Smart Antenna System Implementation under Multipath Propagation Using JADE-MVDR and LMS Algorithms

Ahmad Aminu[1,2]**, Mustafa Secmen**[2]**, Munir Aminu Husein**[2]
[1]Department of Electrical and Electronics Engineering
School of Technology, Kano State Polytechnic
Kano, Nigeria
[2]Department of Electrical and Electronics Engineering
Yasar University
Izmir, Turkey

ahmadkksr@gmail.com, mustafa.secmen@yasar.edu.tr,
munneer@yahoo.com

Abstract— This paper considers the implementation of smart antenna system under multipath propagation. Here, it is considered different non-coherent signal groups each containing direct and multipath signals. The direction of arrival (DOA) of all the signals in each group is estimated using Minimum Variance Distortionless Response (MVDR) in conjunction with joint approximate diagonalization of eigenmatrices (JADE) algorithm. The generalized steering vectors are first estimated using JADE algorithm, and then the MVDR method is realized to estimate the DOA of each signal. The computation times of JADE-MVDR and JADE-MUSIC algorithms are compared for a single iteration and the results show that JADE-MUSIC has slightly lower runtime. Besides, RMSE performances are compared for different scenarios and JADE-MVDR is found to be more effective. The DOAs obtained are then processed using LMS adaptive beamforming algorithm to steer the main lobes of the radiation pattern toward the signal of interest angles and the nulls toward the signals not of interest angles. In addition, a new measure of the power level reduction under different scenarios (snapshots and array elements) is presented. The simulation results reveal that a maximum power drop of 0.4 dB is observed, and adaptive beamforming is successfully done by mitigating the effects of multipath significantly.
Keywords— direction of arrival; adaptive beam forming; joint approximate diagonalization of eigenmatrices; least mean square algorithm; minimum variance distortionless response

I. Introduction

Direction of arrival (DOA) estimation and adaptive beamforming are very crucial in the area of wireless communications (especially smart antenna system application) for the past few decades when there is a strong correlation between signals. Many researches are conducted and several literatures are written concerning suitable methods for estimating coherent signals parameters. Two signals are said to be coherent when a delay replica of the original is produced due to multipath and fading phenomena (Yuen, & Friedlander, 1997). Multipath propagation is occurred due to the multiple reflections caused by reflectors and scatterers in the environment (Al-Zuraiqi, 2004). The separation of main (direct) signals from these reflected (interference) signals usually impinging from different angles than that of direct signal is very critical to increase the performance (signal level) of communication system. Therefore, DOAs and the fading coefficients of these correlated signals should be extracted properly, and the one having highest fading coefficient should be filtered out to arrange a steady communication.

The correlated signals cause spatial covariance matrix to be singular, which is non-invertible due to rank loss. This causes most of the existing classical and second order subspace methods fail to resolve the signals in the correct manner and hence makes DOA estimation impossible. Several methods are developed to restore this rank loss such as spatial smoothing based methods (Pillai, 1989), which are pre-processing schemes that subdivide the array elements into overlapping sub-arrays and then estimate the steering vectors as well as the covariance matrix of each sub-array.

The outlined procedure is followed by estimating DOAs of each sub-array using any DOA estimation algorithm. Matrix-pencil based method (Yilmazer, et al., 2006), (Hua, & Sarkar, 1988), maximum likelihood (Stoica, et al., 1996) and depletion approach (Xu, et al., 2006), where a Toeplitz matrix is constructed for DOA estimation of the coherent sources after the noncoherent sources are estimated with conventional subspace methods. All these methods have in one way or the other some shortcoming(s) (Yuen, et al., 1997) ranging from loss of array aperture, intensive computation, increased number of sensors and some fail in noisy environment as in the case of matrix pencil based methods.

Joint approximate diagonalization of eigenmatrices (JADE) based algorithms have been successfully applied to different DOA estimation as in (Zhang, et al., 2008; Lie, et al., 2006; Xu, et al., 2009; Ye & Zhang, 2009; Jia & Jing-Shu, 2010; Moghaddam, & Nasab, 2013; Moghaddam et al., 2013; Aminu, et al., 2014) since array response vectors estimated without having a prior knowledge of the array manifold (Cardoso & Souloumac, 1993). In this paper, DOA estimation using JADE based MVDR method for signal groups is realized and the results are compared with those of JADE based MUSIC method. The RMSE performance measure is used to evaluate the effectiveness of the proposed method, which shows that JADE based MVDR can estimate DOAs in noisy environment.

