Optimized Controller for Inverted Pendulum

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Abstract: Stability is require in any control system, most system require controller in order to be stable. Tuning is one of the major problems associated with most conventional controllers in existence today. This paper addressesd the difficulties associated with tunning by considering an effective optimized Controller on Inverted pendulum for the control of the angle position. Conventional PID controller was designed separately, to validate the proposed optimized controller. A MATLAB script for genetic algorithm was written with the aim of obtaining optimum PID parameters that would stabilized the pendulum angle at any desired inference inputs (i.e. returns the pendulum to a desire point as quick as possible). This would be achieved by minimizing an objective function (Integral time absolute error ITAE). On the other hand, a convention PID controller was designed using MATLAB/Simulink environment; the PID's gains were manually tuned until an optimum response is achieved. The results obtained in both schemes shows that the optimized controller proves more effective as compared to ordinary conventional PID controller, as optimized controller gives settling time, percentage overshoot of 5.02 seconds and 3% respectively as compared with settling time of 70 seconds and overshoot of 5% for conventional PID controller. Therefore, the proposed optimized controller can serve as a valuable and an effective controller for the control of inverted pendulum.

Keywords/Index Terms: Objective function; Pendulum angle; Genetic Algorithm; Stability, Optimized.

1. Introduction

Inverted pendulum is a pendulum which has its mass above its pivot point. This system is inherently not stable and must be actively balanced by moving the pivot point horizontally which serve as a feedback to the system or by oscillating the support rapidly up and down so that the oscillation is sufficiently strong enough to restore the pendulum from perturbation in a striking counter intuitive manner (Altinoz, Yilmaz and Weber, 2010). Inverted pendulum is used as benchmark for testing control algorithms due to its high degree of instability and non-linearity. Real application of the system can be found in Missiles guidance, Rockets, heavy Crane lifting containers in shipyards, self balancing Robots and etc.

GA is stochastic global search methods based on the mechanics of natural selection and natural genetic. They are iterative method widely used in optimization problems in general branches of science and technology. It was first proposed by Holland in 1976. GA offer some advantages over other search tools in the following ways (Magaji and Mustafa , 2010):

- GAs search from a population of points not a single point
- GAs use probabilistic transition rules not deterministic ones
- GAs work on encoding parameters set rather than the parameter set itself (except where real-valued individuals are used)
- GAs do not require derivative information or other auxiliary knowledge; only the objective function and the corresponding fitness levels influence the directions of the research.

To obtain a solution to a problem through genetic algorithms, the algorithm is started with a set of solutions (represented by chromosomes) termed as the population. This is Initialization. This is follow by Selection, i.e. choosing random solutions of one population forms a new population base on their evaluation on the objective function. This can be done either by Roulette wheel or Stochastic universal sampling. The formal was used because it ensures that each parent chance of being selected is proportional to its fitness value but possibility also exists to choose the worst population member. The new population is formed assuming that the new one will be better than the old one. Parent solutions are selected from the population to form new solutions (offspring) based on their fitness measure through the application of genetic operators such as crossover (exchange of genes from parents), mutation (sudden change in genes, this should however be introduce on a minimum probability) etc. These processes are repeated over several iterations until a stopping criterion is reached (Sumathi and Paneerselian, 2010).

Several control schemes have been designed and implemented by different researchers using different techniques in order to solve the above problem. In (Anguilar, Unpublished), theoretical and experimental results for balancing a single inverted pendulum using approximate input-output linearization and sliding mode control was presented, this however involved complex mathematics and as result affect the response of the system and also, it considered only zero input case. In Wang (2011), control laws such as input-output feedback linearization, Lyapunov second theorem and Lasalle's invariant principle were used, this however considered zero input case and also, settling time of 10 seconds was recorded. Wahida, Banu, and Manoj (2011), present a soft computing method for the controller of the inverted pendulum using Adaptive Neuro Fuzzy Inference System (ANFIS), this also consider a small angle variation, sluggish response and a steady state error of 0.2 radians were recorded. In Pandalai and Kataria (Unpublished), PID controller was designed for linearized model of the inverted pendulum; trial and error method was used in tuning the proportional-Interal-differential (PID) controllers, this waste time due to difficulty in tuning and settling time of 69 seconds was obtained. Another most popular method to tune the PID controllers is the Ziegler and Nichols method; this is a practical method for a single output and stable plants. Heuristic tuning of PID was considered in (Van Overschee et al, 2010). More systematic ways to optimize PID parameters has been proposed in, for instance, (Lee, Park and Brosilow, 1998, Lopez, Murrill and Smith, 2009 and Haupt and Haupt., 2002) uses the deterministic optimization methods base on the integrated Absolute Error (IAE) criterion however, these make use of MATLAB toolbox and their tuned parameters are only optimum in certain operational zones and have unsatisfactory design robustness property. In (Altinoz, Yilmaz, and Weber, 2010), Particle swarm optimizer were used to tuned PID gains, this however takes much time to converge and settling time of 8.2 seconds was recorded. Genetic Algorithm has proven to be a powerful search tools used by many researchers to optimize many complex function as well as PID controllers, most of it focus mainly on Integral of absolute error (IAE) as objective function (Haupt and Haupt, 2002). In this work, comparison between conventional PID and GA- PID controller for linearized model of the inverted pendulum is presented. Here a large variation of angle was considered, and ITAE was used as the objective function to improve the convergence, robustness of the Genetic algorithm. The PID controller was designed using MATLAB/Simulink environment and it's gains were tuned until a optimum response is obtained. On the other hand, the GA-PID is designed by writing an m-file that will automatically obtain PID parameters through the minimization of an objective function Integral time Absolute error (ITAE) using genetic algorithm. The rest of the paper is organized as follows: Section II, presented the model description, Section III, dwells on the controller design, Simulink representation and results comparison are presented in Section IV, and finally Section V concludes the paper.

