terms of network structure, neural networks can be described in various forms based on the arrangement and interaction between the layers of neurons in the network. Some of these neural interactions may be cyclical or feedback, feed forward or recurrent in nature (to mention a few). Neural networks can be trained through either supervised or unsupervised (self-organized) approach. In spite of the structural and learning differences among these neural networks, they all receive information from the outside world, process this information and generate results based on the outcome of the processing. Therefore, the degree of success of a neural approach to solving a problem depends on how well it can learn and approximate relationships between inputs and outputs.

This paper provides an overview of industrial applications of neural networks while also answering pertinent questions some of which include: what problems require a solution by neural networks? What is the best neural model for a specific problem? Why are neural networks better than conventional methods? Neural applications to product development from the authors' research will also be presented with results showing the immense importance of such networks in successful product development.

2. Evolution Of Neural Networks: Discovery To Industrial Application

The advent of the neural network began in 1943 when Walter Pitts and Warren McCulloch proposed that it was possible to mathematically model the behaviour of the biological neural system (McCulloch & Pitts, 1943). This model came to be popularly known as an *artificial* neural network or ANN. Later in 1961, the term *cybernetics* was used by Norbert Weiner to describe this new field which linked engineering, biology, control systems and brain function (Weiner, 1961). This interdisciplinary discovery led to the development of the Von Neumann computer. This computing model led to the development of a learning machine based on Hebb's rule (Hebb, 1949). The idea of the *perceptron* was proposed by Frank

Rosenblatt, head of the cognitive systems division of the Cornell Aeronautical Laboratory in 1958. This perceptron formed the basis of the present day neural networks and led to the realisation of the Mark I perceptron in 1960. The Mark I could learn new skills by trial-and error by means of a neural network capable of simulating human thought processes. The adaptive linear element was proposed by Bernard Widrow and Ted Hoff in 1960 which was similar to the perceptron but included a form of learning via an error correction rule. This technique was based on the least mean square (LMS) algorithm which was successfully used to eliminate echoes in analogue telephony. This was the first documented application of neural networks in industry (Widrow, 1988).

The growing popularity of neural networks particularly between the 1950s and early 60s was thwarted when Marvin Minsky and Seymour Papert published a book titled *Perceptrons* in 1969. In it, they proved mathematically that the perceptron could not be applied in problems that presented as nonseparable logic functions (Minsky & Papert, 1969). Afterwards, the 1970s were a dry period for research involving neural networks with perhaps the most notable being the discovery of back-propagation by Paul Werbos in 1974. This learning algorithm was later rediscovered in the mid-80s by Parker and Rumelhart et al (Parker, 1986; Rumelhart, Widrow, & Lehr, 1994). During this period, various forms of the neural network were realised such as the Boltzmann Machine by Sejnowski, Hinton and Ackley (Hinton & Sejnowski, 1986), radial basis function (RBF) networks by Broomhead and Lowe (Broomhead & Lowe, 1988) and the Hopfield recurrent neural network (Hopfield, 1982).

Renewed interest in the field by the 1980s prompted DARPA to undertake a study of neural applications in industry. The study listed various applications of neural networks including commercial, modelling, control, image and speech recognition, and planning. Fig 1 shows engineering applications of neural networks since 1988. As seen from Fig 1, many control and classification processes have been successfully implemented using a number of neural structures (in some cases a combination of structures). The figure also shows that neural networks are often applied to pattern recognition and classification problems which can either be generic or specific. A generic problem involves methods which are not specific with regard to the parameters and variables used. They are therefore referred to as *domain-free* problems. A specific problem on the other hand, is defined by its parameters, values and constraints which depend on the application area in which the problem arises (Kasabov, 1996). Other application areas of neural networks which involve heuristics include image and speech recognition, character recognition as well as planning and forecasting. The continued success of the neural network in industrial applications is the main motivation behind this paper. In addition to reviewing other work, the paper will present the authors' work in the area of neural simulations and will also discuss how successful results were translated into real-time, functional devices.

3. Choice Of Neural Network Structures And Data Requirements

The success of a neural network's performance depends largely on the structure of the chosen network for the specific problem and also on the parameters used to train the network. This section discusses these important concepts.

3.1 Neural Network Structures

There are several neural network structures that have been successfully implemented in various engineering applications. Table I shows a classification of neural network structures based on their areas of application.

Multi-layer Perceptron (MLP): This is a neural structure in which the output of each element or node is connected to nodes in forward layers in series without any intra-layer connections. This structure was introduced to overcome the limitations of the single-layer perceptron as revealed in *Perceptrons*. They showed that the single-layer perceptron could only solve convex problems. However, Rumelhart et al showed that a 2-layer MLP could solve non-convex problems. Networks with 3 or more layers can essentially solve boundless problems.