The next part in this study is the implementation of the estimated DOAs on the adaptive antenna array beamforming. The current wireless communication systems generally use antennas/antenna arrays having very wide bandwidths to cover the whole space effectively. For instance, base station antennas with 3-sector configuration have almost 120 degrees beamwidth. However, in the cases with strong fading, the undesired signals coming from different DOAs due to multipath can severely reduce the magnitude of desired signal with desired DOA when undesired and desired signals are out of phase. Therefore, the radiation pattern (beam) of the antenna array should be modified (reformed) to get maximum signal for desired angle and minimum signals for undesired angles as possible. By using classical phased array technology (Mailloux, 2005) with the proper arrangement of phase coefficients of the antenna elements, the maximum of the array beam can be directed to the desired angle. Nevertheless, the undesired angles may coincide with the sidelobes of the radiation pattern, and consequently, a reduction of signal power can be still valid. For this purpose, an intelligent adaptive beamforming (radiation pattern) is needed to suppress the levels of undesired signals without changing the power level of the desired signal significantly. The well-known least mean square algorithm (LMS) is used for the adaptive beamforming purpose, and the performance is evaluated with a new measure of “power down in dB”. This measure can be explained as the reduction of power in dB for the worst case where all undesired signals are out of phase to the desired signal. The simulation results present that in spite of challenging environment with strong fading coefficients, the algorithm is able to make a successful beamform adaptively such that the power reduction is observed as 0.4 dB at most.

II. Signal Model

Let us consider a situation where G narrowband, far-field noncoherent sources/groups impinge on a uniform linear and isotropic M element antenna array with element interspacing d equals to half wavelength of the signals. Here, it is assumed that each group contains L coherent signals one of which is handled as “desired” signals (having the highest amplitude), and other L-1 number of delayed and scaled replicas of the original (desired) signal in each group due to multipath and fading phenomena are called as “undesired” signals. Therefore, the total number of signals is $N = L \times G$. The output of the array, which is an M × 1 vector can be written as:

$$\mathbf{X}(k)=\mathbf{As}(k)+\mathbf{n}(k) \quad for\ k=0,\ldots,N_s-1 \quad (1)$$

where $a(\theta_{G,L})$ is and n are the signal sources and additive noise respectively, Ns is the number of snapshots (data) and the matrix A is given by:

$$\mathbf{A}=\left[a(\theta_{1,1})\ a(\theta_{1,2}) \ldots\ a(\theta_{1,L}) \ldots\ a(\theta_{G,L})\right] \quad (2)$$

where $a(\theta_{G,L})$ is the steering vector L in the G^{th} group as

$$a(\theta_{G,L})=\left[1\quad e^{\frac{j2\pi d\sin\theta_{(G,L)}}{\lambda}} \ldots e^{\frac{j2(M-1)\pi d\sin\theta_{(G,L)}}{\lambda}}\right]^T \quad (3)$$

$\theta_{G,L}$ is DOA of signal L in the Gth noncoherent group and λ is the wavelength of the signals. The signal matrix s can be expressed as

$$\mathbf{s}(k)=\left[s_{1,1}(k)\ s_{1,2}(k) \ldots\ s_{1,L}(k) \ldots s_{G,L}(k)\right]^T \quad (4)$$

where each signal contains the information about the fading coefficient. The noise matrix n is assumed to have zero mean entries and spatial covariance matrix being equal to $\sigma_n^2 \mathbf{I}_{\mathbf{M}\times\mathbf{M}}$ where σn2 is the variance of the noise, and $\mathbf{I}_{\mathbf{M}\times\mathbf{M}}$ is unit matrix of size M.

III. Estimation of the Steering Vector Using Jade Algorithm

JADE algorithm is applied to estimate the generalized steering vectors of the matrix X in (1). It is summarized as follows (Moghaddam, et al., 2013).

Step 1: Compute the spatial covariance matrix, Rxx of the signals in (1) with (Chen et al., 2010):

$$\mathbf{R}_{\mathbf{xx}}=E\left\{\mathbf{X}(\mathrm{k})\,\mathbf{X}^H(\mathrm{k})\right\} \quad (5)$$

where E{} is the expected operator, and H is the Hermitian (complex conjugate transpose) operator.

Step 2: Compute a whitening matrix W from the covariance matrix. Then, whitening process can be expressed as:

$$\mathbf{Z}(k)=\mathbf{W}\,\mathbf{X}(k) \quad (6)$$

Step 3: Form fourth order cumulants of Z(k) and compute G most significant eigenpairs

Step 4: Jointly diagonalize the set $\left\{\lambda_{zr}, M_{zr}\ |1\le r\le G\right\}$ by a unitary matrix U.

Step 5: An estimate of the generalized array response matrix:

$$\mathbf{Y}=\mathbf{W}^{\dagger}\mathbf{U}=\left[\mathbf{y_1}\quad \mathbf{y_2}\quad \cdots\quad \mathbf{y_G}\right] \quad (7)$$

where the column vectors y1, y2, …, yG are the generalized steering vectors belonging to each noncoherent source group in the total signal.

IV. DOA Estimation Using Jade Based Spectral MVDR Algorithm

The details of steps of Minimum Variance Distortionless Response (MVDR) can be found in many books and papers of (Foutz, et al., 2008), (Al-Nuaimi, et al., 2004) but are also summarized here.

