Covenant Journal of Informatics & Communication Technology. Vol. 7 No. 1, June, 2019
ISSN: p2354-3566 e 2354-3507 DOI: 10.20370/yhas-n460

An Open Access Journal Available Online

Graphical User Interface (GUI) for Position and Trajectory Tracking Control of the Ball and Plate System Using H-Infinity Controller

Abubakar Umar[1]*, Zaharudden Haruna[1], Umar Musa[2], Shehu Aminu Mohammed[2], Momoh Omuya Muyideen[1]

[1]Computer Engineering Department, Ahmadu Bello University, Zaria, Kaduna, Nigeria
[2]Electrical Engineering Department, Ahmadu Bello University, Zaria, Kaduna, Nigeria
abubakaru061010@gmail.com, elzet2007@gmail.com, umarnadada@yahoo.co.uk
amshehu12@gmail.com, momuyadeen@gmail.com
*Corresponding Author

Received: 05. 02. 2019 Accepted: 16. 04. 2019 Date of Publication: June, 2019

Abstract: In this paper, a graphical user interface (GUI) for position and trajectory tracking of the ball and plate system (BPS) control scheme using the double feedback loop structure i.e. a loop within a loop is proposed. The inner and the outer loop was designed using linear algebraic method by solving a set of Diophantine equations and sensitivity function. The results were simulated in MATLAB 2018a, and the trajectory tracking was displayed on a GUI, which showed that the plate was able to be stabilized at a time of 0.3546 seconds, and also the ball settled at 1.7087 seconds, when a sinusoidal circular reference trajectory of radius 0.4m with an angular frequency of 1.57rad/sec was applied to the BPS, the trajectory tracking error was 0.0095m. This shows that the controllers possess the following properties for the BPS, which are; good adaptability, strong robustness and a high control performance.

Keywords: Ball and plate system (BPS), controller, graphical user interface (GUI), linear algebraic method and Diophantine equation.

1. Introduction

Balancing is amongst the most difficult and interesting aspect in the field of control engineering. There are a lot of benchmark control systems for studying and testing of control algorithms that is used to balance the systems like; the ball and beam system (BBS), double inverted pendulum and the traditional cart-pole system (inverted pendulum) (Cheng & Tsai, 2016). The BPS is a benchmark nonlinear plant that is more complex than the BBS; which is amongst the most pertinent and integral models in control engineering education, and is used at undergraduate and post-graduate studies in teaching and testing of control algorithms (Galvan-Colmenares et al., 2014; Oravec & Jadlovská, 2015). The BPS, an extension of the BBS is a nonlinear and multivariable system (Bigharaz et al., 2013; Cheng & Chou, 2016), which consist of a ball that rolls freely on a flat horizontal plate. The ball is required to follow a given path by manipulating the plates tilt with respect to the two mutually perpendicular directions (Roy et al., 2016b). The control objective of the BPS is to balance and allow the ball to track the reference trajectory on the flat plate, by tilting the plate with respect to the horizontal plane. This BPS is very important to the control engineer, because it allows a user to study and validate different linear and nonlinear systems before applying it to real life problem that exhibits such similar dynamics (Debono & Bugeja, 2015). The BPS has four degrees of freedom (DOF), the tilt of the plate is controlled by two actuating inputs, which indicates that it is an under-actuated system (Das & Roy, 2017). It is an open-loop unstable system, as the balls position becomes uncontrolled when the plate is in a tilted position either on its x or y-axis (Farooq et al., 2013), this results in difficulty in modelling and control of the BPS (Roy et al., 2016a). The system can be applied in areas like unmanned aerial vehicle (UAV), humanoid robots, rocket systems and satellite control (Cherubini et al., 2009; Mukherjee et al., 2002) in the following fields like; friction compensation, trajectory tracking and path planning (Oriolo & Vendittelli, 2005).

Stabilization and trajectory tracking of the BPS has been investigated with the use of different control schemes, some research works has been done using classical controllers like in the work of (Yıldız & Gören-Sümer, 2017) and Dušek et al. (2017), which used the first principle and optimal control approach; the proportional-integral-differential (PID) controllers; Zeeshan et al. (2012) used PID controllers with a discrete phototransistor to design, control and implement the BPS. However, Mohammadi and Ryu (2018) used neural network to tune PID gains of the nonlinear BPS. Also, Aphiratsakun and Otaryan (2016) compared the tuning technique of PID controllers using MATLAB/Simulink with that of manual tuning technique that was on the basis of the trial and error process, and PID tuning methods from Ziegler-Nichols and Tyreus-luyben (closed-loop proportional gain control or P-Control test), and Ali and Aphiratsakun (2015) used PID controller to demonstrate how to make the ball balance on the plate. Others used PID controllers optimized by some meta-heuristic algorithms like Dong et al. (2009), applied genetic algorithm (GA) to on-line updated PID neural network controller for the BPS. However, Fei et al. (2011) tuned the PID controller of the BPS using RBF neural network. The RBF neural network tuned PID controller of the system was compared with the conventional PID and fuzzy controller, Han et al. (2012) tuned the PID controller using an improved particle swarm optimization (PSO) on-line training of the PID neural network controller, also Bang and Lee (2018) implemented the BPS on a steward platform using sliding mode controller (SMC), and compared the result of the SMC with that of linear quadratic (LQ) control and experimental results. Other researchers have modified the PID controller to PD controllers tuned by other meta-heuristics (Borah et al., 2014; Roy et al., 2015). Others used I-PD (Mochizuki & Ichihara, 2013), in which the PID controller was designed based on the generalized Kalman-Yakubovich-Popov lemma, Galvan-Colmenares et al. (2014) modified the normal proportional-diffrential (PD) controller to a new dual form for the control of the BPS. Also, others used fuzzy logic controller (Fan et al., 2004; C. E. Lin et al., 2014; Ming et al., 2011), neural network controller (Singh & Bhushan, 2018; Wang et al., 2012), robust controllers like sliding mode controller (Morales et al., 2017; Suleiman et al., 2018), H-infinity controller (H.-Q. Lin et al., 2014; Umar et al., 2018) and linear quadratic Gaussian (LQG) controller (Jafari et al., 2012; Umar et al., 2019).

The most important control scheme of the BPS is the double feedback loop structure, which is a loop in a loop is adopted (Liu & Liang, 2010) in this research, also researchers have adopted the double feedback loop structure (Das & Roy, 2017; Negash & Singh, 2015; Roy et al., 2016a). The inner loop functions as a position controller that used DC servo motor, while the outer loop functions as a linear controller that controls the position of the ball (Liu & Liang, 2010).

In this paper, the BPS is viewed as a double feedback loop structure for position and trajectory tracking control. The inner feedback loop which is controlled by a DC motor servo controller for positioning of the ball on the plate will be designed using linear algebraic method in which a set of Diophantine equations will be solved, and the outer loop will also be designed using H-infinity controller. A MATLAB based graphical user interface (GUI) will be developed for the trajectory tracking of the BPS.