MLPs have been used in several applications such as speed control of dc motors (Rabaai & Kotaru, 2000; Venyagamoorthy & Harley, 1999), induction motor fault diagnosis (Chow, Mangum, & Yee, 1991) (Chow, Sharpe, & Hung, 1993; Filipetti, Franceschini, Tassoni, & Vas, 2000), induction motor control (Burton & Harley, 1998; Huang, Chen, & Huang, 1999; Wishart & Harley, 1995), feedback control (Er & Liew, 1997; Hashimoto, Kubota, Sato, & Harashima, 1992; Ozaki, Suzuki, Furuhashi, Okuma, & Uchikawa, 1991) and fault diagnosis of robotic systems (Vemuri & Polycarpou, 1997). The topology of the multi-layer perceptron is depicted in Figure 2.

Recurrent neural network (RNN): This neural network structure is realised by feeding the network's output back into the input after a learning session (epoch) has been completed. It was first proposed by

Rumelhart et al in 1986 (Rumelhart, Hinton, & Williams, 1986). The RNN structure can be likened to back-propagation with few hidden layers with each recurrent cycle representing exactly one instance of hidden layer activity. Examples of the RNN include Jordan, Elman and Hopfield networks and Boltzmann machines.

Recurrent neural networks have been used in the spectral analysis of periodic waveforms used to describe engineering phenomena (Bhat, Minderman, McAvoy, & Wang, 1990). Fault detection and isolation in realtime engineering systems has also been achieved using Hopfield networks (Sirinivasan & Batur, 1994).

ARTnet: The adaptive resonance theory network was developed to function as a self-organizing network by retaining knowledge of previously learned patterns. In other words, it closely resembles the biological neural network's capability of learning from past experience. The ART network comprises a comparison layer, a recognition layer and a reset element. The comparison layer consists of three (3) inputs: the recognition layer output, the input vector and a gain g_1. The output of this layer is 1 if and only if at least two of its inputs are 1. The recognition layer is used as a classifier and also has three (3) inputs: the reset element's output, a vector v_j, and a gain g_2. The recognition layer neuron with the winning combination of vector v will output a 1 if and only if $g_2 = 1$. All other combinations will output 0.

Hence, the recognition layer classifies the input vector. The ART network has been successfully applied in sensor pattern interpretation (Whiteley, Davis, Mehrotra, & Ahalt).

Kohonen Network: This is another form of self-organizing network whose topology is different from that of the ART network. In Kohonen networks, the output nodes are ordered in the form of an array determined by the user. The ordering process involves selecting which set of output nodes are neighbours (Krose & Smagt, 1996). When learning patterns are presented to the network, output nodes are adapted in such a way that the order of the input space is replicated at the output. In other words, learning patterns which are near to each other in the input matrix must be matched to output units which are also close to each other. Assume the input vector is $\boldsymbol{I}^{\mathbf{N}}$ and a sample **s**(t) is presented to the network. The winning node **u** is adapted using the Equation (1):

$$w_k(t+1) = w_k(t) + \eta d(k,u)(s(t) - w_k(t)) \quad (1)$$

The collective learning scheme in

Equation (1) ensures that input signals which are near to each other are mapped unto neighbouring neurons. Therefore, the topology in the input signals is preserved in the mapping. The Kohonen network has been extensively used in classification and pattern recognition applications (Sardy & Ibrahim, 1996).

Probabilistic Neural Network (PNN): This neural structure is similar to the MLP. They differ from MLPs in terms of activation functions (usually exponential functions) and synaptic patterns. Unlike the MLP, the hidden neurons are usually not fully connected. This fewer numbers of connections makes this form of neural network easy to train. The PNN operates in parallel with the signal flowing in one direction only (Meireles, Almeida, & Simoes, 2003). PNNs have been used in the identification of transients in nuclear power plants (Attieh, Gribok, Hines, & Uhrig, 2000).

Radial Basis Function (RBF) Networks: This network structure consists of RBF nodes as process units. The input nodes all connect to these hidden units with the output nodes being summations (Meireles, Almeida, & Simoes, 2003). RBF networks are particularly suitable for fault diagnosis applications because they are fast to train compared to MLPs for instance. They have been used to train a robotic hand (Dini & Failli, 2000), for generator system control (Flynn, McCloone, & Irwin, 1997), power electronic drives and digital signal processors (Dote, Strefezza, & Suyitno, 1993).