The main aim of this method is to obtain the possible directions of all the received signals from the peaks of the spectrum of MVDR. Mathematically, MVDR can be expressed as:

$$\min_{w} p(\mathbf{w}) \quad subject\ \ to\ \ \mathbf{w}^{H} a(\theta)=1 \tag{8}$$

where $a(\theta)$ is given in (3), and w is the weight vector. For the first noncoherent source group containing L coherent signals, the corresponding steering column vector is obtained as y1 from previous chapter. Then, the weight vector for this group can be found using (9) below as:

$$\mathbf{w}_{MVDR} = \frac{\hat{\mathbf{R}}_{\mathbf{y}_1\mathbf{y}_1}{}^{-1} a(\theta)}{a^{H}(\theta)\hat{\mathbf{R}}_{\mathbf{y}_1\mathbf{y}_1}{}^{-1} a(\theta)} \tag{9}$$

where $\hat{\mathbf{R}}_{\mathbf{y}_1\mathbf{y}_1}$ is an estimate of the covariance matrix of y1. Finally, the output power spectrum is expressed in (10) where DOAs are estimated from the angles giving peak values at this spectrum.

$$p(\theta) = p_{MVDR} = \frac{1}{a^{H}(\theta)\hat{\mathbf{R}}_{\mathbf{y}_1\mathbf{y}_1}{}^{-1} a(\theta)} \tag{10}$$

This process is repeated for all other possible noncoherent signal groups by just replacing y1 with other generalized column steering vectors of y2,…, yG, and corresponding DOAs are acquired.

V. Adaptive Beamfroming and Power Reduction Measure

After successfully estimating the direction of arrivals of all the signals including the interfering ones using MVDR spectral method, these values and fading coefficients obtained with MVDR are used in the adaptive beamforming part of the study. Adaptive beamforming involves exploiting the arrangement of excitation coefficients of antenna array adaptively in order to achieve optimum reception of the desired signals in one direction and strongly rejecting the interfering ones in any other direction. In this paper, Least Mean Square (LMS) adaptive beamforming algorithm is used due to its simplicity and robustness.

The LMS algorithm was derived by Widow and Hoff (Haykin, 1991) in 1959 and it is widely used in many applications. It involves new observations and iteratively minimizes linearly the mean square error between the estimated and desired signals. In our adaptive antenna array beamforming, the DOA and fading coefficient of desired signal in each group are utilized to acquire the noiseless desired signal d(t) at Np snapshots. Here, Np can be much lower than Ns to save computational time.

Then, for each group, the total signal at each antenna element is calculated by using all DOAs and fading coefficients of the group. Accordingly, these total signals can be considered as the noise-free (clear) version of X in (1) for each group, named as Xg,clear. Then, for the gth noncoherent source group, the LMS algorithm equation to adaptively update the excitation coefficients of the antenna array is expressed as (Haykin, 1991):

$$w(t+1) = w(t) + \mu e(t) \mathrm{X}_{g,clear}{}^{*}(t), \quad t = 0, \ldots, N_p - 1 \qquad (11)$$

where $w(t) = \left[w_1(t) \quad w_2(t) \quad \ldots \quad w_M(t) \right]^T$ are the excitation coefficients of the antenna array at the tth iteration; μ is the step-size parameter which controls the immediate change of the updating factor, and e(t) is the error between the desired and output signal which is given by (Hayes, 1996):

$$e(t) = d(t) - w(t)^H \mathbf{X}_{g,clear}(t) \qquad (12)$$

The step-size parameter has significant effect on the LMS algorithm such that, if it is too small, the convergence to optimal solution takes longer time while if it is high, the stability of the system is affected. For stability, the following condition (Chen et al., 2010) must be satisfied.

$$0 < \mu < \frac{1}{\lambda_{\max}} \qquad (13)$$

where λmax is the maximum eigenvalue of the autocorrelation matrix.

After the optimum excitation coefficients of each antenna elements are obtained by LMS, the normalized array factor (AFn) of the antenna array is calculated. Next, the degradation in desired signal power level is evaluated with a new measure of “power down in dB”. In this measure, the power difference in dB between maximum available power and power with the optimized coefficients in the worst case is used. The received power in dB for each group can be given as

$$P(dB) = 20\log_{10}\left| \rho_d AF_n(\theta_d) + \sum_{i=1}^{L-1} \rho_{i,u} AF_n(\theta_{i,u}) \right| \qquad (14)$$

where ρd and AFn(θd) are fading coefficient and normalized array factor at DOA of the desired signal, respectively; and ρi,u (where |ρi,u| < 1) and AFn(θi,u) for i =1,…, L-1 are those of undesired signals. In mobile wireless communication systems, although the magnitudes of fading coefficients change slowly, the phase terms are very sensitive especially to the relative distances between sources and antennas such that the phase value can jump 180 degrees even with a small change in the distance. Therefore, the phase terms of undesired signals’ contributions in the summation in (14) can all be out of phase relative to desired signal, which results in reduction at the power level of desired signals.