The paper is structures as follows; section 2 gives the mathematical modelling of the BPS, and section three elaborates on the materials and methods that was used for the design of the system. Section four discusses the simulation results of the double feedback loop structure of the BPS, finally the conclusion is given in section five.

Mathematical Model of the BPS

The HUMUSOFT laboratory model of the BPS is shown in Figure 1.

Figure 1: The Humusoft BPS laboratory model (humusoft Ltd., 2012a)

Also, the mathematical schematic model of the BPS is given in Figure 2.

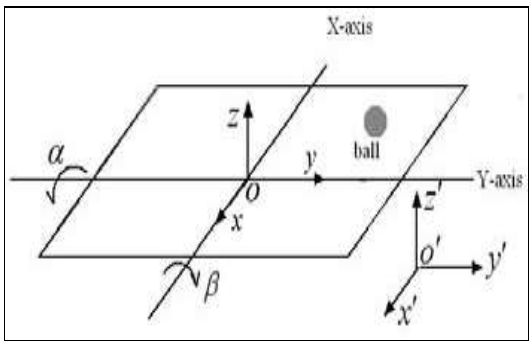

Figure 2: The schematic model for the BPS

The plate rotates in two different axis, which are x-and –y-axis, and are in perpendicular directions. The Kinematic differential equations of the BPS are derived by the use of Euler-Lagrange equation, which is written as (Escobar *et al.*, 2017; Kassem *et al.*, 2015):

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \qquad (1)$$

q_i represents the i^{th} -direction coordinate, T also represents the kinetic energy of the system, V gives the potential energy, and Q is the composite force.

The BPS can be represented in a simple form as a particle system that consist of two rigid bodies; the plate's geometry has limits in translation along the x-y-z axis, and also a geometry limit in rotation about the z-axis. The plate has two DOF in rotation about the x-y axis. The ball's geometry limit in translation is along the z-axis. It has two DOF along the x-y-axis. The BPS system model has four DOF, which has the generalized coordinates as (Hussein *et al*., 2017; Saud & Alwan, 2017):

$$q_1 = x,\ q_2 = y,\ q_3 = \alpha,\ q_4 = \beta$$

The total kinetic energy of the system is represented in equation (2):

$$T = T_{ball} + T_{plate} \tag{2}$$

$$T_{ball} = \frac{1}{2}\left[\begin{array}{r}\left(m_b + \frac{J_b}{R_b^{\ 2}}\right)\left(\dot{x}^2 + \dot{y}^2\right) + J_b\left(\dot{\alpha}^2 + \dot{\beta}^2\right) \\ m_b\left(x\dot{\alpha} + y\dot{\beta}\right)^2\end{array}\right] \tag{3}$$

$$T_{plate} = \frac{1}{2}\left[\begin{array}{r}\left(J_{Px}\dot{\alpha}^2 + J_{py}\dot{\beta}^2\right) + \left(m_b + \frac{J_b}{R_b^{\ 2}}\right)\left(\dot{x}^2 + \dot{y}^2\right) \\ + J_b\left(\dot{\alpha}^2 + \dot{\beta}^2\right) + m_b\left(x\dot{\alpha} + y\dot{\beta}\right)^2\end{array}\right] \tag{4}$$

The potential energies of the system along the x-y-axis is given as:

$$V_x = m_b g x \sin\alpha \tag{5}$$

$$V_y = m_b g y \sin\beta \tag{6}$$

The mathematical equation of the BPS is given as:

$$\left(m_b + \frac{J_b}{R_b^{\ 2}}\right)\ddot{x} - m_b x\left(\dot{\alpha}\right)^2 - m_b y\dot{\alpha}\dot{\beta} + m_b g\sin\alpha = 0 \tag{7}$$

$$\left(m_b + \frac{J_b}{R_b^{\ 2}}\right)\ddot{y} - m_b y\left(\dot{\beta}\right)^2 - m_b x\dot{\alpha}\dot{\beta} + m_b g\sin\alpha = 0 \tag{8}$$

$$\begin{array}{r}\left(m_b x^2 + J_b + J_{Px}\right)\ddot{\alpha} + 2m_b x\dot{x}\dot{\alpha} + m_b xy\ddot{\beta} \\ + m_b\left(\dot{x}y + x\dot{y}\right)\dot{\beta} + m_b g x\cos\alpha = \tau_x\end{array} \tag{9}$$

$$\begin{array}{r}\left(m_b y^2 + J_b + J_{Py}\right)\ddot{\beta} + 2m_b y\dot{y}\dot{\beta} + m_b xy\ddot{\alpha} \\ + m_b\left(\dot{x}y + x\dot{y}\right)\dot{\alpha} + m_b g y\cos\beta = \tau_y\end{array} \tag{10}$$

$m_b\left(kg\right)$ is the mass of the ball, $J_b\left(kgm^2\right)$ is the rotational moment of inertia of the ball, $J_{P_x}\left(kgm^2\right)$ and $R_b\left(m\right)$ are the rotational moment of inertia of the plate and the radius of the ball; $x\left(m\right)$ and $y\left(m\right)$ are the position of the ball along the x-y-axis; $\dot{x}\left(m/s\right)$ and

$\ddot{x}\left(m/s^2\right)$ are the velocity and acceleration along the x-axis; $\dot{y}\left(m/s\right)$ and $\ddot{y}\left(m/s^2\right)$ are the velocity and acceleration along the y-axis; $\alpha\left(rad\right)$ and $\dot{\alpha}\left(rad/\sec\right)$ are the plate deflection angle, and angular velocity about x-axis; $\beta\left(rad\right)$ and $\dot{\beta}\left(rad/\sec\right)$ are the plate deflection angle and angular velocity about y-axis; $\tau_x\left(Nm\right)$ and $\tau_y\left(Nm\right)$ are the torques on the plate in the x-yaxis.

