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A New Method for Cargo-Passenger Vessel Hydrostatic Parameters Analysis

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Abstract **:** In this study, software that can analyze ship hydrostatics parameters using Java programming language is developed. Ship hydrostatic parameters are properties of the immersed portion of the ship in water. The analyses of ship hydrostatic parameters are done by employing computer software, but the cost of acquiring the license of such expert software is expensive. Owing to this fact, this research was carried out to meet the need of analyzing ship hydrostatics parameters for both academic and industrial applications at marginal expense. Java programming language was adopted for this research because of its outstanding features such as internet security, portability, variety of integrated development environments (IDEs), platform independent and free to download on internet. The mathematical approximation or integration method as a technique for formulating ship hydrostatics parameters was adopted for this research. This is because it has the ability to make use of computer technology and it can be used to obtain the area of a ship and other hydrostatics parameters. The results produced in this research are reasonable and acceptable as evidenced from the comparison analysis with existing results of offsets values of a cargo-passenger vessel obtained from SNAME.

Keywords – Cargo-Passenger Vessel, Ship, Hydrostatic Parameters, Java Programming, Software

1. Introduction
The shape of the body of marine vessel influences the draft of the vessel greatly and the vessel is supported by the buoyancy the vessel's draft and shape provides. The cardinal law regulating the static actions of the body or vessel wholly or partially immersed in a particular fluid is known as

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Archimedes principle. It states that a body immersed wholly or partially in a fluid experiences an upward, up thrust or lift force or it is buoyed up by a force that equals the weight of the displaced fluid [1]. Thus, the weight is regarded to be a vertically downward force that is directly proportional to the mass of the body, the equal up thrust is directly proportional to the mass of water that the vessel displaced [1]. When a vessel is floating at a particular draft, the mass of water displaced and the weight of the vessel can be determined, provided the volume displacement of the ship is obtained. Also the area of water plane of the vessel and other properties, particulars or parameters of the immersed portion of the vessel can be determined. These particulars or parameters which are characteristics of the under-water portion of the ship are the hydrostatic parameters. Examples of these parameters are mass displacement (Δ), volume displacement (V), area of water plane (Awp), longitudinal center of buoyancy (LCB), vertical center of buoyancy (VCB or KB), longitudinal center of floatation (LCF), tons per centimeter immersion (TPC), wetted surface area of the ship (WSA), block Coefficient (C_B), coefficient of water plane area (C_W), coefficient of midship area (C_M), prismatic coefficient (C_P), longitudinal metacentric radius (BM_L), transverse metacentric radius

(BM_T), transverse distance of metacenter from keel (KM_T), longitudinal distance of metacenter from keel (KM_L), and moment causing one centimeter trim (MCTIcm) [2].

These parameters define the properties of the immersed part of the vessel at a certain draft. It is related to volume of underwater part of the ship, area of underwater part of the ship, centroid of volume and area of underwater part of the ship and moment of volume and area of the immersed portion. If the vessel is hoisted above water level, and draft gets to zero, the parameters no longer exist. Provided draft and trim remain intact, the magnitude and shape of the portion of the vessel submerged in water are not changed. The volume, area and moment of area and volume are also unchanged, but the moment the draft or trim vary, the parameters will also vary. This variation in draft will often come to play whenever there is variation in total weight of the vessel, or if a force (load) is given to the vessel to increase the draft of the vessel. These hydrostatics parameters are determined at the preliminary design stages of the vessel. They come to play the moment the shape and magnitude of the vessel have been concluded and can be displayed in two formats, either as a set of hydrostatics curves or in the form of a table [2].

2. Determination of Ship Hydrostatics Parameters

It is very important to know the operation of the ship hydrostatics parameters. It is also obvious that if these hydrostatics parameters are displayed in tables or graphs, such information can be used to know the details of the vessel in any floating condition, thus predictions can be made for any particular variation in the operation of the vessel. To determine the hydrostatics parameters of a vessel, different approaches such as graphical, planimeter and mathematical approximation or integration methods can be used [3]. These methods can be described as follows:

- The graphical method is one in which a graph of the ship area is plotted against the draught of the ship, after which the area under the curve is calculated. Its limitations are numerous, ranging from being tedious, not very accurate and the graph must be plot first before the analysis begins.

- The planimeter method is another tedious method where an instrument or equipment called the Planimeter is used to calculate the area of the vessel. It also has its own limitations such as, it is used only when hard cope of water line drawings are available and it requires a lot of manpower to be able to obtain the real or actual values desired of the ship area [4].