By assuming ρd = 1 and AFn(θd)=1, this worst power (Pw) can be expressed as

$$P_w(dB) = \begin{cases} 20\log_{10}\left(1-\sum_{i=1}^{L-1}|\rho_{i,u}||AF_n(\theta_{i,u})|\right), & if \sum_{i=1}^{L-1}|\rho_{i,u}||AF_n(\theta_{i,u})| \le 1 \\ -\infty\ dB & ,if \sum_{i=1}^{L-1}|\rho_{i,u}||AF_n(\theta_{i,u})| > 1 \end{cases} \quad (15)$$

Regardingly, for nonzero fading coefficients of undesired signals, the theoretical maximum available power can be only achieved when a maximum in AF is at the DOA of desired signals, i.e. AFn(θd)=1; and the nulls are at the DOA of undesired signals, i.e. AFn(θi,u)=0. So, according to (15), Pmax(dB) becomes 0 dB, and the power down in dB can be formulated as

$$P_{down}(dB) = P_{\max}(dB) - P_w(dB) = 0 - P_w(dB) \quad (16)$$

Here, for instance, 3 dB of Pdown means the loss of half of the power of the desired signal, and ∞ dB of Pdown corresponds to no received desired signal.

VI. Simulation Results and Discussions

In this part, the simulation of JADE Based Minimum Variance Distortionless Response Algorithm for DOA estimation is first carried out, and then the DOAs obtained are implemented using LMS adaptive beamforming algorithm for smart antenna application. The root mean square error (RMSE) is utilized as performance measure to determine the effectiveness of the method. RMSE is defined in (17) below as follows (Zhang, et al., 2008):

$$RMSE = \sqrt{\frac{1}{NT}\sum_{t=1}^{T}\sum_{n=1}^{N}\left(\hat{\theta}_n(t) - \theta_n\right)^2} \quad (17)$$

where $\hat{\theta}_n(t)$ is the estimate of for tth Monte Carlo trials.

The simulation considers three sources of uncorrelated groups of signals each containing one original signal and three multipath signals. Table I below gives the true directions and fading coefficients of the signals:

TABLE I. True Arrival Angles and Fading Coefficients of the Signals

Group	True DOAs (deg)	True Fading coefficients
First	–41	1
	–14	–0.6426+0.7266j
	12	0.8677+0.0632j
	39	0.7319-0.1639j
Second	–49	1
	–25	0.8262+0.4690j
	1	0.1897-0.8593j
	48	0.2049-0.7630j

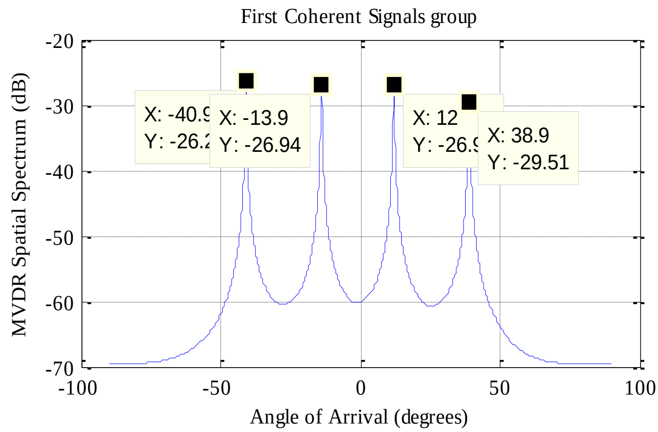
Third	–46	1
	–22	0.1681-0.9045j
	4	-0.7293-0.1750j
	44	0.6102+0.1565j

The angles are deliberately chosen within the range between -60 and 60 degrees to be consistent with base station applications at which the antenna of each sector has 120 degrees beamwidth. In the above table, the fading coefficient of “1” in each group belongs to desired signal and other coefficients are for the undesired ones. Since the sum of magnitudes of the fading coefficients of undesired ones is greater than 1 for each group, by considering (15) there is a possibility of receiving no desired signal (power down of ∞ dB) with the random changes of phases when no adaptive beamforming is employed.

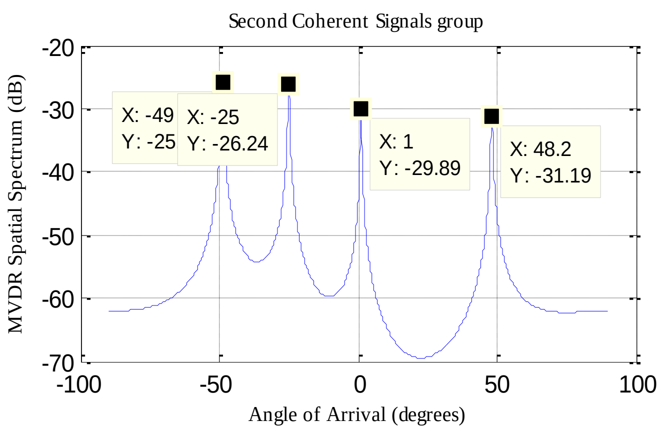
These signals impinge on a uniform linear array with M = 10 array elements with equal distances of d = 0.5λ. The signals having Ns = 2000 snapshots in (1) are corrupted with a Gaussian noise with SNR= 0 dB, and T = 50 trials are performed for each analysis to be described in the next parts.