Polynomial Neural Networks: Polynomial networks are termed 'plastic networks' because their structure is determined during network training. As a result, no two applications can have the same polynomial network structure. The flexible nature of polynomial networks makes them useful in control applications in which the dimension of the plant is not known. Polynomial networks have been used to implement filter design (Silva, Bose, & Pinto, 1999).

3.2 Data Requirements

The data requirements for successful implementation of a neural network answer the important questions of when, how and exactly which problems require a neural-based solution. Problems that are heuristic in nature are typically good candidates for neural solutions. Other neural-solution based problems include those that require classification, regression or pattern recognition of large solution spaces requiring a generic solution. The following are some of the important data requirements which must be ascertained to obtain a neural structure likely to solve the problem:

- Network Structure: This

answers the question regarding whether a single-layer, multi-layer or recurrent network should be employed. A single-layer network (the adaline, for example) is particularly suitable for logic-based applications. Some logic tasks such as the XOR implementation require an additional layer for obtaining accurate output. Multi-layer networks (such as the MLP) are suitable for classification and pattern recognition purposes. Recurrent networks have been used in data recognition applications such as handwritten character recognition (Frinken, Fischer, Manmatha, & Bunke) and the results have been quite encouraging.

- Activation Function: The manner in which neurons interact with each other in the neural network is determined by the choice of activation function. The activation function is usually selected such that the network's overall behaviour remains stable throughout the training process i.e. the neuron outputs are as close as possible to their individual activation levels (Krose & Smagt, 1996). There are several types of activation functions. The most common are the linear, semi-linear and sigmoid functions. The network's transfer function is a ratio of the output function to its activation function (Essiet, 2014).
- Interconnected Weights: These connect the layers of the neural network and are used to adjust layer output(s). Assume two interconnected neurons *m* and *n* have a weight connection W_{mn} such that $\{W_{mn} \in 0.......1\}$. The significance of weight connections is determined by the proximity of their absolute value to unity.
- Data Flow: This specifies the direction in which information flows between network layers. In some neural structures, data flow is from input to output in one forward direction, while in others, it may be fed back from output to input or within the same layer.
- Input Signals: Input signals are of various types such as binary, bipolar or continuous. Binary inputs can be in either one of two states, while bipolar inputs are concerned with the direction of the inputs. In other words, inputs can be in one direction or in the exact opposite. Inputs represented by a string of real values constitute continuous inputs.

Over the years, the MLP has

been the most commonly used NN structure. As at 1995, the ratio of neural network utilisation was: MLPs recorded 81%; Hopfield was 5% and Kohonen, 8% (Haykin, 1995). Figure 3 shows the utilisation in power engineering as at 2007. The MLP utilisation is about 33%, which is the highest percentage.

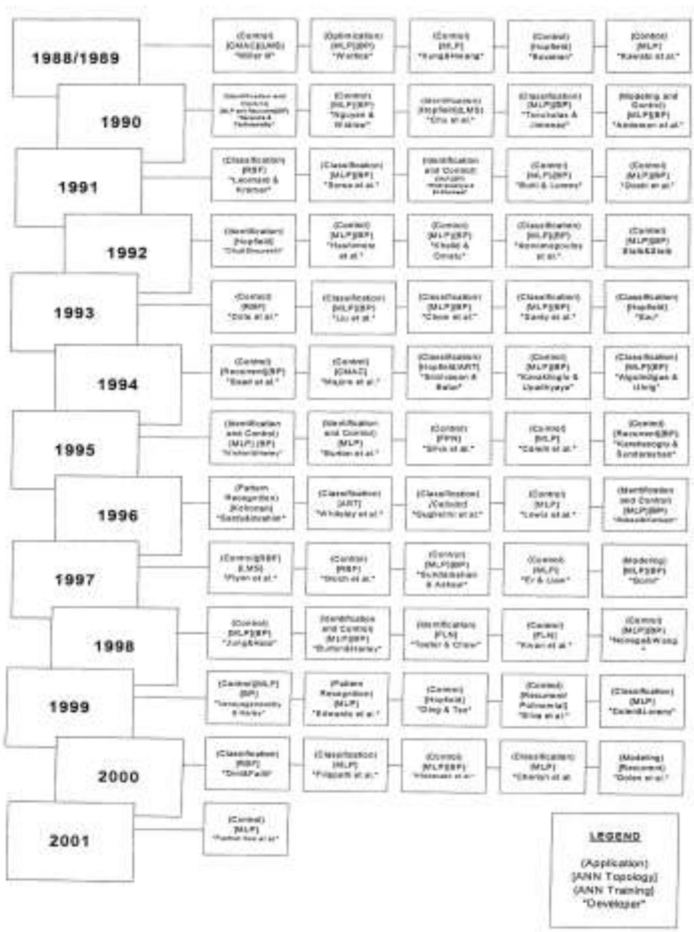

Figure 1. Documented Applications of Neural Networks in Industry since 1980s (Meireles, Almeida, & Simoes, 2003)

4. NN Applications in Engineering

Since the 1980s, application of neural networks to the analysis of engineering-based problems has been on the increase. This phenomenal growth is attributed to the success of results obtained from neural simulations over the years. Today, neural networks are being used in control, power and telecommunications engineering as well as robotics and instrumentation. This section outlines the use of neural networks in power system security assessment and data classification for electronic sensor circuits.