2. Model Description

The model description of the inverted pendulum was obtained using Lagrange Equation, which is one of many methods that can be used to derive a mathematical modeling for a complex mechanical system like inverted pendulum (Katsuhiko, 2010). The free body diagram of the system is first drawn as shown in Figure 1.

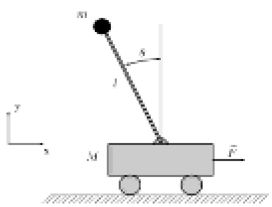


Figure 1.Inverted pendulum on Cart.

The model of inverted pendulum on a cart was derived using Lagrange Equation which base on the difference in Kinetic (K_E) and Potential energy (P_E) of the system. The mathematical model are basically required for the purpose of simulation in MATLAB/Simulink environment and also for the development of controller for the system. The mathematical equation of both the angle of the pendulum and position of the cart are represented in differential equations as:

Langragian (L) natural form is given by

$$L = K_E - P_E \quad (1)$$

$$L = \frac{1}{2}(M+m)\dot{x}^2 - ml\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta \quad (2)$$

Where: x denote the position of the cart. Using equations (1) and (2)

$$F = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} \quad (3)$$

$$0 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} \quad (4)$$

We have:

$$\ddot{\theta} = \frac{F\cos\theta - (M+m)g\sin\theta + ml(\sin\theta\cos\theta)\dot{\theta}^2}{ml\cos^2\theta - (M+m)l} \quad (5)$$

$$\ddot{x} = \frac{u + ml(\sin\theta)\dot{\theta}^2 - mg\cos\theta\sin\theta}{M + m - m\cos^2\theta} \quad (6)$$

Linearizing (5) and (6) about equilibrium points

$$(\theta = 0, \sin\theta \to \theta, \cos\theta \to 1 and \dot{\theta}^2 \to 0)$$

Equations becomes:

$$\ddot{\theta} = \frac{F - (M+m)g\theta}{Ml} \quad (7)$$

$$\ddot{x} = \frac{F - mg\theta}{M} \quad (8)$$

In state space:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (9)$$

$$y(t) = Cx(t) + Du(t) \qquad (10)$$

$$\frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-mg}{M} & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} F \qquad (11)$$

$$y = \begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} \qquad (12)$$

TABLE I SYSTEM PARAMETERS

Parameter	***Value***
Length of the pendulum, L	0.35 m
Mass of the cart, M	1.2 kg
Mass of the pendulum bob, m	0.2 kg
Acceleration due to gravity, g	9.8 ms^{-2}

Source [1]

After substation we have

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 32.667 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -1.633 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ -2.381 \\ 0 \\ 0.833 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

TABLE I. SYSTEM PARAMETERS

3. Controller Design

In this section the two proposed controller design are carried out which are conventional PID and GA-PID controllers. In this section a design procedure of model base PID is presented.

A. PID controller design

The PID Controller is incorporated in the system as shown in Fig. 3,. The general transfer function of the controller is given as:

$$C = K_p + \frac{K_i}{s} + K_d s \qquad (13)$$

Where: K_p, K_i and K_d are the controller gains.

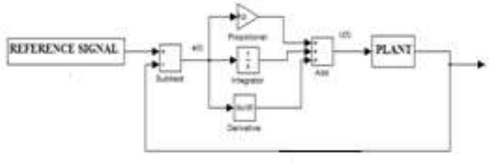
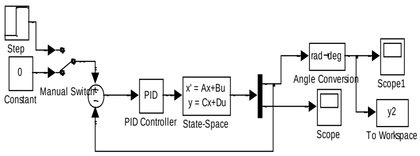


Figure 2.PID Controller Block Diagram

The controller gains were tuned to obtain an optimum response for the system, putting in mind the following guides in TABLEII.