The JADE-MVDR spectrums for each coherent signal group are shown in Fig. 1 for a sample trial. Here, the sharp peaks indicate the angle of arrival (DOA) of each signal. From the results, it can be observed that the method succeeds in resolving coherent signals correctly with 0.2 degrees error at most for this trial.

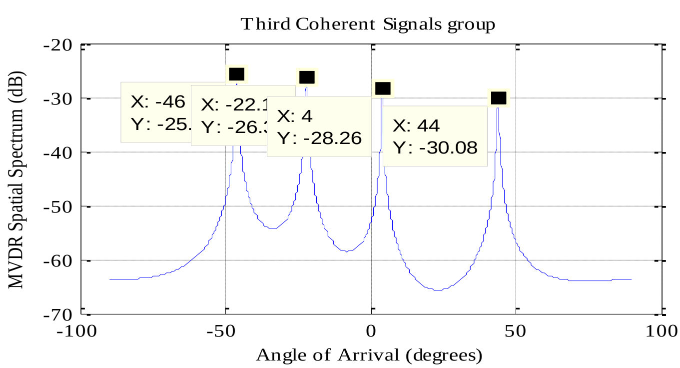
In this simulation, the proposed algorithm is compared with JADE based MUSIC (Zhang, et al., 2008). The RMSE performances of two algorithms are analyzed in terms of the parameters of number of array elements (M), number of snapshots (Ns), and signal to noise ratio (SNR).



(a)



(b)



(c)

Fig. 1.The estimation of JADE base MVDR spectrum for each coherent signals group with the sharp peaks indicating the estimated DOAs of the coherent signals.

In the first analysis, the RMSE performance is compared for different M numbers by fixing SNR = 0 dB and Ns = 2000. The corresponding results are depicted in Fig. 2 such that JADE-MVDR has less RMSE at low M values meaning to have better performance than JADE-MUSIC. However, as the number of antenna increases 16 and beyond, two algorithms have equal performances with less than 0.04 degrees error.

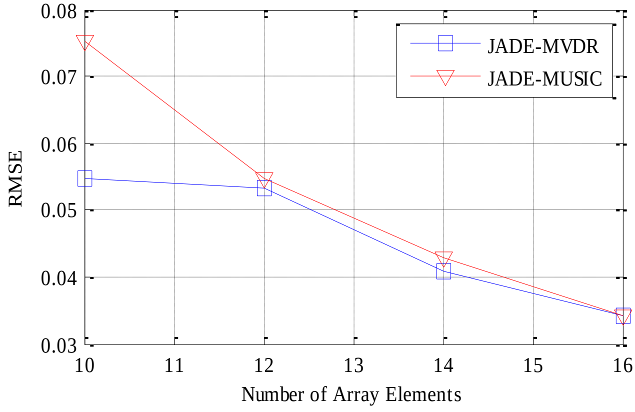


Fig. 2.Variation of RMSE with Number of Array Elements for JADE-MVDR and JADE-MUSIC for SNR = 0 dB and Ns = 2000.

In the second analysis, the RMSE performance is compared for different Ns numbers by fixing SNR = 0 dB and M = 10, and the corresponding results are given in Fig. 3. As it can be seen from Fig. 3 that JADE-MVDR has superior performance compared to JADE-MUSIC even at the low number of snapshots.

The last analysis of SNR is realized by fixing Ns = 500 and M = 10 which are the worst cases of the previous analyses. The results can be seen in Fig. 4 that both methods have almost equal performance for the worst case with minimum number of snapshots and antenna elements. RMSE value lower than 0.8 degrees is achieved even for the case of SNR = -10 dB, Ns = 500 and M =10, which shows the effectiveness of the proposed JADE based MVDR algorithm.

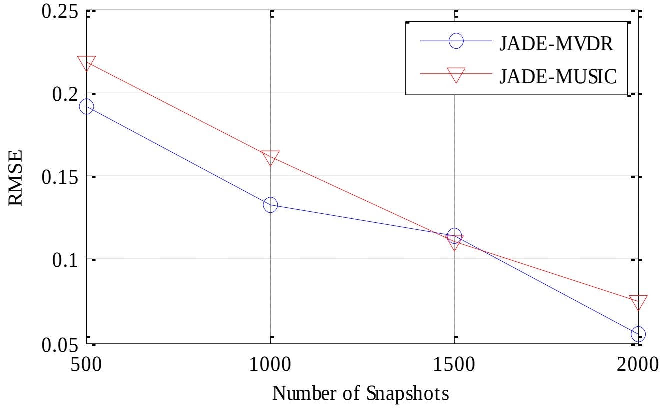


Fig.3 Variation of RMSE with Number of Snapshots for JADE-MVDR and JADE-MUSIC for *SNR* = 0 dB and *M* = 10.