Equation (7) and (8) governs the movement of the ball on the plate; it also states how the effect of the ball's acceleration depends on the plate's deflection angle and also it's angular velocity; Equation (9) and (10) relates to how the dynamics of the plate's deflection depends on the external driving forces, and the position of the ball (Duan *et al.*, 2009):

Consider the assignment of the state variable as (Fan *et al.*, 2004):

$$X = \left(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\right)^T = \left(x, \dot{x}, \alpha, \dot{\alpha}, y, \dot{y}, \beta, \dot{\beta}\right)^T \quad (11)$$

The state space equation of the BPS is given as (Fan *et al.*, 2004):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} x_2 \\ B\left(x_1 x_4{}^2 + x_4 x_5 x_8 - g\sin x_3\right) \\ x_4 \\ 0 \\ x_6 \\ B\left(x_5 x_8{}^2 + x_1 x_4 x_8 - g\sin x_7\right) \\ x_8 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix} \quad (12)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \left[X\right] \quad (13)$$

Where:

$$B = \frac{m_b}{\left(m_b + \frac{J_b}{R_b{}^2}\right)} \quad (14)$$

In its steady state, the plate is in the horizontal position where the inclination of both angles are equal to zero, if the change of the inclination angles is not much, i.e. $\pm 5^0$, then, the sine function should be represented by its argument (Fabregas *et al.*, 2017). The BPS can be simplified and decomposed into x-y axis as (Umar *et al.*, 2019):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ B\left(x_1 x_4^{\,2} - g \sin x_3\right) \\ x_4 \\ 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \left[u_x\right] \tag{15}$$

$$\begin{bmatrix} \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} x_6 \\ B\left(x_5 x_8^{\,2} - g \sin x_7\right) \\ x_8 \\ 0 \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \left[u_y\right] \tag{16}$$

3. Materials and Methods

The step by step procedures followed in actualizing the trajectory tracking of the BPS is:

3.1. Double Feedback Loop Structure

The double feedback loop structure can be regarded as two distinct systems that operates simultaneously with each other. Hence, similar but different controllers can be applied for controlling each coordinate of the ball's motion (Awtar et al., 2002). The design of the inner loop is done first, this is where the encoder feedback is sent to the dc servo motors for the position control, and the outer loop is designed next based on the transfer function formed between the inner and outer loop. The inner loop and outer loop controllers designed using linear algebraic method by solving a set of Diophantine equation and controller. The inner loop controls the tilt of the plate, and the outer loop controls the ball's position on the plate. Figure 3 shows the double loop feedback structure of the control scheme.

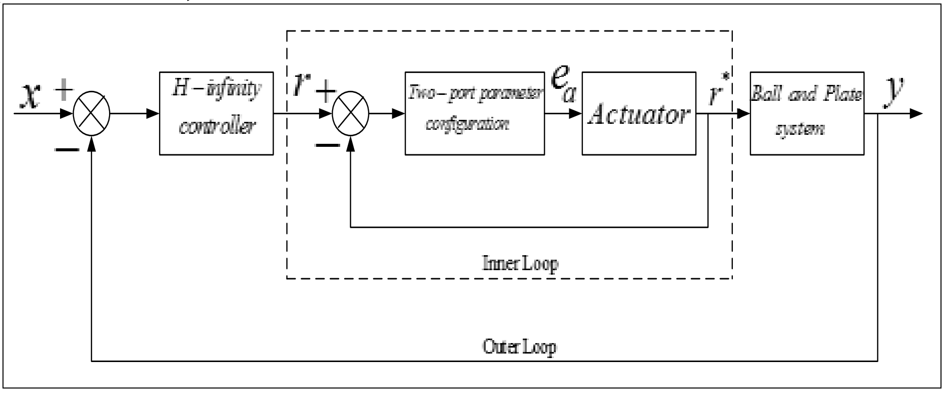

Figure 3: Double feedback loop structure of the control scheme

3.2. Inner Loop Design

The design procedure carried out for the inner loop are:

I. Determination of Actuator Parameters

A specified actuator with a permanent DC motor parameters is considered for the design of the inner loop, the relationship between and is (Golnaraghi & Kuo, 2010):

$$\frac{\theta_L}{e_a} = \frac{K_t \frac{N_1}{N_2}}{\left[J_{eq}L_a s^3 + \left(J_{eq}R_a + D_{eq}L_a\right)s^2 + \left(D_{eq}R_a + K_t K_e\right)s\right]} = T_L \quad (17)$$

The DC servo motors parameters are given based on the following requirements which are; the moment of inertia, the load torque and the speed of the motor. This is given in Table I.

Table I: Parameters of the BPS (HUMUSOFT Ltd., 2012b)

S/N	Description	Symbol	Unit	Value
1	Mass of the ball	m	kg	0.11
2	Radius of the ball	R	m	0.02
3	Dimension of the plate (square)	lxb	m^2	0.16
4	Mass moment of inertia of the plate	$J_{Px,y}$	kgm^2	0.5
5	Mass moment of inertia of the ball	J_b	kgm^2	1.76e-5
6	Maximum velocity of the ball	V	m/s	0.04

Equation (17) is given as (Umar, 2017):

$$\frac{\theta_L}{e_a} = \frac{0.105}{\left[0.47005s^2 + 421.113s\right]} \quad (18)$$

$$= \frac{0.2234}{s\left(s+895.89\right)} \approx \frac{2.49x10^{-4}}{s} \quad (19)$$

II. Two-Port Parameter Configuration

The parameter of the two-port configuration was used for the design of the inner loop. A step response which could settle in 0.4 seconds was required in order to find the value of , making use of the Humusoft BPS manual, and also through simulation, the steady state response which is was found to be .The overall optimal ITAE transfer function of the system with a zero position error is given as (Chen, 1995):

$$G_0\left(s\right) = \frac{\omega_0^{\,2}}{s^2 + 1.4\omega_0 s + \omega_0^{\,2}} \quad (20)$$

In order to limit the step response using a preamplifier, an additional gain of 494 was provided, and $G_0(s)$ is implemented using the two-port configuration as shown in Figure 4.

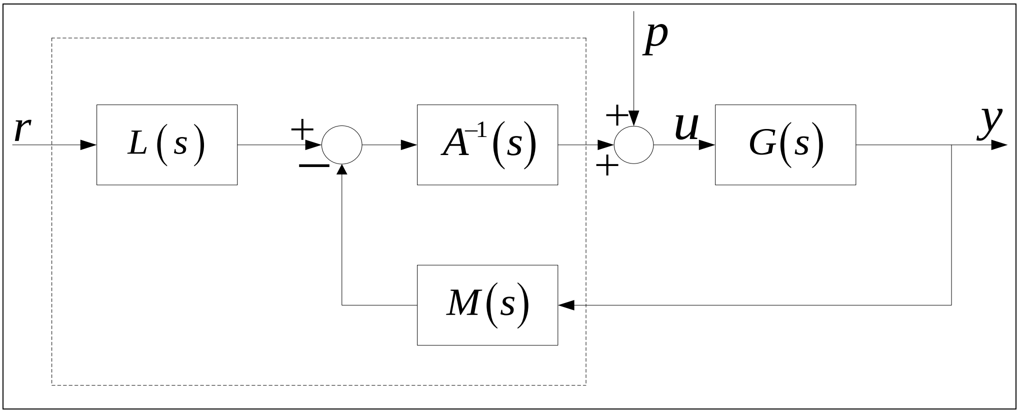

Figure 4: Two-port parameter configuration

Where $L(s)$, $M(s)$and $A(s)$ are the polynomials that describes the compensator, and p is the input disturbance. By solving the Diophantine equation, the values of the compensator and the DC servo actuator has the values as: (Umar *et al.*, 2018):

$$A(s) = 28 + s \tag{21}$$

$$M(s) = 3252 + s \tag{22}$$

$$L(s) = 3252 \tag{23}$$

1.1. Outer Loop Design

The outer loop design of the BPS involves the following:

I. H_∞ Controller

The augmented plant model for H_∞ controller can be constructed as (Dingyu *et al.*, 2007):

$$P(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \tag{24}$$

And the augmented state description is:

$$\dot{x} = Ax + \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{25}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{26}$$

The Straight forward manipulation of the closed loop transfer function gives:

$$Ty_1u_1(s) = P_{11}(s) + P_{12}(s)\left[I - F(s)P_{22}(s)\right]^{-1} F(s)P_{21}(s) \tag{27}$$

Equation (27) is called the linear fractional transformation (LFT) of the interconnected system. For a robust control, the aim is to get a controller that stabilizes the system, such that:

$$u_2(s) = F(s)y_2(s) \tag{28}$$

$$\|Ty_1u_1\|_\infty < 1 \tag{29}$$

The objective of the design is to obtain a robust controller $F_c(s)$ that will guarantee the closed-loop system with an H_∞-norm that bound a positive number γ, given by (Dingyu *et al.*, 2007):

$$\|Ty_1u_1\|_\infty < \gamma \tag{30}$$

Which implies that the controller can be represented by (Dingyu *et al.*, 2007):

$$F_c(s) = \left[\begin{array}{c:c} A_f & -ZL \\ \hdashline F & 0 \end{array}\right] \quad (31)$$

Where:

$$A_f = A + \gamma^{-2} B_1 B_1^T X + B_2 F + ZLC_2 \quad (32)$$

$$F = -B_2^T X \quad (33)$$

$$L = -YC_2^T \quad (34)$$

$$Z = \left(I - \gamma^{-2} YX\right)^{-1} \quad (35)$$

X and Y are the solutions of the two Algebraic Riccati Equations (AREs), which are (Dingyu *et al.*, 2007):

$$A^T X + XA + X\left(\gamma^{-2} B_1 B_1^T - B_2 B_2^T\right)X + C_1 C_1^T = 0 \quad (36)$$

$$AY + YA^T + Y\left(\gamma^{-2} C_1^T C_1 - C_2^T C_2\right)Y + B_1^T B_1 = 0 \quad (37)$$

The conditions for the existence of an H_∞ controller are as follows (Dingyu *et al.*, 2007):

a. D_{11} is small enough that $D_{11} < \gamma$

b. X is the solution of the controller ARE, which is positive-definite;

c. Y is the solution of the observer ARE, which is positive-definite;

d. $\lambda_{\max}\left(XY\right) < \gamma^2$ shows that the eigenvalues of the product of the two Riccati equation solution matrices are all less than γ^2.

The optimal criterion in the design of the optimal H_∞ controller, is given as (Dingyu *et al.*, 2007):

$$\max_{\gamma} \| Ty_1u_1 \|_\infty < \frac{1}{\gamma}$$

(38)

II. H_∞ Mixed Sensitivity Function

In the design of the H_∞ optimal control, using the mixed sensitivity problem, the weighting functions $\mathrm{W}_1(\mathrm{s})$, $\mathrm{W}_2(\mathrm{s})$ and $\mathrm{W}_3(\mathrm{s})$ are used for shaping the plant model $G(s)$. The weighting function $\mathrm{W}_1(\mathrm{s})$, penalises the error signal, $\mathrm{W}_2(\mathrm{s})$ penalises the input signal and $\mathrm{W}_3(\mathrm{s})$ penalises the output signal (Hossain, 2007). This is shown in Figure 5.

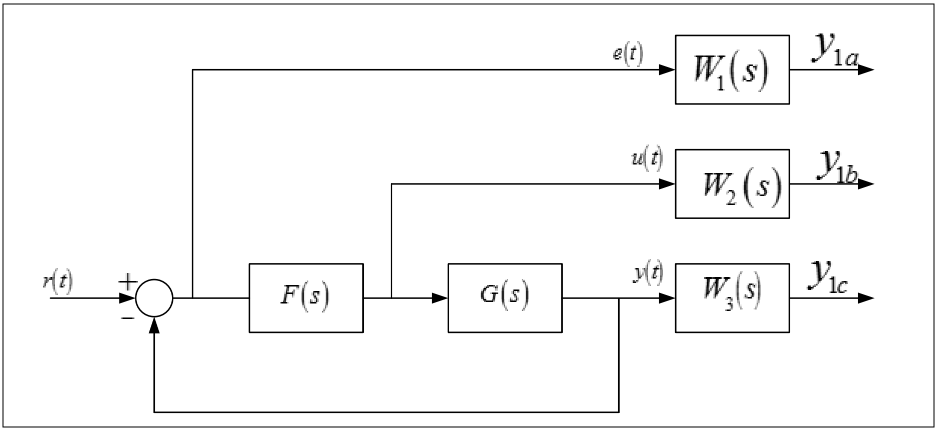

Figure 5: Mixed sensitivity problem

The augmented plant model $P(s)$ is:

$$P(s)=\left[\begin{array}{c:c} W_1 & -W_1G \\ 0 & W_2 \\ 0 & W_3G \\ \hdashline I & -G \end{array}\right] \qquad (39)$$

The linear fractional transformation (LFT) of the mixed sensitivity problem Ty_1u_1, the sensitivity transfer function $S(s)$ and the complementary sensitivity transfer function $T(s)$ are given as (Dingyu *et al.*, 2007):

$$Ty_1u_1=\begin{bmatrix} W_1S \\ W_2FS \\ W_3T \end{bmatrix} \qquad (40)$$

$$S(s)=\left[1+F(s)G(s)\right]^{-1} \qquad (41)$$

$$T(s)=1-S(s)=F(s)G(s)\left[1+F(s)G(s)\right]^{-1} \qquad (42)$$

$F(s)$ is the controller, $S(s)$ and $T(s)$ are known as the sensitivity and complementary transfer functions.

II. Determination of the H_∞ Controller

The weighting functions $W_1(s)$, $W_2(s)$ and $W_3(s)$ for the control of the ball on the plate were selected after extensive simulation and fine tuning as:

$$W_1(s) = \frac{100\left(1.5s^2 + 12.64s + 18.49\right)}{100\left(1.5s^2 + 103.2s + 18.49\right)} \quad (43)$$

$$W_2(s) = 1 \quad (44)$$

$$W_3(s) = \frac{s^2}{100} \quad (45)$$

4. Graphical Use Interface

Graphical User Interface (GUI), refers to the universal idea of buttons, icons, etc., that can be visually presented to a user as a "front-end" of a software application (Patrick & Thomas, 2003). GUI contains a figure window which consists of menus, texts, graphics, buttons etc., that a user can make use of interactively with the aid of the mouse and keyboards. There two main procedures in creating a GUI are (Hunt *et al.*, 2014):

i. Design of its layout, and
ii. To write callback functions which performs the desired operations when different features are selected by the user.