4.1 Power Systems Security Assessment

Security assessment with regard to power systems is defined as the process of determining the presence and extent of interference to the normal operation of power systems. It also involves determining system stability in response to certain external perturbations in either its present or future state (Kalyani & Swarup, 2009). There are basically two forms of security assessment: static and transient. The neural approach was proposed because traditional methods of security evaluation based on load flow and transient stability analysis are unrealistic for real-time implementation (Kalyani & Swarup, 2009). Table IV shows the simulation results.

Training and testing vectors for the NN training were obtained by sequential forward selection (SFS) method. NN model implementation was simulated in IEEE 14 bus, 30 bus and 57 bus test systems respectively (Haque & Kashtiban, 2007). Results obtained showed that PNN and ARTNN gave high classification accuracies of 100% and 96% respectively. Mean squared error (MSE) and classification accuracy are evaluated according to Equations (2) and (3).

$$MSE = \frac{1}{n}\sum_{k=1}^{n}(E_k)^2 \quad ; E_k = \left|DO_k - AO_k\right| \qquad (2)$$

Where n= no of data test samples

DO_k= target output

AO_k= actual neural network output as given by the neural network

$$CA(\%) = \frac{no\ of\ correctly\ classified\ samples}{total\ test\ samples} \times 100\% \qquad (3)$$

4.2 Neural Simulation of Food Classification and Tooth Decay Sensor Circuits

*T*his section focuses on the authors' research in the application of neural network to the real-time realisation of electronic circuits for the classification of food condition and identification of tooth decay respectively. The neural software used in the simulation is NeuroSolutions version 5 training software. The purpose of the neural simulations is to select an appropriate sensor capable of yielding the most accurate data classification for the applications mentioned above. In both cases, an MLP was used to obtain the best simulation results. The results obtained in this work answer the three most important questions raised in the introduction section viz: the task associated with both applications is that of data classification, which is particularly suited to neural networks. From both literature and experimentation, the MLP has been found to give the most accurate classification results. The neural approach has been found to be better than conventional methods in these applications because it is cost-effective in terms of both time and resources. Conventional approach would basically involve a trial-and-error approach to selecting best-performing sensor(s). Normalized data samples were obtained from constructed ammonia sensor circuits using a TGS 2602 metal oxide semiconductor (MOS) sensor. Tables II and III show simulation results for food and tooth decay data classifications respectively. Overall accuracy of 92.3% and 85% for the food and tooth decay classification neural networks respectively have resulted in the real-time implementation of electronic circuits capable of performing these classifications as shown in Figure 4.

Further research is presently being carried out to improve the accuracy of the constructed circuits by fine tuning training data for the neural network model. From research carried out so far, the MLP has yielded the most accurate classification results. It was also observed that MLP models with more than two process layers resulted in delayed convergence, likely due to over fitting of training data points in relation to the target data.

Table I. Classification of Neural Network Structures based on their application areas (Meireles, Almeida, & Simoes, 2003)

Functional Characteristics	Structure
Pattern Recognition	MLP, Hopfield, Kohonen, PNN
Associative Memory	Hopfield, recurrent MLP, Kohonen
Optimization	Hopfield, ART, CNN
Function Approximation	MLP, CMAC, RBF
Modeling and Control	MLP, recurrent MLP, CMAC, FLN, FPN
Image Processing	CNN, Hopfield
Classification (including Clustering)	MLP, Kohonen, RBF, ART, PNN

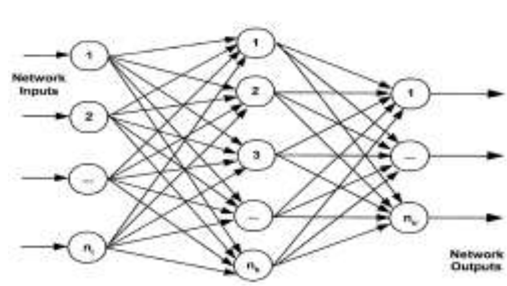

Figure 2. Structure of the MLP (Meireles, Almeida, & Simoes, 2003)

