

# Optimization of Production Plan of Hebron Drinks using Operational Research Technique 

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#### Abstract

Manufacturing companies frequently face challenging operational problems. In such business environment, operations that compete for the same resources must be planned in a way that deadlines are met. Certain expertise in optimization is often required for successful solution of these problems. In this paper, we attempted to optimize the production plan of a manufacturing company - Hebron Drinks, by minimizing the Labour hours, Marching hours and Materials used in producing six different types of products. Linear programming technique was use to model the production plan of Hebron Drinks. The resulted model was solved using simplex method with the aid of computer software (LIP Solver 1.11.1 and 1.11.0). The optimal value obtained shows a reduction in the total cost of production for the period considered.


Keywords: Linear programming, Simplex method, Optimization, Operational research, Production plan.

## 1. Introduction

Production systems are area of operations research that concentrates on real-word operational problems. Production planning is concerned primarily with the adaptation of the industrial limited resources of the firm in order to satisfy demand for its product. (Bruce R. Feiring, 1991; Shapiro, J.F., 1993). The following settings, among others, usually produce production systems problems: manufacturing, telecommunications, health-care delivery, facility location and
layout, and staffing (Hillier, F., \& Lieberman, G., 2001). Operations research fundamentals are required to solve production problems since they are operations research problems. Additionally, the solution of production systems problems frequently draws on expertise in more than one of the primary areas of operations research, implying that the successful production researcher cannot be one-dimensional (Banks, J., Carson II J., \& Nelson, B., 1995: Taha, H. A., 2002).. In order to solve production
problems, an in-depth understanding of the real problem is required, since invoking assumptions that simplify the mathematical structure of the problem may lead to an elegant solution for the wrong problem. Common attributes of successful production planners are common sense and practical insight (Hillier, F., \& Lieberman, G., 2001; Epply, T., 2004). Design objective could be simply to minimize the cost of production or to maximize the efficiency of production (Agarana, M.C. Anake T.A., and Adeleke O.J., 2014).

An optimization algorithm is a procedure which is executed iteratively by comparing various solutions till an optimum or a satisfactory solution is found. Optimization or mathematical programming is the selection of a best element (with regard to some criteria) from some set of available alternatives (Ofori S., 2013). In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function (Voss, C., Tsikriktsis, N. \& Frohlich, M. 2002). The generalization of optimization theory and techniques to other formulations comprises a large area of applied mathematics. More generally, optimization includes finding "best available" values of some objective functions
given a defined domain (or a set of constraints), including a variety of different types of objective functions and different types of domains (Fagotinbo, I. S., Akinbo, R. Y., Ajibode, I. A., Olaniran Y. O. A., 2011) A production plan is the administrative process that takes place within a manufacturing business and which involves making sure that sufficient raw materials, staff and other necessary items are procured the schedule specified (Graves, C. S., 1999)..

The Hebron drinks is a unit under Strategic Business Unit (SBU) of Covenant University Consultancy Services. It was established in December 2005, to cater, primarily, for worshippers at Canaan land and Covenant University students. The unit adequately provides Sachet water and Bottle water to Canaan land community. Hebron drinks is driving towards having distributors nationwide and maintaining high quality products at all times. Products range from Sachet water, Bottle water, Black currants flavour drink, Orange flavour drink. In March 2014, new products were added to the production line, these are: Hebron yoghurt and apple drink. In the near future, more products are expected to be added to the production, this will include pineapple drink and kola drink which are already being experimented to determine its demand and acceptance. (Covenant

University Student Handbook, 2013-2017). A typical manufacturing business engaging in production planning would usually aim at maximizing profit while maintaining a satisfied customer base (Voss, C., Tsikriktsis, N. \& Frohlich, M., 2002). A planning problem exists because there are limited production resources that cannot be stored from period to period. Choices must be made as to which resources to include and how to model their capacity, behavior, and their costs (Unti, J. G., 1968). Also, there may be uncertainty associated with the production function, such as uncertain yields or lead times. One might only include the most critical or limiting resource in the planning problem, e. g., a bottleneck (Unti, J. G., 1968). Alternatively, when there is not a dominant resource, then one must model the resources that could limit production. We describe two types of production functions can be described as follows: The first assumes a linear relationship between the production quantity and the resource consumption. The second assumes that there is a required fixed charge or setup to initiate production and then a linear relationship between the production quantity and resource usage (Voss, C., Tsikriktsis, N. \& Frohlich, M. 2002; Unti, J. G., 1968). In this paper attempt is made to maximize the production
plan of Hebron drinks using Linear programming model.