The Simulink block diagram is shown in Fig. 3. The simulation was run under various input conditions.



Controller response	***Rise time***	***Over shoot***	***Settling time***	***Steady state error***
K_p	Decrease	Increase	Small change	Decrease
K_i	Decrease	Increase	Increase	Eliminate
K_d	Small change	Decrease	Decrease	No change

Figure 3. PID Smulink block diagram for the system

B. GA-PID Controller Design

In this section a design procedure of model base GA-PID is presented. The GA-PID Controller is incorporated in the system as shown in Fig. 3,.

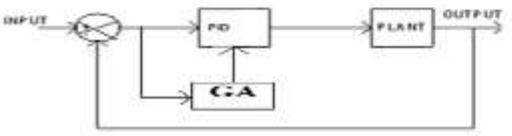


Figure 4. Block diagram of System with GA-PID controller

The errors from the summer serve as the inputs to the genetic Algorithm (GA), the integral time absolute errors of thse error were obtained which serve as the functions needed to be minimized through GA. This is achieved by searching for the controller gains that will best minimized the objective function based on stopping criteria set.

GA-PID controller was designed by writing a program using m-file in MATLAB that will minimize an objective function (ITAE), the program is run under various Input disturbances. The error from the system is fed to GA for minimization. The flow chat for the GA process is shown in Fig. 5. GA is a stochastic algorithm, that is, the result obtained in each time the codes are run is not always the same (Yusuf and Magaji, 2014).

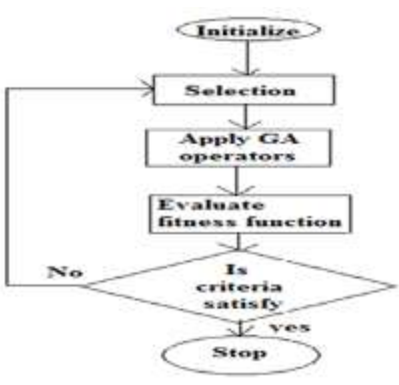


Figure 5. Flow process in application of GA.

4. Results and Discussion

The conventional PID control scheme is implemented in Simulink, and GA base on PID script was written and each of the control schemes was tested under different input conditions.

The result from the two controller schemes are compared in this section. The responses of conventional PID and GA-PID control for pendulum angle are shown in Fig. (6, 7, 8 & 9) under various input steps value. TABLE (III, IV, V and VI) summarizes performance index used for the two controller schemes in two decimal places. The performance indices uses are: Settling time(t_s), Overshoot(M_p), Rise time(t_r).

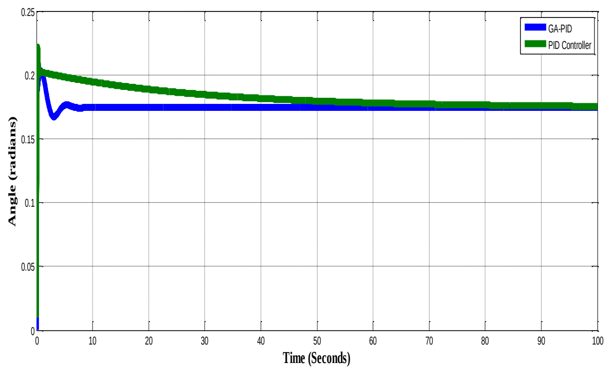


Fig. 1. *Response of the two controller schemes under 10-degree.*

The responses of the system with GA-PID and PID controllers under input of 0.175 radians is shown in Fig. 6., the GA-PID shows sharper response and settled much faster than the conventional PID controller as shown in TABLE III.

TABLE II. RESPONSE UNDER 10-DEGREE OF INPUT

Performance index	***GA-PID***	***PID***
Settling Time (sec)	5.02	70.00
Overshoot	3.00%	5.00%
Rise time (sec)	0.00	0.00

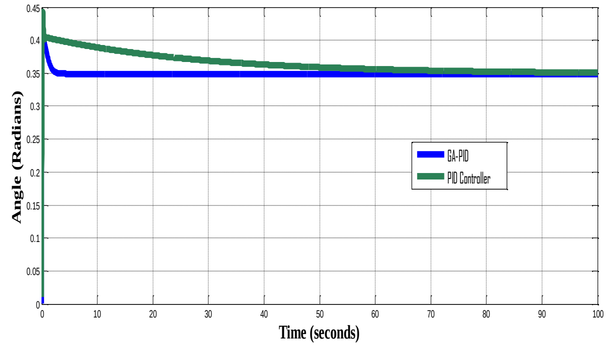


Fig. 2. *Response of the two controller schemes under 20-degree.*

The responses of the system with GA-PID and PID controllers under input of 0.349 radians is shown in Fig. 7, the GA-PID still show sharper response and settled quicker than the conventional PID controller. This is summarized in TABLEIV.