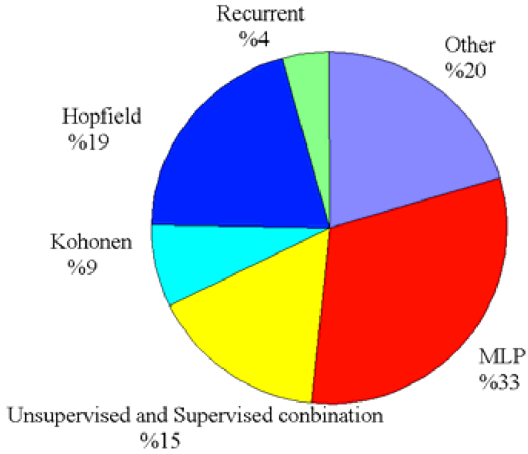

Figure 3. Proportional usage of NN types in Security Assessment of Power Systems (Haque & Kashtiban, 2007)

Table II. Neural Simulation Results for Food Classification NN

	Bad (predicted)	Not Bad (predicted)	Accuracy %
Bad (actual)	92.3	7.3	92.3
Not Bad (actual)	7.7	92.3	92.3
Overall Accuracy			**92.3**

Table III. Neural Classification Results for Tooth Decay Breath Samples

	Kidney failure (predicted)	Non kidney failure (predicted)	Accuracy %
Kidney failure (actual)	9	1	90.0
Non kidney failure (actual)	2	8	80.0
Overall Accuracy			85.0

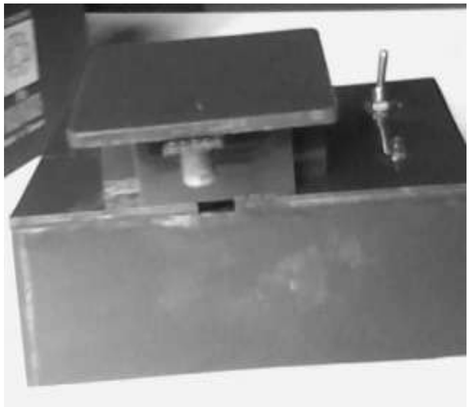

Figure 4. Real-Time Implementation of Electronic Circuit for Food and Tooth Decay Sample Classification

5. Conclusion

The paper has examined the applicability of neural networks in engineering research and practice. It has been shown that the level of confidence in the results of neural simulations in engineering-based applications is on the increase. As a result, neural networks are expected to become more commonplace in years to come. Results from the authors' research also show that neural simulation results can also be used to realise reliable devices for real-time application in health and food security. One important advantage of the neural approach is the prediction of the performance of the physical system via simulation.

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Statistical Analysis on Students' Performance

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Abstract: This research uses Cohen's Kappa to examine the performance of students in the Faculty of Science, University of Ilorin. The data was collected from eight departments in the faculty and it covers the performance of students measured by their Grade Point Average (GPA) and Cumulative Grade Point Average (CGPA) in both their first and final year between 2000-2006 academic sessions. It is of interest to determine the proportion of students that improved on their performance, dropped from the class of grade point which they started with and those that maintained their performance using psychometrics approach. Also, the strength of agreement that exist between the first and the final year was examined.

Keywords: Cohen's Kappa, Intra-class Kappa, Agreement, Raters.

1. Introduction

Education in a broad sense is the process of exposing the individuals to concepts and activities which physically, mentally, morally and spiritually help equip him/her with the knowledge of things around him. Education also exposes the individual to further knowledge by means of books, mass media and academic institutions.

From the foregoing, thousands of people normally apply into the Nigeria universities-the peak of tertiary institutions. The search for knowledge and the little recognition of certificates of the lower tertiary institution in Nigeria labor market has subjected the university into an over-crowded community with many still outside, eager to add to the congestion. In order to bring about fair play and to exercise justice in the admission of candidates into the universities, the National University Commission (NUC) was set up to look into the affairs of the universities. The commission was established in 1978 and it has since embarked on some policies so as to ensure that admission processes are conducted without duplication of admission.

The information used for this research was obtained by the method of transcription from records and they are all secondary type of data. Data were collected from eight departments in Faculty of Science: Biochemistry, Chemistry, Physics, Geology, Computer Science, Mathematics, Statistics and Microbiology. The data include students' performance measured by their GPA and CGPA as appropriate, in both their first and final year.