## 2. Model Formulation

Production plan of Hebron Drinks is modeled using linear programming technique. The model include the objective function, the constraints; including the non-negativity constraints. Decision variables, constraints, per unit usage of resources. The available resources are labour hours, machine hours and materials. They are represented by the variables and parameters involved in the model formulation.

### 2.1 Decision Variables

Decision variables are a set of quantities that need to be determined in order to solve the linear programming problem. They are so called because the problem is to decide what value each variable should take. Typically, the variables represent the amount of a resource to use or the level of some activity. (Agarana, M. C., Anake T.A., and Adeleke O.J.,2014)
For this paper, let the decision variables be represented as follows:
$x_{i j}$ : Number of product j produced by employing resource i
$x_{1 j}$ : Number of product j produced by employing resource 1
$x_{2 j}$ : Number of product j produced by employing resource 2
$x_{3 j}$ : Number of product j produced by employing resource 3

Specifically, we have the following representations:
$x_{11}$ : Number of sachet water produced by employing machine hour.
$x_{12}$ : Number of bottle water produced by employing machine hour
$x_{13}$ : Number of apple juice produced by employing machine hour
$x_{14}$ : Number of orange juice produced by employing machine hour
$x_{15}$ : Number of yoghurt produced by employing machine hour
$x_{16}$ : Number of communion drink produced by employing machine hour.
$x_{21}$ : Number of sachet water produced by employing labour hour.
$x_{22}$ :Number of bottle water produced by employing labour hour.
$x_{23}$ :Number of apple juice produced by employing labour hour.
$x_{24}$ :Number of orange juice produced by employing labour hour.
$x_{25}$ :Number of yoghurt produced by employing labour hour.
$x_{26}$ : Number of communion drink produced by employing labour hour.
$x_{31}$ :Number of sachet water produced by employing material.
$x_{32}$ :Number of bottle water produced by employing material $x_{33}$ :Number of apple juice produced by employing material $x_{34}$ : Number of orange juice produced by employing material $x_{35}$ : Number of yoghurt produced by employing material
$x_{36}$ : Number of communion drink produced by employing material $x_{32}$ : Number of bottle water produced by employing material $x_{33}$ :Number of apple juice produced by employing material $x_{34}$ : Number of orange juice produced by employing material $x_{35}$ : Number of yoghurt produced by employing material
$x_{36}$ :Number of communion drink produced by employing material

### 2.2 Resources Utilization

The resources used for production are represented as follows:
$a_{i j}$ :Amount/Number of resource i used to produce a unit of product j $a_{1 j}$ : Number of labour hour used to produce one unit of product $j$ $a_{2 j}$ : Number of machine hour used to produce one unit of product j $a_{3 j}$ : Amount, in kilogram, of material used to produce one unit of product j

### 2.2.1 Labour Hours used:

$a_{11}$ : Number of labour hour used to produce one unit of sachet water $=1.5$ seconds
$a_{12}$ : Number of labour hour used to produce one unit of bottle water $=7.083$ seconds
$a_{13}$ : Number of labour hour used to produce one unit of apple juice $=$ 8.5714seconds
$a_{14}$ : Number of labour hour used to produce one unit of orange juice $=8.5714$ seconds
$a_{15}$ : Number of labour hour used to produce one unit of yoghurt $=$ 75seconds
$a_{16}$ : Number of labour hour used to produce one unit of apple juice $=$ 14.094 seconds

### 2.2.2 Machine Hours used:

$a_{21}$ : Number of machine hour used to produce one unit of sachet water $=1$ second
$a_{22}$ : Number of machine hours used to produce one unit of bottle water $=5$ seconds
$a_{23}$ : Number of machine hours used to produce one unit of apple juice $=8.75$ seconds
$a_{24}$ : Number of machine hours used to produce one unit of orange juice $=8.75$ seconds
$a_{25}$ : Number of machine hours used to produce one unit of yoghurt $=50$ seconds
$a_{26}$ : Number of machine hours used to produce one unit of apple juice $=12.5$ seconds