- The mathematical approximation or integration is a method in which the curve or shape of the vessel is represented by mathematical expressions and application of calculus. This facilitates determination of the area and moment of area bounded by the curve using piecewise integration.

In this research, the method of mathematical approximation or integration is utilized because it is more accurate when compared with graphical and planimeter methods. There is no need for hard copies of the ship's area curves. Offset tables are often produced from the ship design. In calculating the ship hydrostatics parameters, offset tables are used or applied directly. Computer technology can equally easily be applied to calculate the entire ship hydrostatics parameters with this method. The method also enables the determination of other ship hydrostatics parameters, including the area easily unlike the other two methods [5]. This method entails the development and applications of mathematical formula in other to calculate ship hydrostatics parameters.

Offset tables are tables containing the half breadth values of a ship at every station point. They are the basic inputs to the formulae to determine the hydrostatics parameters. Station points of a ship are regular intervals along the length of the ship upon which the half breadth values of the ship are

taken. They are represented by the symbols 'h' or 's'. In calculating these hydrostatics parameters using mathematical approximation or integration methods, some fundamental methods are applied, such as Trapezoidal, Newton-cotes, Simpson rules one, two and three and Tche-bycheff method [6]. In this research, concentration is given to the two most popular methods such as Trapezoidal and Simpson rules. This is because of their flexibilities and accuracies. Presently, the existing software in use for the purpose of ship hydrostatics and stability analysis are Wolfson hydrostatics software, Rhino software, Delftship software and Orca3D software etc [7]. These softwares are very efficient in operations but they were all developed from different programming languages excluding Java. Java as a third generation and high level programming language is used in this research because of its special features. This is the novelty of this research.

3. Methodology of the Research

The ship hydrostatics parameters models were developed individually using mathematical integration method as seen in the subsections 3.1 - 3.12 and the sums of products linking ship hydrostatics parameters to each other is described/established in subsection 3.13, so as to facilitate JAVA application.

3.1. Ship Water Plane Area

The first ship hydrostatics parameter upon which others are derived or determined is the waterplane area of the ship. The water plane area is the area of the plane surface of a vessel's hull at a particular waterline. The ship hull shape is not a regular shape or body, so to determine the area of waterplane Trapezoidal and Simpson's rules were used as the mathematical approximation or integration method. The curve or shape of a ship waterplane can be considered to be a set of trapezoid, when this is assumed, the area under the curve is expressed using Trapezoidal rule as seen below,

$$\text{Area} = \frac{1}{2}\sum_{i}^{n}(\text{Parallel sides}) \times \text{Hieght}$$
(1)

Simpson's first rule for area of waterplane is predominantly applied and the formulation is given below.

$$\text{Area} = \frac{1}{2}h\left[\frac{1}{2}y_0 + 2y_{\frac{1}{2}} + y_1 + 2y_{1\frac{1}{2}} + \frac{3}{2}y_2 + 4y_3 + 2y_4 + 4y_5 + 2y_6 + 4y_7 + \frac{3}{2}y_8 + 2y_{8\frac{1}{4}} + y9 + 2y_{9\frac{1}{2}} + \frac{1}{2}y_{10}\right]$$
(2)

3.2. Waterplane Areas within the Fore and Aft Peak regions of Ship Hull

The fore peak region is the area between the ship's Fore Perpendicular (FP) and the fore extreme. The Aft peak region is the area between the ship's Aft Perpendicular (FP) and the aft extreme. The waterplane areas within these regions are added to the ship waterplane area gotten from the length between perpendiculars (LBP) at a particular draft. This is done when the ship waterline at a particular draft is above the point of intersection of the ship Aft Perpendicular and the stern. It is also done when the ship waterline at a particular draft is above the point of intersection of the ship Fore Perpendicular and the stem. As can be seen in Figure 1, the fore and aft peak regions are the regions between AP and the Aft extreme of the ship and the distance between FP and the Fore extreme of the ship. Mathematically it can be said that from the Figure 1, the sum of aft and fore peak regions is equal to the difference between the ship length overall (L_OA) and length between perpendiculars (LBP or LPP).

The area of waterplane within these region is developed by concatenating Simpson's first rule and trapezoidal rule to account for the extended stations fore and aft of fore perpendicular and aft perpendicular respectively up to the extremes of the vessel and the derivation is given below.