The aim of this research is to use Cohen's Kappa to study students' performance of some departments in the Faculty of Science, University of Ilorin and objectives are: to know the strength of agreement that exist between student grade point in both their first and final year, to know the proportion of students that maintained their CGPA (i.e. those that maintained what they started with), to know the proportion of students that dropped from the class of grade point they started with and to know the proportion of students that improved on their performance.

In a study conducted by (Akinrefon & Balogun, 2014), control chart was used to monitor students' performances, the causes underlying the charting statistics that are less than the lower control limits were identified which indicate a negative shift in students CGPA. Also, the reason for charting statistics falling above the upper control limit, which indicate the positive shift in student CGPA was identified. Then, a solution to correct students' poor performance and was suggested. If the charting statistics for all the semester fall within the control limits, the student has maintained the desire target GPA value.

According to (Balogun *et al.,* 2014) the main focus of their research is to develop models that can be used to study the trend of graduate emigration in Nigeria using log-linear modeling based on the results from of likelihood ratio (G^2), Akaike Information Criteria (AIC) , Saturated model has a perfect fit for modeling graduate emigration in Nigeria. This implies all the three factors involved (discipline, year and sex) has to be included in the model in order to have an appropriate result.

Since its introduction Kappa statistics, several authors have applied the concept in different field, for instance; (Zeeshan *et al.,* 2015) carried out an initial audit for evaluating the case notes for each team against the TONK score. In order to evaluate the producibility of this score, the Cohen's kappa was used and substantial agreement was noted. The article by (Viera & Garret, 2015) provided a basic overview of Kappa statistics as one measure of inter-observer agreement. They concluded that "Kappa is affected by prevalence but nonetheless kappa can provide more information than a simple calculation of the raw proportion of agreement".

(Kilem, 2002) established that the unpredictable behavior of the PI and Kappa statistics is due to a wrong method of computing the chance agreement probability. (Warrens, 2015) reviewed five ways to look at Cohen's kappa. Nevertheless, the five approaches illustrate the diversity of interpretations available to researchers who use kappa. In (Wang *et al.*, 2015) Cohen's kappa coefficient was used to assess between raters agreement, which has the desirable property of correcting for chance agreement. It was concluded that despite the limitations, the kappa coefficient is an informative measure of agreement in most circumstance that is widely used in clinical research.

1.1 Categorical Response Data

A categorical variable is one which the measurement scale consists of set of categories. For instance, political philosophy may be measured as "liberal", "moderate", or "conservative" also smoking status might be measured using categories "never smoked", "former smoker" and "current smoker" etc. Though categorical scales are common in the social and biomedical science, they occur frequently in the behavioral sciences, public health, ecology education and marketing. They even occur in highly quantitative field such as engineering science and industrial quality control.

1.2 Categorical Data Analysis

These are data consisting of a classification of the behavior or subjects into a number of mutually exclusive and exhaustive corresponding categories. A multivariate quantitative data is one in which each individual is described by a number of attributes. All individuals with the same description are enumerated and count is entered into a cell of the resulting contingency table

1.3 Contingency Tables

The multidimensional table in which each dimension is specified by discrete variable or grouped continuous (range) variable gives a basic summary for multivariate discrete and grouped continuous data. If the cell of the table are number of observation in the corresponding values of the discrete variables then it is CONTINGENCY TABLES. The discrete or grouped continuous variables that can be used to classify a table are known as FACTORS. Examples include Sex (Male or Female), religion (Christianity, Islam, Traditional etc.).

Types of Contingency table:

One dimensional $(1\times J)$ tables

Two dimensional (I×J) tables

Square tables (I×I)

Multidimensional tables

1.4 Measures of Agreement

Agreement is a special case of association which reflects the extent to which observers classify a given subject identically into the same category. In order to assess the psychometric integrity of different ratings, inter-raters agreement is computed. Inter-rater reliability coefficients reveal the similarity or consistency of the pattern of responses, or the ranking-ordering of responses between two or more raters (or two or more rating sources), independent of the level or magnitude of those ratings. For example, let us consider the table 1.

Ratings of three subjects by three raters, one observes from the table 1 that all the raters were consistent in their ratings, rater 2 maintained his leading ratings followed by rater 1 and rater 3 respectively.

Inter-rater agreement on the other hand is to measure that ratings are similar in level or magnitude. It pertains to the extent to which the raters classify a given subject identically into the same category. (Kozlowski & Haltrup, 1992) noted that an inert-rater agreement index is designed to "reference the interchangeability among raters: it addresses the extent to which raters makes essentially the same ratings". Thus, theoretically, obtaining high levels of agreement should be more difficult than obtaining high levels of reliability or consistency.