### 2.2.3 Amount of Materials used:

$a_{31}$ : Amount of material used to produce one unit of sachet water $=$ 0.12
$a_{32}$ : Amount of material used to produce one unit of bottle water $=$ 0.34 kg
$a_{33}$ : Amount of material used to produce one unit of apple juice $=$ 3.49 kg
$a_{34}$ : Amount of material used to produce one unit of orange juice $=$ 2.01 kg
$a_{35}$ : Amount of material used to produce one unit of yoghurt $=$ 1.24 kg
$a_{36}$ : Amount of material used to produce one unit of communion drink $=3.40 \mathrm{~kg}$

Table 1: Hebron Drinks' Daily Production Data

| Days | $x_{1}$ <br> (bag) | $x_{2}$ <br> (tray) | $x_{3}$ <br> (tray) | $x_{4}$ <br> (tray) | $x_{5}$ <br> (tray) | $x_{6}$ <br> (tray) | Available <br> resources |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | :--- |
| Day I | 700 | 700 | 41 | 80 | 240 | 260 | 2400 |
| Day II | 600 | 600 | 79 | 84 | 240 | 149 | 2000 |
| Day III | 500 | 300 | 39 | 99 | - | 220 | 1700 |

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| Day IV | 700 | 700 | 26 | 64 | 215 | 190 | 2125 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| Day V | 750 | 600 | 60 | 100 | 200 | 149 | 2318 |
| Cost per <br> week | $\# 85427$ | $\# 664.630$ | $\# 185,220$ | $\# 240,828$ | $\# 676,620$ | $\# 789,880$ | $\# 2,629,565.40$ |

Where $x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}$ are defined as follows:
$x_{1}$ : Number of sachet water $x_{4}$ : Number of orange juice produced per day using labour hour, machine hours and materials
$x_{2}$ : Number of bottle water produced per day using labour hour, machine hours and materials
$x_{3}$ : Number of apple juice produced per day using labour hour, machine hours and materials
produced per day using labour hour, machine hours and materials
$x_{5}$ : Number of yoghurt produced per day using labour hour, machine hours and materials
$x_{6}$ : Number of communion produced per day using labour hour, machine hours and materials

## Table 2: Cost of Producing One Unit of Product at Hebron Drinks

| $c_{j}$ | cost per unit of producing product j | Cost in naira(\#) |
| :--- | :--- | :--- |
| $c_{1}$ | cost per unit of producing sachet water | 1.314 |
| $c_{2}$ | cost per unit of producing bottle water | 19.1 |
| $c_{3}$ | cost per unit of producing apple juice | 51 |
| $c_{4}$ | cost per unit of producing orange juice | 47 |
| $c_{5}$ | cost per unit of producing yoghurt | 63 |
| $c_{6}$ | cost per unit of production communion drink | 68 |

From the information gathered at Hebron Drinks, the total cost of producing all the products, on a weekly basis, is 2.63 million naira. Also the minimum required capacity, in terms of, labour hour, machine hour and material are 12600 minutes, 2160 minutes and 149 kilograms respectively.

### 2.1 The Model

The resulting linear programming model from the above representations and formulation is as follows:
$\operatorname{Minimize} \mathrm{Z}=\sum_{j=1}^{6} c_{j} x_{j}$
Subject to

$$
\begin{aligned}
& \sum_{j=1}^{6} a_{1 j} x_{j} \geq 12600 \\
& \sum_{j=1}^{6} a_{2 j} x_{j} \geq 2160 \\
& \sum_{j=1}^{6} a_{3 j} x_{j} \geq 149 \\
& \sum_{i=1}^{3} a_{i 1} x_{1} \geq 65000 \\
& \sum_{i=1}^{3} a_{i 2} x_{2} \geq 34800 \\
& \sum_{i=1}^{3} a_{i 3} x_{3} \geq 4165 \\
& \sum_{i=1}^{3} a_{i 4} x_{4} \geq 5124 \\
& \sum_{i=1}^{3} a_{i 5} x_{5} \geq 10740 \\
& \sum_{i=1}^{3} a_{i 6} x_{6} \geq 11580 \\
& x_{j} \geq 0, j=1,2, \ldots, 6
\end{aligned}
$$

Substituting the values of Cj 's and $a_{i j}{ }^{\prime} s$, the model becomes;