If Simpson's first rule is used between stations 1 to 10 of a ship waterplane with internal multipliers as 0.5, 2, 1,- - - ,2, 1 2, 0.5 and external multiplier as 1/3, to concatenate the rules, consider the points f1, f2, f3, and so on and points a1, a2, a3, and so on.

Station point 0 is the point on the fore perpendicular.

Station point 10 is the point on the aft perpendicular.

Point f1 is station point 1, a distance of Sf1 from station point 0, towards the fore extreme.

Point f2 is station point 2, a distance of Sf2 from station point f1, towards the fore extreme.

Point f3 is station point 3, a distance of Sf3 from station point f2, towards the fore extreme.

Point 'a1' is station point 1, a distance of Sa1 from station point 10, towards the aft extreme.

Point 'a2' is station point 2, a distance of Sa2 from station point 'a1', towards the aft extreme.

Point a3 is station point 3, a distance of Sa3 from station point a2, towards the aft extreme. and so on. Consider the area between station points 0 and f1.

$$Area_{0-f1} = \frac{1}{2} Sf1(y0 + yf1) \qquad (3)$$

Let "w" be a value which when multiplied by $\frac{1}{2} Sa$

will give $\frac{1}{3} S$. Therefore,

$$\frac{1}{2} Sf1 * w = \frac{1}{3} S \qquad (4)$$

such that $w = \frac{1}{3} S * \frac{2}{Sf1} = \frac{2S}{3Sf1}$ (5)

Now, substituting "w" in equation (5) into (4),

$$\frac{1}{2} Sf1 \left(\frac{2S}{3Sf1} \right) = \frac{1}{3} S \qquad (6)$$

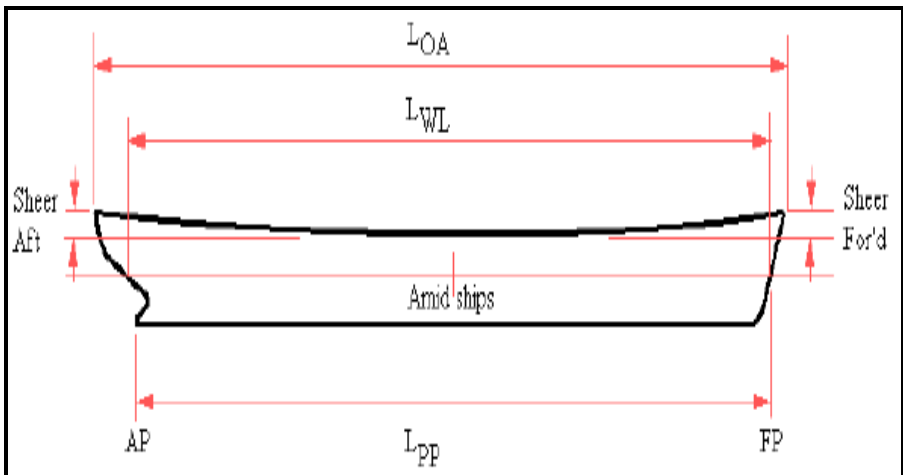Make $\frac{1}{2} Sf1$ the subject of equation   (4)

Figure 1: Ship Hull showing Linear Distances and Perpendiculars [8]

$$\frac{1}{2} Sf1 = \frac{1}{3} S * \left(\frac{3Sf1}{2S} \right) = \frac{1}{3} S \left(\frac{3Sf1}{2S} \right) \qquad (7)$$

Equation (3) becomes,

$$Area_{0-f1} = \frac{1}{2} Sf1(y0 + yf1)$$

$$Area_{0-f1} = \frac{1}{3} S \left(\frac{3Sf1}{2S} \right) (y0 + yf1) \qquad (8)$$

$$= \frac{1}{3} S \left(\frac{3Sf1}{2S} y0 + \frac{3Sf1}{2S} yf1 \right)$$

If an offset table gives half breadth values for a vessel up to upper stations, like station f3, the formulation becomes the sum of Area 0 to f1, Area f1 to f2 and Area f2 to f3 as the case may be.

$$Area_{0-f3} = \frac{1}{3}S \begin{bmatrix} \frac{3Sf1}{2S}y0 + \left(\frac{3Sf1}{2S} + \frac{3Sf2}{2S}\right)yf1 \\ + \left(\frac{3Sf2}{2S} + \frac{3Sf3}{2S}\right)yf2 \\ + \frac{3Sf3}{2S}yf3 \end{bmatrix} \qquad (9)$$