2. Materials and Method

2.1 *Kappa Statistics*

There is wide disagreement about the usefulness of Kappa statistics to assess rater agreement. At the least, it can be said that: kappa statistics should not be viewed as the unequivocal standard or default way to quantity agreement, it should be concerned about using a statistics that is the source of so much controversy and it should consider alternatives and make an informed choice.

One can distinguish between two possible uses of kappa as a way to test rater independence (that is, as a test statistic), and as a way to qualify the level of agreement (that is, as an effect-size measure).

2.2 Cohen's Kappa Coefficient

Cohen's kappa is one of the most commonly used statistic for assessing the nominal agreement between two raters (Warrens, 2010; 2011).

(Cohen, 1960) proposed a standardized coefficient of raw agreement for nominal scales in terms of the proportion of the subjects classified into the same category by the two observers. However, the idea of having an agreement measure was anticipated before 1960. For example, decades earlier Corrado Gini already considered measures for assessing agreement on a nominal scale (Warrens, 2013). The proportion is estimated as;

$$\pi_i = \sum_{i=1}^{I} \pi_{ii} \qquad (1)$$

And under the baseline constraints of complete independence between ratings by the two observers, which is the expected agreement proportion estimated as;

$$\pi_0 = \sum_{i=1}^{I} \pi_{i.}\pi_{.j} \quad (2)$$

The kappa statistics can now be written as;

$$K_c = \frac{\pi_i - \pi_0}{1-\pi_0} \quad (3)$$

where π_i and π_0 are as defined above

(Landis & Koch, 1977a) have characterized different ranges of values for kappa with respect to the degree of agreement they suggest. Although, these original suggestions were admitted to be "clearly arbitrary", they have become incorporated into the literature as standards for the interpretation of the kappa values. For most purposes, values greater than 0.75 or so may be taken to represent excellent agreement beyond chance, values below 0.40 or so may be taken to represent poor agreement beyond chance, values between 0.40 and 0.75 may be taken to represent fair to good agreement beyond chance and this is clearly shown in table 2. Bias of one rater relative to another refers to discrepancies between these marginal distributions. Bias decreases as the marginal distributions becomes more nearly equivalent. The effect of rater bias on kappa has been investigated by (Feinstein & Ciccheti 1990) and (Bryt *et al.*, 1993).

Early approaches to this problem have focused on the observed proportion of agreement; see (Goodman & Kruskal, 1954), this suggests the chance that the agreement can be ignored. Later, Cohen's kappa was introduced for measuring nominal scale chance-corrected agreement. (Scott, 1955) defined π_e using the underlying assumption that the distribution of proportion over the Ith categories for the population is known, and is equal for the two raters. Therefore, if the two raters are interchangeable, in the sense that the marginal distributions are identical, then Cohen's and Scott's measures are equivalent because Cohen's kappa is an extension of Scott's index of chance-corrected measures. To determine whether K differs significantly from zero, one could use the asymptotic variance formulae given by (Fleiss *et al.*, 1969) for the general $I \times I$ table. For large n, Fleiss' formulae is practically equivalent to the exact variance derived by (Everitt, 1968) based on the central hypergeometric distribution. Under the hypothesis of only chance agreement, the estimated large-sample variance of K is given by;

$$\tilde{V}ar_0(K_e) = \frac{\pi_e - \pi_e^2 - \sum_{i=1}^{I} \pi_{i.}\pi_{.i}(\pi_{i.} + \pi_{.i})}{n(1-\pi_e)^2} \ldots\ldots(4)$$

Assuming that

$$\frac{\tilde{K}}{\sqrt{\tilde{V}ar_0(\tilde{K})}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Follows a normal distribution, we can test the hypothesis of the chance agreement with reference to the standard normal distribution

2.3 *Intraclass Kappa*

Intraclass kappa was defined for data consisting of blind dichotomous ratings on each of n subjects by two fixed raters. It is assumed that the ratings on a subject are interchangeable, that is, in the population of subject; the two ratings for each subject have a distribution that is invariant under permutations of the raters to ensure that there is no rater bias (Scott, 1955), (Bloch & Kraemer, 1988), (Donner & Eliasziw, 1992) and (Banergee *et al.,* 1999). The Intraclass is estimated as;

$$K_i = \frac{\pi_0^e - \pi_e^*}{1 - \pi_e^*} \qquad (6)$$

where

$$\pi_e^* = \sum \pi_{ii}$$

$$\pi_0^e = \sum \left[\frac{\frac{\pi_{i.} + \pi_{.j}}{\pi_{..}}}{2} \right]^2$$

Furthermore, to obtain the proportion of those that maintained the performance or grade they started with, the proportion of those that improved on their performance and also the proportion of those that dropped from the class of grade point they started with. Let

P_1= the proportion of those that maintained what they started with, that is, the diagonal table

P_2 = the proportion of those that improved on their performance, that is, those below the diagonal table

P_3= the proportion of those that dropped from the class of grade point they started with, that is, those above the diagonal table.