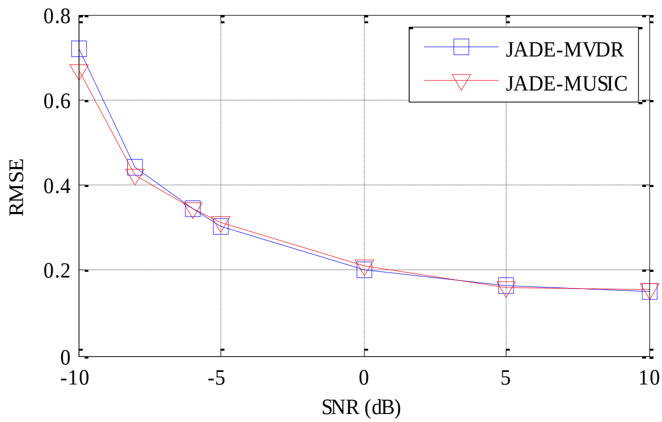


Fig. 4 Variation of RMSE with SNR for JADE-MVDR and JADE-MUSIC for *Ns* = 500 and *M* = 10.

The estimated DOA angles in the above simulation results are well separated. Therefore, the calculated RMSE is below 0.8 degrees even for this challenging case. When the angles belonging to coherent signals are closer to each other, the results may degrade (Yuen, et al., 1997); however, the JADE based MVDR is still expected to give sufficient results.

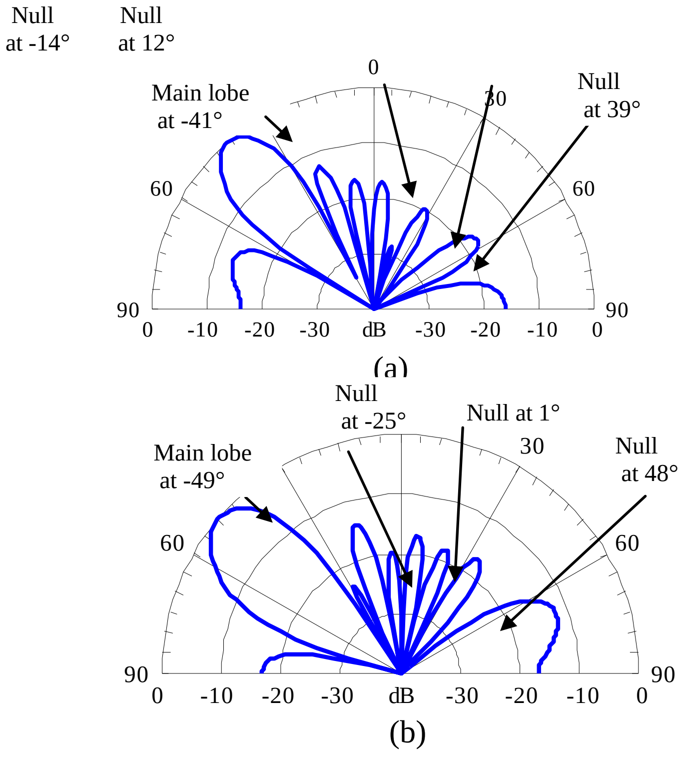
The computation times of JADE-MVDR and JADE-MUSIC algorithms are also compared for a single iteration, and the results are shown in Table II. These results are obtained in MATLAB environment with a HP Personal Computer, which has Intel Core i3-2328M processor at 2.2GHz and 4GB (929 usable) RAM.

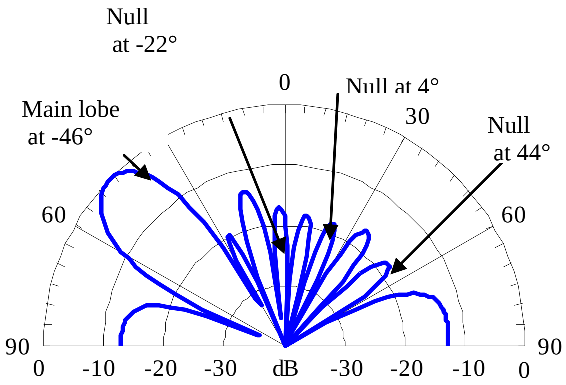
TABLE II. COMPARISON OF COMPUTATION TIME OF JADE-MVDR AND JADE-MUSIC ALGORITHMS

	JADE-MVDR	**JADE-MUSIC**
Run Time/Iteration (sec)	6.53	5.6

The results show that JADE-MUSIC has slightly lower run time than JADE-MVDR, and it is probably due to the additional time spent by the MVDR to take the inverse of the covariance matrix.