In same way, Equation (9) is applied for Areas between the AFT Perpendicular and the AFT extreme of the vessel as seen below.

$$Area_{0-a3} = \frac{1}{3}S \begin{bmatrix} \frac{3Sa1}{2S}y0 + \left(\frac{3Sa1}{2S} + \frac{3Sa2}{2S}\right)ya1 \\ + \\ \left(\frac{3Sa2}{2S} + \frac{3Sa3}{2S}\right)ya2 \\ + \frac{3Sa3}{2S}ya3 \end{bmatrix} \qquad (10)$$

Whenever half breadth of the Fore or Aft peak regions are given, internal multipliers for station point "0" and "a1" or "f1" becomes

$$0.5 + \frac{3Sa1}{2S} \text{ And } \frac{3Sa1}{2S} \text{ or } \frac{3Sf1}{2S} \text{ respectively} \qquad (11)$$

Also, the levers or moment arms corresponding to the fore and aft region station points are given below.

At station point 'a1', lever $La1 = 5 + \dfrac{Sa1}{S}$ $\qquad (12)$

At station point a2, lever $La2 = La1 + \dfrac{Sa2}{S}$ $\qquad (13)$

At station point a3, lever $La3 = La2 + \dfrac{Sa3}{S}$ $\qquad (14)$

The same formulations are applied to levers at station points fore of the fore perpendiculars.

3.3. Volumes Displacements of Ship Shapes

The volume displacements of a ship at any draft can be calculated if the waterplane areas at various waterlines up to the required draft and the sectional areas up to the required draught at various station point are known. Figure 2 shows how to compute the volume displacement using both the transverse and longitudinal sections of the ship hull.

For the purpose of this research work, method (b) which is integration of sectional areas was used base on the fact that the draft values for waterlines may not be equal, as such, integrating along waterlines may be complex to apply.

$$Volume\,Displacement = \frac{h}{3}\left(\begin{array}{c} A + 4B + 2C \\ +4D + E \end{array}\right) \qquad (15)$$

Where A, B, C and so on are the sectional areas at each station point and 'h' is the equal intervals of the station points.

3.4. Centroids: Longitudinal Center of Floatation, Longitudinal Center of Buoyancy and Longitudinal Second Moment of Area

The center of floatation also known the center of gravity or centroid of the waterplane area is a point in the ship's waterplane area about which the vessel heels or trims when given an angular displacement. It can also be defined as a point in the waterplane area of a ship at which when load or weight is added on the ship, will produce a parallel sinkage. This point normally lies along the longitudinal center line but not so for the transverse center line because it may drift a little forward or aft of amidship, normally it may lie a distance of say three percent length forward and say three percent length aft of amidships for oil tankers and containerships respectively [9]. Similarly, the longitudinal center of buoyancy of a ship is the centroid of the underwater shape of the ship, so to obtain the area of a waterplane by Simpson's rules, the halfbreadths are used as station points. If the moment of the halfbreadths about any point are used as station points, then the total moment of the area about that point will be gotten. If the total moment obtained is then divided by the total area, the quotient will yield the distance of the centroid of the area from the point about which the moments were taken [10].

The following formulae were adopted from reference material.

$$Area = \int y\,dx$$

$$= \frac{1}{3} \times \Sigma_{PRODUCT_A}\; xh \qquad (16)$$

1st moment

$$= \int X^A y\, dx$$

$$= \frac{1}{3} \Sigma_{PRODUCT_{1st\ moment}}\ x\ \ h^2 \tag{17}$$

2nd moment

$$= \int X^2 y\, dx$$

$$= \frac{1}{3} \Sigma_{PRODUCT_{2nd\ moment}}\ x\ \ h^3 \tag{18}$$

Where values of 'X' are given in multiples of 'h', the common interval. If the product for area is multiplied by multiples of 'h' called levers, the sum of products can be used to find the first moment and hence the longitudinal position of the centroid [11].

$$LCF = \frac{\frac{1}{3} \times h^2 \times \Sigma_{PRODUCT\ 1st\ moment}}{\frac{1}{3} \times h\ \times \Sigma_{PRODUCT_{Area}}}$$

$$= \frac{\Sigma_{PRODUCT\ 1st\ moment}\ *\ h}{\Sigma_{PRODUCT_{Area}}} \tag{19}$$

If the offsets are halfbreadths, the centroid is longitudinal center of floatation (LCF). If the offsets are sectional areas, the centroid is center of volume that is longitudinal center of buoyancy (LCB). The longitudinal center of floatation (LCF) is measured from the axis where levers are taken.