The main advantage of GUIs is its provision of a means through which the individuals can communicate with the aid of a computer without the use of a programming commands. The GUI graphics objects components can be classified in two classes (Hunt *et al.*, 2014):

i. User interface controls *(uicontrols)*
ii. User interface menus *(uimenus)*

The *uicontrols* and *uimenus* is used to create an intuitive, informative and a visually pleasing GUIs when in combination with other graphic objects. In order to have access to them, two approaches are employed; the low-level, bottom-up approach in which use of skills to handle graphics and write M-files that implement the GUI. The other approach is in the use of MATLAB's Graphical User interface Development Environment (GUIDE), which is high-level, extremely easy to use and a powerful technique; an outstanding tool that aids in the development of GUIs , and also takes into consideration of much of the "bookkeeping" which is regularly associated with the development of the GUI (Hunt *et al.*, 2014).

5. Results and Discussions

Simulations results for the inner loop design, outer loop design and trajectory tracking of the ball-on-the- plate is given in the following sub-section. However, the simulation results were obtained from MATLAB 2018a simulation environment.

5.1 Inner Loop Results

The actuator's step response is shown in Figure 6.

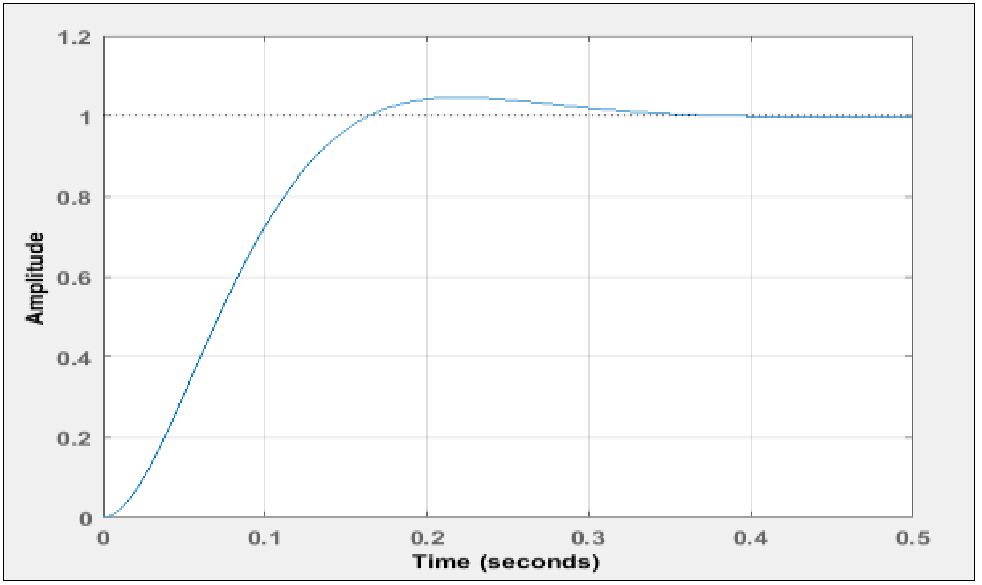

Figure 6: Actuator Step Response

Table II shows the properties of the actuator

Table II: Properties of the actuator

System Response	**Value**
Settling Time (sec)	0.2989
Overshoot (%)	4.5986

From Table II, the actuator's settling time of the actuator is 0.2989 seconds, this shows that the plate should settle before 0.4 seconds that was set for it. However, the actuator $U(s)$ due to a step input should not exceed the DC motor rated voltage, which is 75V. Also, the actuator with open parameter (rated power and voltage) step response is shown in Figure 7.

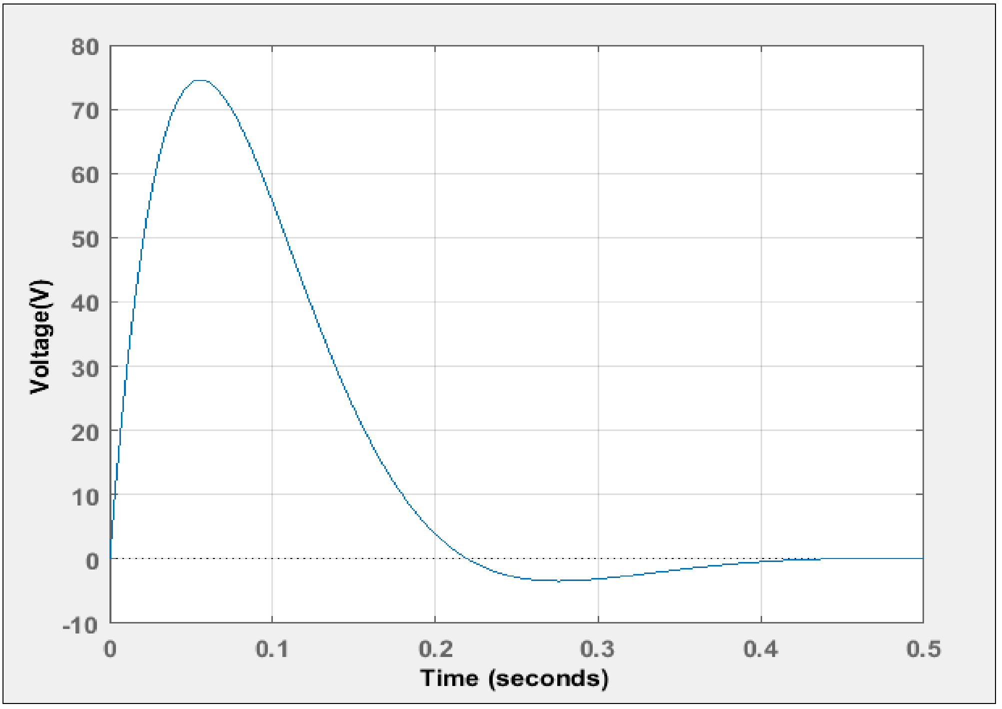

Figure 7: Actuator with open parameters Step Response

The properties of the actuator with open parameters is shown in Table III

Table III: Properties of the Actuator with Open Parameters

Actuator System Response	Value
Settling Time (sec)	0.3546
Peak Voltage (V)	74.5631

From Table III, it can be seen that the stability of the plate is at 0.3546 seconds. The peak voltage which is 74.5631V, is closer to the rated voltage of 75V, which invariably showed that for a proper design, the DC motor actuator should have the following properties as given in Table III.