After the estimation of DOAs and fading coefficients with MVDR is completed for the parameters of SNR = 0 dB, N_s = 2000 and M = 10, these values are used to adaptively optimize the excitation coefficients of the antenna array. In the beamforming part, a small portion of entire signal is used (N_p = 300 snapshots) to reduce computational complexity. In all simulations the step-size of LMS is fixed to μ = 0.0014. The normalized array factors (radiation patterns) for a sample trial are shown in Fig. 5 .





(c)

Fig. 5 Polar radiation plot of the adaptive beamforming for (a) the first signal group (b) the second signal group (c) the third signal group.

In Fig. 5, it can be clearly seen that the main lobes of the adaptive beamforming patterns are directed toward the angles of desired signals (θ_d) in all three groups, which are -41°, -49° and -46° in the first, second and third groups, respectively; while all other angles of undesired signals ($\theta_{i,u}$) are directed toward the nulls. For this sample trial, the maximum power reduction is calculated to be at most 0.4 dB for all three groups.

The additional simulation involves the analysis of measure of the power reduction, P_{down} (dB) in (16), for different scenarios (different number of snapshots and array elements). Figure 6 shows the results of the variation of power reduction with respect to snapshots used in the beamforming part, N_p. As it can be seen in Fig. 6, even if 50 snapshots are used for beamforming part, maximum power drop is found as 0.4 dB, which is reasonable. Besides, it is clear that the results are not significantly affected by either increase or decrease in the number of snapshots.

Similarly, Fig. 7 shows the power reduction with respect to number of antenna elements by fixing N_p = 300. The related results indicate that the power level down in dB remains slightly constant for all the three groups as the number of antenna elements increasing from 10 to 14, while it increases with the number of antenna elements above 16 for first and second groups. First and second groups have minimum power reductions when 14 and 16 antenna elements are used, respectively. Again, the maximum power level drop is found to be no more than 0.4 dB meaning a very negligible loss in the desired signal power level.

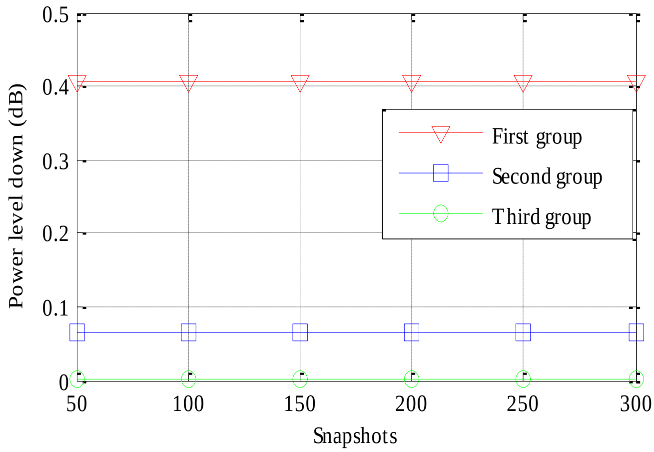


Fig. 6 Variation of Power level down with N_p for the signal groups.

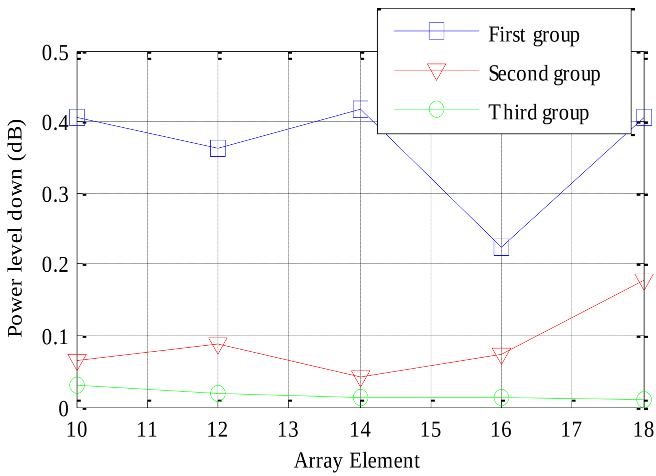


Fig. 7 Variation of Power level down with array element for the signal groups.

VII. Conclusions

In this paper, DOAs in multipath propagation are examined and estimated using two-step approach, which involves estimating the generalized steering vectors using JADE algorithm followed by estimating the angle of arrival using MVDR algorithm. The case of noncoherent signal groups with each containing coherent signals having strong multipath effects is used throughout the simulations. The performance of JADE-MVDR algorithm is compared with that of JADE-MUSIC in different scenarios and simulation results show that JADE-MVDR algorithm which is emphasized in this paper has slightly better performance than JADE-MUSIC. The DOAs and fading coefficients obtained by the JADE-MVDR algorithm are processed using LMS adaptive beamforming algorithm. The main lobes of the adaptive beamforming patterns are successfully steered to the desired signal and the nulls to the undesired signals in each noncoherent group giving the maximum power reduction of 0.4 dB with the new measure of "power down in dB". As a conclusion, the proposed method can be used effectively for the smart antenna system applications and implementations.