For second moment, Simpson's product for area are multiplied twice with levers. Again the second moment are taken about the axis from where levers are taken [12].

$$I_L = \frac{1}{3} \times h^3 \times \Sigma_{PRODUCT_{2nd\ moment}} \tag{20}$$
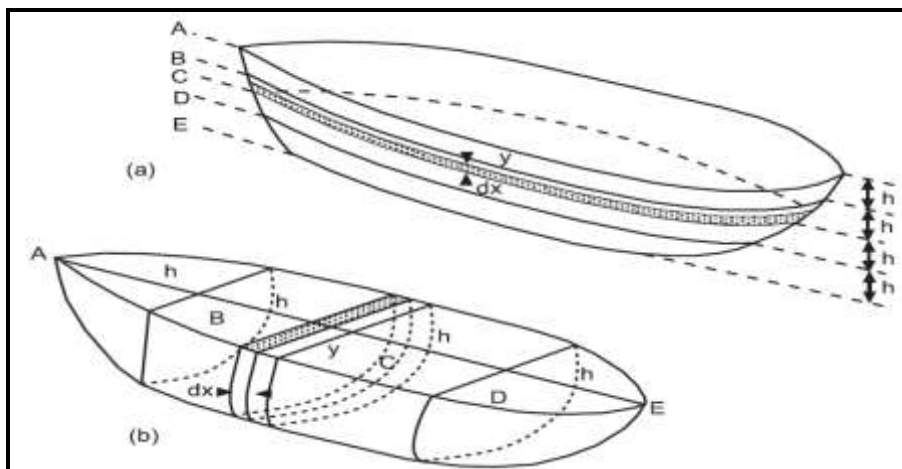
Figure 2: Volume of Ship Shapes (a) Longitudinal Sectioning (b) Transverse Sectioning [9]

3.5. Coefficients of Form

In comparing ship's hull forms, displacements and dimensions, a number of coefficients are use in naval architecture. These coefficients are useful in power estimates and in expressing the fullness of a ship's overall form and those of the body plan sections and waterlines. Also, in ship design it is often necessary to classify the hulls and to find relationships between forms and their properties, especially the hydrodynamic properties. The coefficients of form are the most important means of achieving this. By their definition, the coefficients of form are non-dimensional numbers [13].

3.6. Block Coefficient

This is defined as the ratio of the volume of displacement of the molded form up to any water line of a ship to the volume of a circumscribing rectangular block. [14].

$$C_M = \frac{\text{Im} mersed\ area\ of\ midship\ \sec tion}{Breadth \times Draught}$$

(21)

$$C_B = \frac{\nabla}{L * B * T}$$

Where L, B, T and ∇ are length, beam and draft of waterline and the volume displacement respectively.

3.7. Waterplane Area Coefficient

The waterplane coefficient is defined as the ratio between the area of the waterplane (Awp) and the area of a circumscribing rectangle [14].

$$C_W = \frac{Awp}{L * B}$$ (22)

Where L and B are length and beam of waterline respectively.

3.8. Midship Area Coefficient

This midship section coefficient (C_M) sometime called simply midship coefficient at any draught is the ratio of the immersed area of the midship station to the area of a circumscribing rectangle [14].

$$C_M = \frac{Amidship}{B * T}$$ (23)

Where B and T are beam and draft of waterline respectively.

3.9. Prismatic Coefficient

This is the ratio of the ship's volume displacement at a particular waterline to the product of the midship sectional area and the length (Bagahari and hassan , 2014).

$$C_p = \frac{\nabla}{A_M L} = \frac{C_B * LBT}{C_M BTL}$$ (24)

Where L, B, T, A_M, C_M, C_B and ∇ are length, beam and draft of waterline, area about midship section, coefficient of midship, block coefficient and the volume displacement respectively.

3.10. Wetted Surface Area

For a vessel floating at a given waterline, the total area of its hull in contact with the surrounding water is known as its 'wetted surface'. The wetted surface may be used in estimating the amount of paint required to coat the vessel's bottom up to a given waterline. Also the wetted surface bellow the waterline may be added to area of the topsides above the waterline to obtain the total area of the shell plating. Thus, the approximate weight of the shell may be estimated as well as the paint required for it.