Minimize $\mathrm{Z}=1.314 x_{1}+19.1 x_{2}+51 x_{3}+47 x_{4}+63 x_{5}+68 x_{6}$ $0.025 x_{1}+0.118 x_{2}+0.143 x_{3}+0.143 x_{4}+1.25 x_{5}+0.235 x_{6} \geq 12600$
$0.016 x_{1}+0.083 x_{2}+0.146 x_{3}+0.146 x_{4}+0.83 x_{5}+0.21 x_{6} \geq 2160$ $0.12 x_{1}+0.39 x_{2}+3.49 x_{3}+2.01 x_{4}+1.24 x_{5}+3.4 x_{6} \geq 149$ $0.1617 x_{1} \geq 65000$
$0.540 x_{2} \geq 34800$
$5.114 x_{3} \geq 4165$
$3.634 x_{4} \geq 5124$
$3.32 x_{5} \geq 10740$
$3.843 x_{6} \geq 11580$
$x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6} \geq 0$

### 2.3 The Dual

In this section we find the dual of the above minimization problem, resulting in the following maximization problem, with
A,B,C,D,E,F,G,H,I,J,K,L,M,N,O as the new decision variables.

Max imize $P=12600 A+2160 B+149 C+65000 D+$ $34800 E+4165 F+5124 G+10740 H+11580 I$
Subject to:
$0.025 A+0.016 B+0.12 C+0.1617 D 34$
$0.118 A+0.083 B+0.39 C+0.54 E \leq 19.1$
$0.143 A+0.146 B+3.49 C+5.114 F \leq 51$
$0.143 A+0.146 B+2.01 C+3.634 G \leq 47$
$1.25 A+0.83 B+1.24 C+3.32 H \leq 63$
$0.235 A+0.21 B+3.4 C+3.843 I \leq 68$
$A, B, C, D, E, F, G, H, I \geq 0$

### 2.4 The Standardized Form of the Dual Problem

In order to form the initial tableau, we standardized the above dual problem by introducing slack variables as follows (M.C. Agarana and T.O. Olokunde, 2015)

Max imize $P=12600 A+2160 B+149 C+65000 D+$ $34800 E+4165 F+5124 G+10740 H+11580 I$

Subject to:
$1.25 A+0.83 B+1.24 C+3.32 H+N=63$
$0.025 A+0.016 B+0.12 C+0.1617 D+J=1.314$
$0.118 A+0.083 B+0.39 C+0.54 E+K=19.1$
$0.235 A+0.21 B+3.4 C+3.843 I+O=68$
$A, B, C, D, E, F, G, H, I, J, K, L, M, N, O \geq 0$
$0.143 A+0.146 B+3.49 C+5.114 F+L=51$
$0.143 A+0.146 B+2.01 C+3.634 G+M=47$

Table 3: Initial Simplex Tableau

|  | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | RHS |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| J | 0.025 | 0.016 | 0.12 | 0.1617 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1.314 |
| K | 0.118 | 0.083 | 0.39 | 0 | 0.54 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 19. <br> 1 |
| L | 0.143 | 0.146 | 3.49 | 0 | 0 | 5.114 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 51 |
| M | 0.143 | 0.146 | 2.01 | 0 | 0 | 0 | 3.634 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 47 |
| N | 1.25 | 0.83 | 1.24 | 0 | 0 | 0 | 0 | 3.32 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 63 |
| O | 0.235 | 0.21 | 3.4 | 0 | 0 | 0 | 0 | 0 | 3.843 | 0 | 0 | 0 | 0 | 0 | 1 | 68 |
| P | 12600 | 2160 | 149 | 65000 | 34800 | 4165 | 5124 | 10740 | 11580 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

This initial simplex tableau was solved using computer application software (LIP SOLVER), and the following results obtained. It was observed that six (6) iterations were
involved. The tables resulting from the iterations give better results as the iterations progresses. The sixth iteration therefore gives the best result shown in table 9 .

Table 4: Simplex Tableau 2 (First iteration)

|  | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | RHS |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| D | 0.155 | $16 / 1617$ | 0.742 | 1 | 0 | 0 | 0 | 0 | 0 | 6.18429 | 0 | 0 | 0 | 0 | 0 | 8.1262 |
| K | 0.21 <br> 9 | 0.083 | 0.39 | 0 | 0.54 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 19.1 |
| L | 0.14 <br> 3 | $73 / 500$ | 3.49 | 0 | 0 | 5.114 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 51 |
| M | 0.143 | $73 / 500$ | 2.01 | 0 | 0 | 0 | 3.634 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 47 |
| N | 1.25 | 0.83 | 1.24 | 0 | 0 | 0 | 0 | 3.32 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 63 |
| O | 0.23 <br> 5 | 0.21 | 3.4 | 0 | 0 | 0 | 0 | 0 | 3.843 | 0 | 0 | 0 | 0 | 0 | 1 | 68 |
| P | $2550.53-4271.66$ | -48088.5 | 0 | 34800 | 4165 | 5124 | 10740 | 11580 | -401979 | 0 | 0 | 0 | 0 | 0 | 528200 |  |