References

Al-Nuaimi M. A., Shubair R. M., & Al-Midfa K. O. (2004). "Direction of Arrival Estimation in Wireless Mobile Communication Using Minimum Variance Distorsionless Response" The Second International Conference on Innovations in Information Technology (IIT'05), Etisalat University College, U.A.E.

Al-Zuraiqi F. A. K. (2004). ''Analysis, Simulation and Modelling of Mobile and Fixed Fading Channels'' BSc Thesis, Eastern Mediterranean University.

Aminu A., Secmen M., & Poyraz S. (2014). "The Comparison of JADE Based DOA Estimation Methods for Unknown Noncoherent Source Groups Containing Coherent Signals with Frequency Matching," 20th International Conference on Microwaves, Radar and Wireless Communications (MIKON), pp. 397-402, Gdansk, Poland.

Cardoso J. F., & Souloumac A. (1993). "Blind Beamforming for Non-Gaussian Signals," IEE Proceedings-F, vol. 140, pp. 362-370.

Chen Z., Gokeda G., & Yu Y. (2010). Introduction to Direction-of-Arrival Estimation, Artech House, Boston, London.

Foutz J., Spanias A., & Banavar M. K. (2008). Narrowband Direction of Arrival Estimation for Antenna Arrays, Morgan & Claypool, Ontario, Canada.

Hayes M. H. (1996). Statistical Digital Signal Processing and Modeling, John Wiley & Sons, Inc.

Haykin S. (1991). Adaptive Filter Theory, Second Edition, Prentice-Hall Inc., New Jersey.

Hua Y. & Sarkar T. K. (1988). "Matrix Pencil Method and Its Performance," Proc. ICASSP-88, vol. 4, pp. 2476-2479, New York.

Jia Z., & Jing-Shu Y. (2010). "Blind DOA Estimation Based on JADE Algorithm in Multipath Environment" CCIE 2010 International Conference on Computing, Control and Industrial Engineering, Wuhan, pp. 141-144.

Lie S., Leyman A. R. & Chew Y. H. (2006). "Fourth-Order and Weighted Mixed Order Direction-of-Arrival Estimators," IEEE Signal Process. Lett., vol. 13, no. 11, pp. 691-694.

Mailloux R. J. (2005). Phased Array Antenna Handbook, Second Edition, Artech House, Norwood, MA, USA.

Moghaddam S. S., Ehadi Z., & Vakili V. T. (2013). "Modified FOC-based Root-MUSIC algorithm for DOA estimation of coherent signal groups," Life Science Journal, vol. 10, no. 1, pp. 843-846.

Moghaddam S. S., & Nasab A. K. (2013). "A New Threshold-Based JADE-MUSIC Algorithm for DOA Estimation of Unknown Signal Groups," JECE, vol. 3, no. 6, pp. 17-22.

Pillai S. U. (1989). "Forward/Backward Spatial Smoothing Techniques for Coherent Signal Identification," IEEE Transactions on Signal Processing, vol. 37, pp. 8-15.

Stoica P., Ottersten B., Viberg M., Moses R. L. (1996). "Maximum Likelihood Array Processing for Stochastic Coherent Sources," IEEE Transactions on Signal Processing, vol. 44, pp. 96-105.

Xu X., Ye Z., Zhang Y., & Chang C. A. (2006). "Deflation Approach to Direction of Arrival Estimation for Symmetric Uniform Linear Array" IEEE Antennas and Wireless Propagation Letters, vol. 5, pp. 486-489.

Xu Y. L., Zhang H. Sh. & Zhan J. S., (2009). "A Signal-Subspace-Based ESPRIT-Like Algorithm for Coherent DOA Estimation" WiCom '09 International Conference on Wireless Communications, Networking and Mobile Computing, Beijing, pp. 1-4.

Ye Z., & Zhang Y. (2009). "DOA Estimation for Non-Gaussian Signals Using Fourth-Order Cumulants" IET Microwaves, Antennas and Propagation, vol. 3, pp. 1069-1078.

Yilmazer N., Koh J., & Sarkar T. K. (2006). "Utilization of a Unitary Transform for Efficient Computation in the Matrix Pencil Method to Find the Direction of Arrival," IEEE Transactions on Antennas and Propagation vol. 54, pp. 175- 181.

Yuen, N., & Friedlander, B. (1997). "DOA Estimation in Multipath: An Approach Using Fourth-Order Cumulants" IEEE Transactions on Signal Processing vol. 45, pp. 1253-1263.

Zhang Y., Ye Z., & Xu X. (2008). "Multipath DOA and Fading Coefficients Estimation Using Fourth-Order Cumulants," ICSP 2008 International Conference on Signal Processing, Beijing, pp. 374-377.