TABLE IV RESPONSE UNDER 20-DEGREE INPUT

Performance index	***GA-PID***	***PID***
Settling Time (sec)	7.00	85.00
Overshoot	6.00%	10.00%
Rise time (sec)	0.00	0.00

Fig. 3. *Response of the two controller schemes under 30-degree*

The responses of the system with GA-PID and PID controllers under input of 0.524 radians is shown in Fig. 8, the GA-PID shows sharper response and settled much faster than the conventional PID controller as its settling time is much shorter. This is shown TABLE V.

TABLE V RESPONSE UNDER 30-DEGREE INPUT

Performance index	***GA-PID***	***PID***
Settling Time (sec)	7.00	68.00
Overshoot	6.00%	16.00%
Rise time (sec)	0.00	0.00

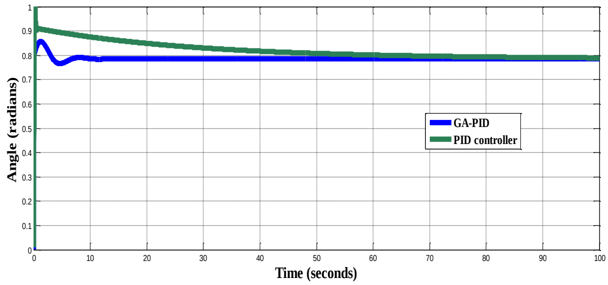


Fig. 4. Response of the two controller schemes under 45-degree.

The responses of the system with GA-PID and PID controllers under 0.7855 radians input is shown in Fig. 9, the GA-PID still demonstrated superiority in term of settling time and overshoot. This is shown in TABLE VI.

TABLE VI RESPONSE UNDER 45-DEGREE INPUT

Performance index	***GA-PID***	***PID***
Settling Time (sec)	6.90	72.00
Overshoot	7.00%	21.00%
Rise time (sec)	0.00	0.00

5. Conclusion

It was observed that the two proposed control schemes(Optimized PID and Conventional PID) performed well in the control of pendulum angle of an inverted pendulum however, the optimized PID controller performs much better than the conventional PID controller when considering settling time, rise time, overshoot as illustrated in TABLE (III to VI). The result shows that the optimized controller gives settling time, percentage overshoot of 5.02 seconds and 3% respectively as compared with settling time of 70 seconds and overshoot of 5% for conventional PID controller. More so, The problems associated with manual tunning of the gains for the PID has been eliminated in Optimized PID controller since it's gains were obtained automatically through an optimization process with Genetic Algorithm. Therefore, the optimized (GA-PID) controller can serves as valuable, easily tunned and effective controller for the system.

References

Altinoz, O.T., Yilmaz, A.E. and Weber, G.W. (2010). 2nd int. Conference on Eng. Optimization.

Anguilar, C. (Unpublished). Approximate Feedback Linearization and Sliding Mode Control for Single Inverted Pendulum.

Haupt, L.R. and Haupt, S.E. (2002). Practical Genetic Algorithms. John Wiley and Sons.

Katsuhiko, O. (2010). Modern Control Engineering. Boston: Prentice Hall.

Lee, Y. Park, S. and Brosilow, C. (1998). PID Controller Tuning for Desired Closed-Loop Resonses for SI/SO Systems . AIChEJ , 106-115.

Lopez, A., Murrill, P. and Smith, C. (2009). Tuning PI and PID Digital Controllers. Instruments and Control , 89-95.

Magaji, N. and Mustafa, M.W. . (2010). Optimal Location and Signal Selection of SVC device for Damping Oscillation. Int. Rev. on Modeling and Simulations , 56-59.

Pandalai, K. and Kataria, M. (Unpublished). Inverted Pendulum System.

Sumathi, S. and Paneerselian, S. (2010). Computational Intelligence Paradigms. Taylor and Francis Group.

Van Overschee, P. et al. (2010). RAPID the end of heuristic PID Tuning. Journal A , 6-10.

Wahida, R.S.D, Banu, B.S.A. and Manoj, D. (2011). Identification and Control of Non-linear System using Soft Computing Technique. International Journal of Modeling and Optimization , 32-35.

Wang, Y. (2011). Nonlinear Control of Cart Pendulum Systems. Int. Rev. on Modeling and Similations , 22-28.

Yusuf, L.A. and Magaji, N. (2014). Comparison of Fuzzy Logic and GA-PID Controller for Position Control of Inverted Pendulum. International Review of Automatic Control , 380-385.