1.2 Outer Loop Results

The designed H_{∞} controller is:

$$Gc = \frac{23622(s+1.631)(s+0.2213)(s+0.1794)}{(s+1109)(s+25.82)(s+0.9012)(s+0.3308)} \quad (46)$$

The H_∞ controller step response is shown in Figure 8.

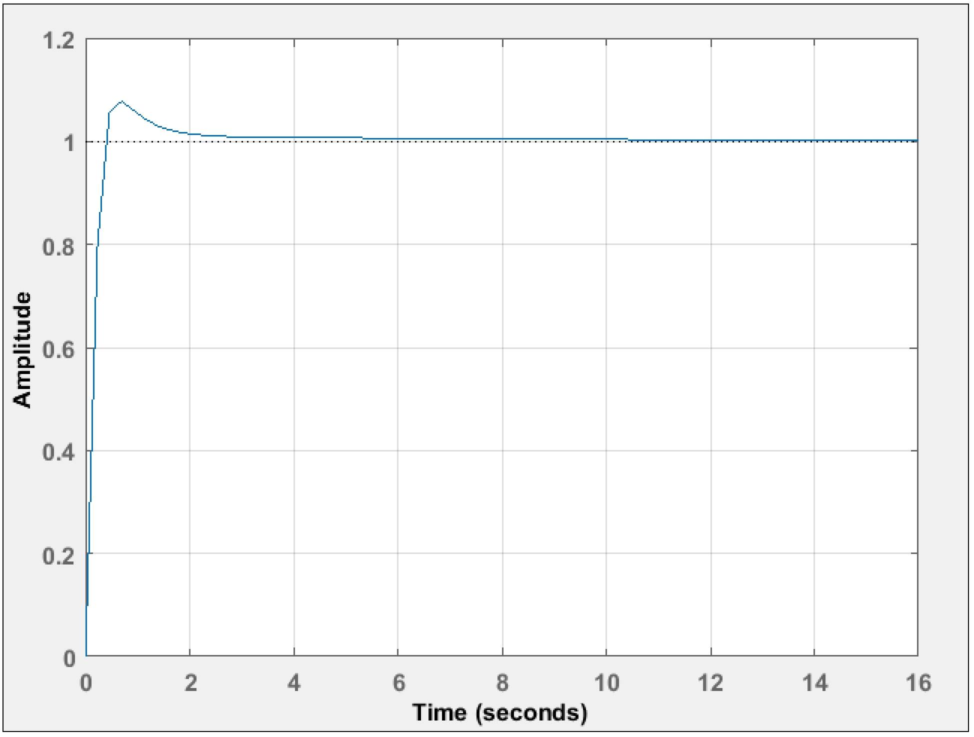

Figure 8: controller step response

From Figure 8, H_∞ controller has the following properties given in Table IV.

Table IV: Properties of the H-infinity Controller

H-infinity Controller System Response	Value
Settling Time (sec)	1.7087
Overshoot (%)	7.7246

From Table IV, the H_∞ controller stabilized the ball at 1.7087 seconds, with an overshoot of 7.7246 %. This gives a good indication that the ball was tracked on the reference path on the plate.

5.3 Trajectory Tracking Results

Figure 9 shows a figure interface of the GUI of the plot of the trajectory of X-position with respect to the Y-position. A circular trajectory of radius 0.4 m, and a sinusoidal signal of a reference input of $x = 0.4(1 - \cos\omega t)$ and

$y = 0.4(\sin \omega t)$ was considered, and also used for the demonstration of the trajectory tracking performance of the ball. The angular frequency of the referenced sinusoidal signal used was 1.57 rad/sec.

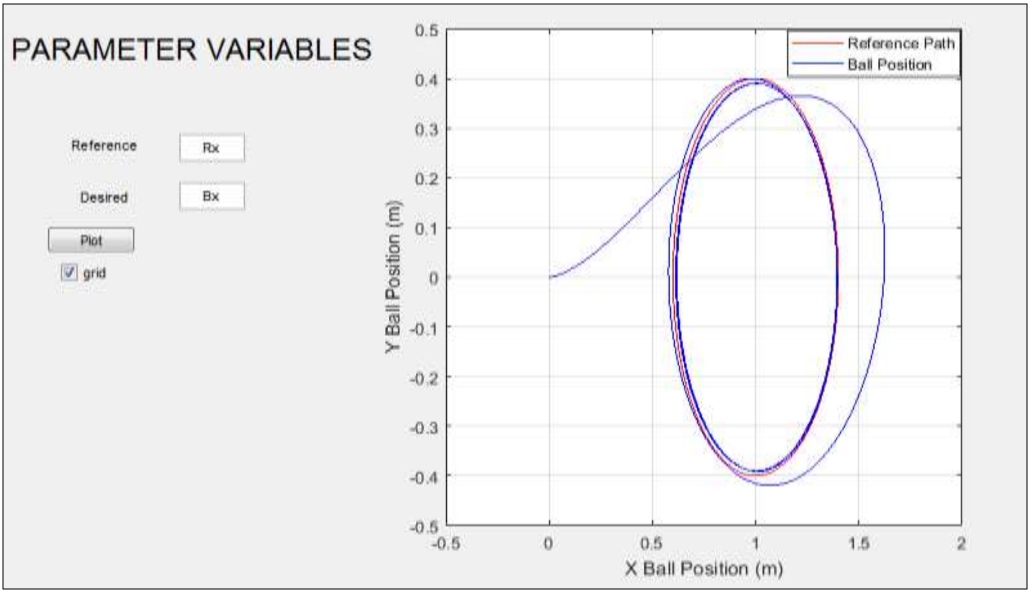

Figure 9: GUI circular trajectory tracking using H-infinity controller

The ball was allowed to track a circular trajectory at a frequency of 0.52 rad/sec at a complete revolution of 12 seconds. Increasing the speed to 0.9 rad/sec, at a complete revolution of 7 seconds, the trajectory tracking error also increased.

However, it was observed that using the H-infinity controller, the steady state tracking error of the ball is 0.0095 m, which depicts that the ball was able to track the referenced sinusoidal signal with a trajectory tracking error of 0.0095 m.

6. Conclusion

The position and trajectory tracking control of the Ball and Plate System (BPS) based GUI using a double feedback loop structure i.e. a loop within a loop has been presented. The inner loop was designed using linear algebraic method by solving a set of Diophantine equation, while outer loop was designed using sensitivity function. Simulation results showed that the controllers has strong adaptability, robustness and high control performance for the BPS. The ball was able to settle with a good settling time and overshoot. However, future research work will be in the direction of optimizing the weighting parameters of the controller optimally by using artificial intelligent technique like bat algorithm (BA) and cuckoo search optimization algorithm (CSO).