Table 5: Simplex tableau 3 (Second Iteration)

|  | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | RHS |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| D | 0.155 | 0.099 | 0.742 | 1 | 0 | 0 | 0 | 0 | 0 | 6.18429 | 0 | 0 | 0 | 0 | 0 | 8.1262 |
| E | 0.219 | 0.083 | 0.722 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1.85 | 0 | 0 | 0 | 0 | 35.37 |
| L | 0.143 | 0.146 | 3.49 | 0 | 0 | 5.114 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 51 |
| M | 0.143 | 0.146 | 2.01 | 0 | 0 | 0 | 3.634 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 47 |
| N | 1.25 | 0.83 | 1.24 | 0 | 0 | 0 | 0 | 3.32 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 63 |
| O | 0.235 | 0.21 | 3.4 | 0 | 0 | 0 | 0 | 0 | 3.843 | 0 | 0 | 0 | 0 | 0 | 1 | 68 |
| P | -5053.92 | -9620.55 | -73221.8 | 0 | 0 | 4165 | 5124 | 10740 | 11580 | -401979 | -64444.4 | 0 | 0 | 0 | 0 | 1759090 |

## Table 6: Simplex tableau 4 (Third Iteration)

|  | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | RHS |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| D | 0.155 | 0.099 | 0.742 | 1 | 0 | 0 | 0 | 0 | 0 | 6.18429 | 0 | 0 | 0 | 0 | 0 | 8.1262 |  |
| E | 0.219 | 0.154 | 0.722 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1.85 | 0 | 0 | 0 | 0 |  | 35.37 |
| L | 0.143 | 0.146 | 3.49 | 0 | 0 | 5.114 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 51 |  |
| M | 0.143 | 0.146 | 2.01 | 0 | 0 | 0 | 3.634 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 47 |  |
| N | 1.25 | 0.83 | 1.24 | 0 | 0 | 0 | 0 | 3.32 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 63 |  |
| O | 0.235 | 0.055 | 0.88473 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0.26 | 17.6945 |  |
| P | -5762.04 | -10253.3 | -83466.9 | 0 | 0 | 4165 | 5124 | 10740 | 0 | -401979 | -64444.4 | 0 | 0 | 0 | -3013.27 | 1963990 |  |

Table 7: Simplex tableau 5 (Fourth Iteration)

|  | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | RHS |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| D | 0.155 | 0.099 | 0.742 | 1 | 0 | 0 | 0 | 0 | 0 | 6.18429 | 0 | 0 | 0 | 0 | 0 | 8.1262 |
| E | 0.219 | 0.154 | 0.722 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1.85 | 0 | 0 | 0 | 0 | 35.37 |
| L | 0.143 | 0.146 | 3.49 | 0 | 0 | 5.114 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 51 |
| M | 0.143 | 0.146 | 2.01 | 0 | 0 | 0 | 3.634 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 47 |
| N | 0.377 | 0.25 | 0.373 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.3012 | 0 | 18.9759 |
| I | 0.061 | 0.055 | 0.88473 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0.26 | 17.6945 |
| P | -9805.71 | -12938.3 | 7478.3 | 0 | 0 | 4165 | 5124 | 0 | 0 | 401979 | 64444.4 | 0 | 0 | -3234.94 | -3013.27 | 2167790 |

Table 8: Simplex tableau 6 (Fifth iteration)

|  | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | RHS |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| D | 0.155 | 0.099 | 0.742 | 1 | 0 | 0 | 0 | 0 | 0 | 6.18429 | 0 | 0 | 0 | 0 | 0 | 8.1262 |
| E | 0.219 | 0.154 | 0.722 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1.85 | 0 | 0 | 0 | 0 | 35.37 |
| L | 0.143 | 0.146 | 3.49 | 0 | 0 | 5.114 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 51 |
| G | 0.143 | 0.0402 | 0.55311 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0.2752 | 0 | 0 | 12.9334 |
| H | 0.377 | 0.25 | 0.373 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.3012 | 0 | 18.9759 |
| I | 0.061 | 0.055 | 0.88473 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0.26 | 17.6945 |
| P | 10007.3 | -13144.2 | -90312.4 | 0 | 0 | 4165 | 0 | 0 | 0 | -401979 | -64444.4 | 0 | -1410.02 | -3234.94 | -3013.27 | 2167790 |