3. Data Analysis

This section shows the analysis on Cohen kappa, Intra-class kappa statistic and the proportion of students who maintained, dropped and improved on their performance as shown in table 3 and 4 below.

This calculation is done to demonstrate the percentage of students who maintained, improved and dropped in the CGPA they started with

$$P_1 = \frac{4.3538}{7.9998} \times 100 = 54.42$$

$$P_2 = \frac{2.7513}{7.9998} \times 100 = 34.39$$

$$P_3 = \frac{0.8947}{7.9998} \times 100 = 11.18$$

Discussion, Summary and Conclusion

4.1 *Discussion*

For Physics Department, 55.56% of the students were able to maintain their grade point, 35.56% of the students improved while 8.89% dropped from the class of grade point they started with.

For Statistics Department, 74.04% of the students were able to maintain their grade point, 18.52% of the students improved while 7.4% dropped from the class of grade point they started with.

For Microbiology Department, 48.98% of the students were able to maintain their grade point, 44.89% of the students improved while 6.12dropped from the class of grade point they started with.

For Mathematics Department, 36% of the students were able to maintain their grade point, 64% of the students improved while none dropped from the class of grade point they started with.

For Geology Department, 46.15% of the students were able to maintain their grade point, 49.23% of the students improved while 4.62% dropped from the class of grade point they started with.

For Computer Science Department, 51.49% of the students were able to maintain their grade point, 1.79% of the students improved while 46.71% dropped from the class of grade point they started with.

For Biochemistry Department, 60.17% of the students were able to maintain their grade point, 29.66% of the students improved while 10.17% dropped from the class of grade point they started with.

For Chemistry Department, 62.96% of the students were able to maintain their grade point, 31.46% of the students improved while 5.56% dropped from the class of grade point they started with.

4.2.1 Summary

From the above interpretation, we could see that 54.42% of the students were able to maintain their CGPA that they started with, 34.39% of the students improved and 11.18% of the students dropped from the class of grade point they started with. Also, the strength of agreement between the first and the final year result is on the 0.40%.

4.3 *Conclusion*

It can be observed that Mathematics department has the highest number of students that improved on their performance, Statistics department had the highest number of students that maintained their grade point and Computer Science department had the highest number of students that dropped from the grade point they started with. Also, the strength of agreement that exist between the first and the final year result is on Average, that is, “fair”.

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Appendix

Table 1: Example of Raters

Subject	Rater 1	Rater 2	Rater 3
1	5	6	2
2	3	4	2
3	1	2	1

Table 2: The Range Of Kappa Statistic with the Respective Strength of Agreement

Kappa statistic	Strength of Agreement
<0.00	Poor
0.00-0.20	Slight
0.21-0.40	Fair
0.41-0.60	Moderate
0.61-0.80	Substantial
0.81-1.00	Almost perfect

Table 3: Cohen's and Intra-Class Kappa Estimates

S/No	**Department**	**Cohen's Kappa**	**Intra-class Kappa**
1	Physics	0.3410	0.3280
2	Statistics	0.6291	0.6279
3	Microbiology	0.2409	0.2214

4	Mathematics	0.0315	0.035
5	Computer Science	0.2861	0.2615
6	Geology	0.1955	0.1710
7	Biochemistry	0.4169	0.6017
8	Chemistry	0.3865	0.3822

Table 4: The proportion of Students that improved, maintained and dropped

S/No	**Department**	P_1	P_2	P_3	**Sum**
1	Physics	0.5556	0.3556	0.0889	1.0001=1
2	Statistics	0.7407	0.1852	0.074	0.9999=1
3	Microbiology	0.4898	0.4489	0.0612	0.9999=1
4	Mathematics	0.3600	0.6400	0	1.0000=1
5	Computer Science	0.5149	0.0179	0.4671	0.9999=1
6	Geology	0.4615	0.4923	0.0462	0.9999=1
7	Biochemistry	0.6017	0.2966	0.1017	0.9999=1
8	Chemistry	0.6296	0.3148	0.0556	0.9999=1

Table 5: The Strength of Agreement for each Department

S/No	**Department**	**Strength of Agreement**
1	Physics	Fair
2	Statistics	Substantial
3	Microbiology	Fair
4	Mathematics	Slight
5	Computer Science	Fair
6	Geology	Moderate
7	Biochemistry	Slight
8	Chemistry	Fair