Acknowledgment

The authors express their deep

appreciation to the Control/Computer Research Group of the Computer Engineering Department, Ahmadu Bello University, Zaria, Nigeria. And all those in Tuesdays meetings, where academic freedom and discourse are at its best.

References

Ali, E., & Aphiratsakun, N. (2015). AU Ball on Plate Balancing Robot. Paper presented at the IEEE International Conference on Robotics and Biomimetics (ROBIO), Zhuhai, China

Aphiratsakun, N., & Otaryan, N. (2016). Ball On The Plate Model Based on PID Tuning Methods. Paper presented at the 13th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI-CON), Chiang Mai, Thailand.

Awtar, S., Bernard, C., Boklund, N., Master, A., Ueda, D., & Craig, K. (2002). Mechatronic design of a ball-on-plate balancing system. Mechatronics, 12(2), 217-228.

Bang, H., & Lee, Y. S. (2018). Implementation of a Ball and Plate Control System using Sliding Mode Control. IEEE Access, 14(8), 1-8. doi: 10.1109/ACCESS.2018.2838544

Bigharaz, M., Safaei, F., Afshar, A., & Suratgar, A. (2013). Identification and Nonlinear Control of a Ball-Plate System Using Neural Networks. Paper presented at the 3rd International Conference on Control, Instrumentation, and Automation (ICCIA). , Tehran, Iran.

Borah, M., Majhi, L., Roy, P., & Roy, B. K. (2014). Design of a Fractional Order PD Controller Tuned by Firefly Algorithm for Stability Control of the Nonlinear Ball and Plate System. Paper presented at the IEEE International Conference on Advanced Communication Control and Computing Technologies, Ramanathapuram, India.

Chen, C.-T. (1995). Analog and Digital Control System Design: Transfer-function, State-space, and Algebraic Methods. New York, USA: Oxford University Press, Inc.

Cheng, C.-C., & Chou, C.-C. (2016). Fuzzy-Based Visual Servo with Path Planning for a Ball-Plate System. International Symposium on Intelligent Computing Systems (ISICS), 97-107. doi: 10.1007/978-3-319-30447-2_8

Cheng, C.-C., & Tsai, C.-H. (2016). Visual Servo Control for Balancing a Ball-Plate System. International Journal of Mechanical Engineering and Robotics Research, 5(1), 28-32.

Cherubini, A., Colafrancesco, M., Oriolo, G., Freda, L., & Chaumette, F. (2009). Comparing appearance-based controllers for nonholonomic navigation from a visual memory. Paper presented at the ICRA 2009 Workshop on safe navigation in open and dynamic environments: application to autonomous vehicles, Kobe, Japan.

Das, A., & Roy, P. (2017). Improved Performance of Cascaded Fractional-Order SMC over Cascaded SMC for Position Control of a Ball and Plate System. IETE Journal of Research, 63(2), 238-247. doi: 10.1080/03772063.2016.1258336

Debono, D., & Bugeja, M. (2015). Application of Sliding Mode Control to the Ball and Plate Problem. Paper presented at the 12th International Conference on Informatics in Control, Automation and Robotics (ICINCO), 2015, Colmar, Alsace, France.

Dingyu, X., YangQuan Chen, & Derek P., A. (2007). Linear Feedback Control; Analysis and Design with MATLAB (Vol. 3). 3600 Market Street, 6th floor, Philadelphia, PA 19104-2688, USA: Society for Industrial and Applied Mathematics.

Dong, X., Zhang, Z., & Chen, C. (2009). Applying genetic algorithm to on-line updated PID neural network controllers for ball and plate system. Paper presented at the Innovative Computing, Information and Control (ICICIC), 2009 Fourth International Conference on.

Duan, H., Tian, Y., & Wang, G. (2009). Trajectory Tracking Control of Ball and Plate System Based on Auto-Disturbance Rejection Controller. Paper presented at the 7th Asian Control Conference, ASCC 2009. , Hong Kong, China.

Dušek, F., Honc, D., & Sharma, K. R. (2017). Modelling of Ball and Plate System Based on First Principle Model and Optimal Control. Paper presented at the 21st International Conference on Process Control (PC), 2017 Strbske Pleso, Slovakia

Escobar, L. A. M., Almeida, M. A. G., Quintero, O. E. C., Acosta, J. A. R., & Espin, D. F. P. (2017). A comparative analysis among different controllers applied to the experimental ball and plate system. Paper presented at the 2017 International Conference on Information Systems and Computer Science (INCISCOS), Quito, Ecuador

Fabregas, E., Chacon, J., Dormido-Canto, S., Farias, G., & Dormido, S. (2017). Virtual and Remote Laboratory with the Ball and Plate System. IFAC-PapersOnLine, 50(1), 9132-9137. doi: https://doi.org/10.1016/j.ifacol.2017.08.1716

Fan, X., Zhang, N., & Teng, S. (2004). Trajectory planning and tracking of ball and plate system using hierarchical fuzzy control scheme. Fuzzy Sets and Systems, 144(2), 297-312.

Farooq, U., Gu, J., & Luo, J. (2013). On the Interval Type-2 Fuzzy Logic

Control of Ball and Plate System. Paper presented at the 2013 IEEE International Conference on Robotics and Biomimetics (ROBIO).

Fei, Z., Xiaolong, Q., Xiaoli, L., & Shangjun, W. (2011). Modeling and PID Neural Network Research for the Ball and Plate System. Paper presented at the 2011 International Conference on Electronics, Communications and Control (ICECC).

Galvan-Colmenares, S., Moreno-Armendáriz, M. A., Rubio, J. d. J., Ortíz-Rodriguez, F., Yu, W., & Aguilar-Ibáñez, C. F. (2014). Dual PD Control Regulation with Nonlinear Compensation for a Ball and Plate System. Mathematical Problems in Engineering, 2014, 1-10. doi: 10.1155

Golnaraghi, F., & Kuo, B. C. (2010). Automatic Control Systems (9th ed. Vol. 2): John Wiley & Sons, Inc.

Han, K., Tian, Y., Kong, Y., Li, J., & Zhang, Y. (2012). Tracking control of ball and plate system using a improved PSO on-line training PID neural network. Paper presented at the 2012 IEEE International Conference on Mechatronics and Automation.

Hossain, S. (2007). Design of a robust controller for a magnetic levitation system. Wichita State University.

HUMUSOFT Ltd. (2012a). Educational Manual for CE 151 Ball and Plate Apparatus: Humusoft.

HUMUSOFT Ltd. (2012b). User's Manual for CE151 Ball and Plate Apparatus.

Hunt, B. R., Lipsman, R. L., Rosenberg, J. M., Coombes, K. R., Osborn, J. E., & Stuck, G. J. (2014). A Guide to MATLAB®: for beginners and experienced users (2 ed.). Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo: Cambridge University Press.