Table 9: Simplex tableau 7 (Sixth iteration)

|  | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | RHS |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| D | 0.155 | 0.099 | 0.742 | 1 | 0 | 0 | 0 | 0 | 0 | 6.18429 | 0 | 0 | 0 | 0 | 0 | 8.126 |
| E | 0.219 | 0.154 | 0.722 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1.85 | 0 | 0 | 0 | 0 | 35.37 |
| F | 0.027 | 0.02855 | 0.68244 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.195 | 0 | 0 | 0 | 51 |
| G | 0.039 | 0.0402 | 0.5531 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0.2752 | 0 | 0 | 12.93 |
| H | 0.377 | 0.25 | 0.373 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.3012 | 0 | 18.97 |
| I | 0.061 | 0.055 | 0.88473 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0.26 | 17.6945 |
| P | -10123.8 | -13263.1 | -93154.8 | 0 | 0 | 0 | 0 | 0 | 0 | 401979 | -64444.4 | -814.423 | -1410.02 | -3234.94 | -3013.27 | 2,275600 |

## 4. Interpretation of Result and Discussion

From the final tableau, the decision variables values are as follows:
$x_{1}=401979$
$x_{2}=64444.4$
$x_{3}=814.431$
$x_{4}=1410.02$
$x_{5}=3234.94$
$x_{6}=3013.27$

Substituting these values into the objective function of the primal problem, we have:
$1.314(41979)+19.1(6441.4)+5(1814.431)+47(1410.02)+63(3234.44)+68(3031.27)$
$=2,276,822.477$ naira
This value is approximately the same as the optimal feasible solution shown in the final tableau (the sixth iteration). This means that the cost of production of the six (6) products in HEBRON DRINKS can
be reduced to 2.276822 .947 million naira as against the prevailing total cost of production of 2.63 million naira per week. This implies that if the management of Hebron Drinks actually wants to maximize their profit, by efficiently putting the scarce available resources into use, the following must be put into consideration: Number of sachet water produced per day using labour hour, machine hours and materials must be 401979 . Number of bottle water produced per day using labour hour, machine hours and materials must be 64444 . Number of apple juice produced per day using labour hour, machine hours and materials must be 814. Number of orange juice produced per day using labour hour, machine hours and materials must be 1410 .
Number of yoghurt produced per day using labour hour, machine hours and materials must be 3235 .
Number of communion produced per day using labour hour, machine hours and materials must be 3013 .
Looking at the values of the decision variables closely, we realized that less of apple juice should be produced while more of sachet water and bottled water should be produced. Doing this will minimize the usage of the available resources and maximize the company's profit.

### 4.1 Sensitivity Analysis

Sensitivity analyses were performed to identify key factors affecting the behaviour of the model. The sensitized parameters include:

- $C_{J}$ ' $s$ and
- $a_{I J}$ 's

We had a new set of decision variables, as a result of varying the sensitized parameters, which when substituted into the objective function gave 2,037,604 and 2,168,249 respectively.
We can see that both the $C_{J}{ }^{\prime} s$ and $a_{I J}$ 's are sensitive to the result of the model. The management of Hebron Drinks can manipulate these parameters to their benefit without necessarily altering their organizational policy or set goals in terms of expected income.

## 5. Conclusion

The data collected from HEBRON DRINKS was modeled into a Linear Programming Problem.
A primal problem was formed, and being a minimization case that would be solved using simplex method, it was imperative to find the Dual of the primal problem; hence a dual problem was developed out of the Linear Programming Problem. Upon obtaining the initial simplex tableau, the data was then run on a lip solver (linear integer problem solver) software application. The analysis done, using operations research technique (the linear programming problem, solved by using the simplex method) showed that average weekly cost of production at Hebron Drinks was 2,629,565.40 naira. Based on this research work, the company can reduce its weekly
cost of production to $2,227,822.95$ naira, which represents $13.41 \%$ decrease in the weekly production cost. The results also revealed that, for the decrease to be achieved there has to be an adherence to a new production routine. We can see that both the $C_{J}$ 's and $a_{I J} ' s$ are sensitive to the result of the model.

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The management of Hebron Drinks can manipulate these parameters to their benefit without necessarily altering their organizational policy or set goals especially as it affects the contributions and amount of resources used in producing one unit of each of the products in order to maximize their expected income.

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