Wetted surface area is customarily calculated at various waterlines for a new ship and appears as one of the curves of form. In some cases, addition is made for appendages, such as stern, stern frame, rudder, propeller shaft bossing and bilge keels.

The wetted surface area of the molded form may be obtained by calculating various portions directly from the lines drawing, and estimating other portions closely. The calculation method has traditionally been that of drawing an expansion of the molded surface up to the desired waterline, and measuring the area enclosed by the expansion, it being assumed the area of the expansion is virtually that of the molded surface.

Several approximate formulae for wetted surface area, amongst which the Denny's is adopted for this research. It is given below [8].

Wetted Surface Area $= 1.7Ld + \dfrac{\nabla}{d}$

(25)

3.11. Metacentric Heights of Vessels

The metacentric height (*GM*) is another important ship hydrostatics parameter that influence the stability of a ship in water. This parameter tells the distance between the point of the center of gravity of the ship (G) and the point of the metacenter (M). The metacenter of a ship is simply a point in the ship about which the ship oscillates when given an angular moment. Metacentric heights can be transverse or longitudinal, their values can be positive, negative or neutral (zero). When the value is positive, it implies that the vessel is in stable equilibrium at the individual waterline but when the values become negative, it means that the vessel is in an unstable condition. This may lead to capsize if the negativity increases to a certain critical value. Also when the GM value is zero, it implies that the vessel is in a neutral equilibrium that is the vessel will continue in the tilted or heeled condition until an external righting moment restores it to stable equilibrium. It is worth noting that different sizes of vessel have their corresponding metacentric heights and periods of roll or capsize, the period of roll reduces as metacentric heights increases, at high metacentric height when the ship is inclined, she tends to return quickly and violently to the initial positon (stiff ship), but when the metacentric height is small and the ship is inclined, the ship tends to return slowly to the initial position (tender ship) [15]. Conversely an extreme stiff or tender ship is not ideal for marine operations, therefore, there should be a compromise in loading the vessel so as to get a moderate metacentric height for effective and efficient marine operations. The mathematical expressions for both longitudinal and transverse metacentric heights (GM) are given below.

Longitudinal Metacentric Height
$GM_L = KB + BM_L - KG$
(26)
Transverse Metacentric Height
$GM_T = KB + BM_T - KG$
(27)

3.12. Tons Per Centimeter Immersion of Vessels

The Tons per centimeter immersion (TPC) is a hydrostatic parameter that is dependent on mass displacement and density of water the vessel is moving in, it is very significant in ship stability because it tells how much or to what extent a ship can be submerged or sink under the addition of load and to what extent the ship is buoyed at the removal of load. This parameter enables Naval architects to determine the stability of vessels as they add or remove weights or loads from vessels during ship design [16-18]. The mathematical expression of the parameter is given below.

Tons per centimeter Immersion,

$$TPC = \frac{Awp \times Density\,of\,Water}{100}$$

(28)

3.13. Sums of Products for Linking Ship Hydrostatics Parameters to each other

It is very important to note that ship hydrostatics parameters are dependent on one another. It simply means that some parameters are derived from others. For instance, mass displacement is dependent on volume displacement, which in turn is derived from area of waterplane. Therefore, in configuring the Java based software for ship hydrostatics parameters analysis in this research work, the "Sum of products" were used to link the parameters to each other in a chronological order as seen below.

Sum1 is the sum of product of areas

Sum2 is the sum of (product of areas) and Lever

Sum3 is the sum of (product of areas) and Lever-squared

Sum4 is the sum of product of internal multipliers and the cube of half breadths

Sum5 is the sum of product of sectional areas and internal multipliers.

Sum6 is the sum of (product of sectional areas and internal multipliers) and lever.