Hussein, S., Muhammed, B., Sikiru, T., Umoh, I., & Salawudeen, A. (2017). Trajectory Tracking Control of Ball on Plate System Using Weighted Artificial Fish Swarm Algorithm Based PID. Paper presented at the IEEE 3rd International Conference on Electro-Technology for National Development (NIGERCON), 2017 Owerri, Nigeria

Jafari, H., Rahimpour, A., Pourrahim, M., & Hashemzadeh, F. (2012). Linear Quadratic Gaussian Control for ball and plate system. Paper presented at the 2012 International Conference on Computer, Control, Education and Management (CCEM 2012), Dubai, United Arab Emirates.

Kassem, A., Haddad, H., & Albitar, C. (2015). Commparison Between Different Methods of Control of Ball and Plate System with 6DOF Stewart Platform. IFAC-PapersOnLine, 48(11), 47-52.

Lin, C. E., Liou, M.-C., & Lee, C.-M. (2014). Image Fuzzy Control on

Magnetic Suspension Ball and Plate System. International Journal of Automation and Control Engineering, (IJACE), 3(2), 35-47. doi: 10.14355/ijace.2014.0302.02

Lin, H.-Q., Cui, S.-G., Geng, L.-H., & Zhang, Y.-L. (2014). H∞ Controller Design for A Ball and Plate System Using Normalized Coprime Factors. Paper presented at the The 26th Chinese Control and Decision Conference (CCDC), , Changsha, China

Liu, H., & Liang, Y. (2010). Trajectory tracking sliding mode control of ball and plate system. Paper presented at the 2nd International Asia Conference on Informatics in Control, Automation and Robotics (CAR), 2010, Wuhan, China.

Ming, B., Yantao, T., & Yongxiang, W. (2011). Decoupled fuzzy sliding mode control to ball and plate system. Paper presented at the Intelligent Control and Information Processing (ICICIP), 2011 2nd International Conference on.

Mochizuki, S., & Ichihara, H. (2013, July 17-19). I-PD controller design based on generalized KYP lemma for ball and plate system. Paper presented at the European Control Conference (ECC), 2013 Zurich, Switzerland.

Mohammadi, A., & Ryu, J.-C. (2018). Neural network-based PID compensation for nonlinear systems: ball-on-plate example. International Journal of Dynamics and Control, 1-11. doi: https://doi.org/10.1007/s40435-018-0480-5

Morales, L., Camacho, O., Leica, P., & Chávez, D. (2017). A Sliding-Mode Controller from a Reduced System Model: Ball and Plate System Experimental Application. Paper presented at the 14th International Conference on Informatics in Control, Automation and Robotics (ICNCO 2017), Madrid-España.

Mukherjee, R., Minor, M. A., & Pukrushpan, J. T. (2002). Motion Planning for a Spherical Mobile Robot: Revisiting the Classical Ball-Plate Problem. Journal of Dynamic Systems Measurement and Control, 124, 502-511. doi: 10.1115/1.1513177

Negash, A., & Singh, N. P. (2015). Position Control and Tracking of Ball and Plate System Using Fuzzy Sliding Mode Controller. Paper presented at the Afro-European Conference for Industrial Advancement.

Oravec, M., & Jadlovská, A. (2015). Model Predictive Control of a Ball and Plate laboratory model. Paper presented at the IEEE 13th International Symposium on Applied Machine Intelligence and Informatics (SAMII), 2015 Slovakia.

Oriolo, G., & Vendittelli, M. (2005). A Framework for the Stabilization of General Nonholonomic Systems With an Application to the Plate-Ball Mechanism. IEEE Transactions on Robotics, 21(2), 162-175.

Roy, P., Acharjee, S., Ram, A., Das, A., Sen, T., & Roy, B. K. (2016a). Cascaded Sliding Mode Control for Position Control of a Ball in a Ball and Plate System. Paper presented at the IEEE Students' Technology Symposium (TechSym), India.

Roy, P., Das, A., & Roy, B. K. (2016b). Cascaded fractional order sliding mode control for trajectory control of a ball and plate system. Transactions of the Institute of Measurement and Control, 1-11. doi: 10.1177/0142331216663826

Roy, P., Kar, B., & Hussain, I. (2015). Trajectory Control of a Ball in a Ball and Plate System Using Cascaded PD Controllers Tuned by PSO. Paper presented at the Proceedings of Fourth International Conference on Soft Computing for Problem Solving.

Saud, L. J., & Alwan, M. M. (2017). Design and Implementation of Classical Sliding Mode Controller for Ball and Plate System. Journal of Engineering, 23(6), 74-92.

Singh, R., & Bhushan, B. (2018). Real-time control of ball balancer using neural integrated fuzzy controller. Artificial Intelligence Review, 1-18. doi: https://doi.org/10.1007/s10462-018-9658-7

Suleiman, H. U., Mu'azu, M. B., Zarma, T. A., Salawudeen, A. T., Thomas, S., & Galadima, A. A. (2018). Methods of Chattering Reduction in Sliding Mode Control: A Case Study of Ball and Plate System. Paper presented at the 2018 IEEE 7th International Conference on Adaptive Science & Technology (ICAST), Accra, Ghana

Umar, A. (2017). Development of a Position and Trajectory Tracking Control of Ball and Plate System using a Double Feedback Loop Structure. (Masters of Science Thesis), Department of Electrical and Computer Engineering, Faculty of Engineering, Ahmadu Bello University, Zaria, Published.

Umar, A., Mu'azu, M. B., Usman, A. D., Musa, U., Ajayi, O.-o., & Yusuf, A. M. (2019). Linear Quadratic Gaussian (LQG) Control Design for Position and Trajectory Tracking of the Ball and Plate System. Computing & Information Systems, 23(1).

Umar, A., Muazu, M. B., Aliyu, U. D., Musa, U., Haruna, Z., & Oyibo, P. O. (2018). Position and Trajectory Tracking Control for the Ball and Plate System using Mixed H∞ Sensitivity Problem. Covenant Journal of Informatics and Communication Technology, (CJICT), 6(1), 1-15.

Wang, Y., Li, X., Li, Y., & Zhao, B. (2012). Identification of ball and plate system using multiple neural network models. Paper presented at the International Conference on System Science and Engineering

(ICSSE), 2012.

Yıldız, H. A., & Gören-Sümer, L. (2017). Stabilizing of Ball and Plate System Using an Approximate Model. IFAC-PapersOnLine, 50(1), 9601-9606. doi: https://doi.org/10.1016/j.ifacol.2017.08.1688

Zeeshan, A., Nauman, N., & Khan, M. J. (2012). Design, Control and Implementation of a Ball on Plate Balancing System. Paper presented at the IEEE 9th International Bhurban Conference on Applied Sciences & Technology (IBCAST), Islamabad, Pakistan.