Area of water plane $Awp = \frac{2}{3} \times Sum1 \times S$ (29)

Volume displacement $Vol.Displ. = \frac{2}{3} \times Sum6 \times S$ (30)

Mass displacement Δ = Volume Displacement * Density of water   (31)

Longitudinal Center of Buoyancy $LCB = \frac{Sum6}{Sum5} \times S$ (32)

Longitudinal Center of Floatation $LCF = \frac{Sum2}{Sum1} \times S$ (33)

Beam of waterline B = maximum half breadth * 2   (34)

Longitudinal moment of Inertia about

midship, $Im\,idship = \frac{2}{3} \times Sum3 \times (S\wedge3)$ (35)

Longitudinal moment of Inertia about longitudinal center of floatation

 Ilcf = Imidship – Awp (LCF^2)   (36)

Transverse moment of Inertia $It = \frac{2}{9} \times Sum4 \times S$ (37)

The mathematical expressions for other ship hydrostatics parameters such as longitudinal and transverse metacentric radii, longitudinal and

transverse metacenter above keel and moment causing one centimeter trim are compiled from Longitudinal metacentric radius

$$BMl = \frac{Ilcf}{Volume\,Displacement} \qquad (38)$$

Transverse metacentric radius

$$BMt = \frac{It}{Volume\,Displacemænt} \quad (39)$$

Longitudinal metacenter above keel $KM_L = KB + BM_L$ (40)

Transverse metacenter above keel $KM_T = KB + BM_T$ (41)
Moment Cause 1cm Trim

$$(MCT1cm) = \frac{(Mass\,Displ. \times GMl)}{(100 \times L)} \qquad (42)$$

3.14. Application Programming Interface (API) and Data Base Incorporated in the Development of the Ship Hydrostatics Analysis Software

Application programming interface is an interface to a software component that can be invoked at a distance over a communication network using standard base technologies. In this research work, three APIs were used to develop the software such as Java Matrix API, Java Excel API and J Free Chart API. Java Matrix API is used to interface all processes in matrix (rows and volumes) arrangements, while Java Excel API is used to interface the software with all excel files. J Free Chart API is used for plotting graphs. A data base is a separate application that stores a collection of data. Each data base has one or more distinct API for creating,

accessing, managing, searching and replicating the data it holds. Examples of data base are Oracle, my structured query language (MySQL) etc. For the purpose of developing the ship hydrostatics analysis software in this research work, MySQL was used. This is because it is free to download unlike Oracle which license must be purchased before use.

3.15. Algorithms for Implementing the Analysis of Ship Hydrostatics Parameters

Using Java high level programming language, the algorithm (Pseudo code and Flow chart) of the program that will analyze the Hydrostatics Parameters of vessels at different draughts is given below and the flow chart is depicted in Figure 3.

3.16. Pseudo Codes for Implementing the Analysis of Ship Hydrostatics Parameters

Remark: Program that will compute ship hydrostatic parameters and display results on tables and graphs

(1) Enter values for LBP, NFE, NAE, Sf, Sa, KG in meters, Density of Water in Tons per meters.

(2) Compute

(i) Area of Water plane $Awp = \frac{2}{3} \times Sum1 \times S$

(ii) Volume Displacement $Vol.Displ. = \frac{2}{3} \times Sum6 \times S$

(iii) Mass Displacement Δ = Volume Displacement * Density of water

(iv) Longitudinal Center of Buoyancy $LCG = \frac{Sum6}{Sum5} \times S$

(v) Longitudinal Center of Floatation $LCF = \frac{Sum2}{Sum1} \times S$

(vi) Tons per centimeter Immersion TPC,

$$TPC = \frac{Awp \times Density\,of\,Water}{100}$$

(vii) Beam of Water Line B = maximum Half Breadth * 2

(viii) Longitudinal moment of Inertia about midship.

$$Im\,idship = \frac{2}{3} \times Sum3 \times (S\textasciicircum 3)$$

(ix) Longitudinal moment of Inertia about Longitudinal Center of Floatation

$$Ilcf = Imidship - Awp\,(LCF^2)$$

(x) Transverse moment of Inertia $It = \frac{2}{9} \times Sum4 \times S$

(xi) Longitudinal Metacentric Radius $BMl = \frac{Ilcf}{Volume\,Displacement}$

(xii) Transverse Metacentric Radius $BMt = \frac{It}{Volume\,Displacement}$

(xiii) Longitudinal Metacenter above keel $KM_L = KB + BM_L$

(xiv) Transverse Metacenter above keel $KM_T = KB + BM_T$

(xv) Longitudinal Metacentric Height $GM_L = KB + BM_L - KG$

(xvi) Transverse Metacentric Height $GM_T = KB + BM_T - KG$

(xvii) Moment Cause 1cm Trim $MCT1cm = \frac{(Mass\,Displ. \times GMl)}{(100 \times L)}$

(2)   Display: Results in Tables and Graphs
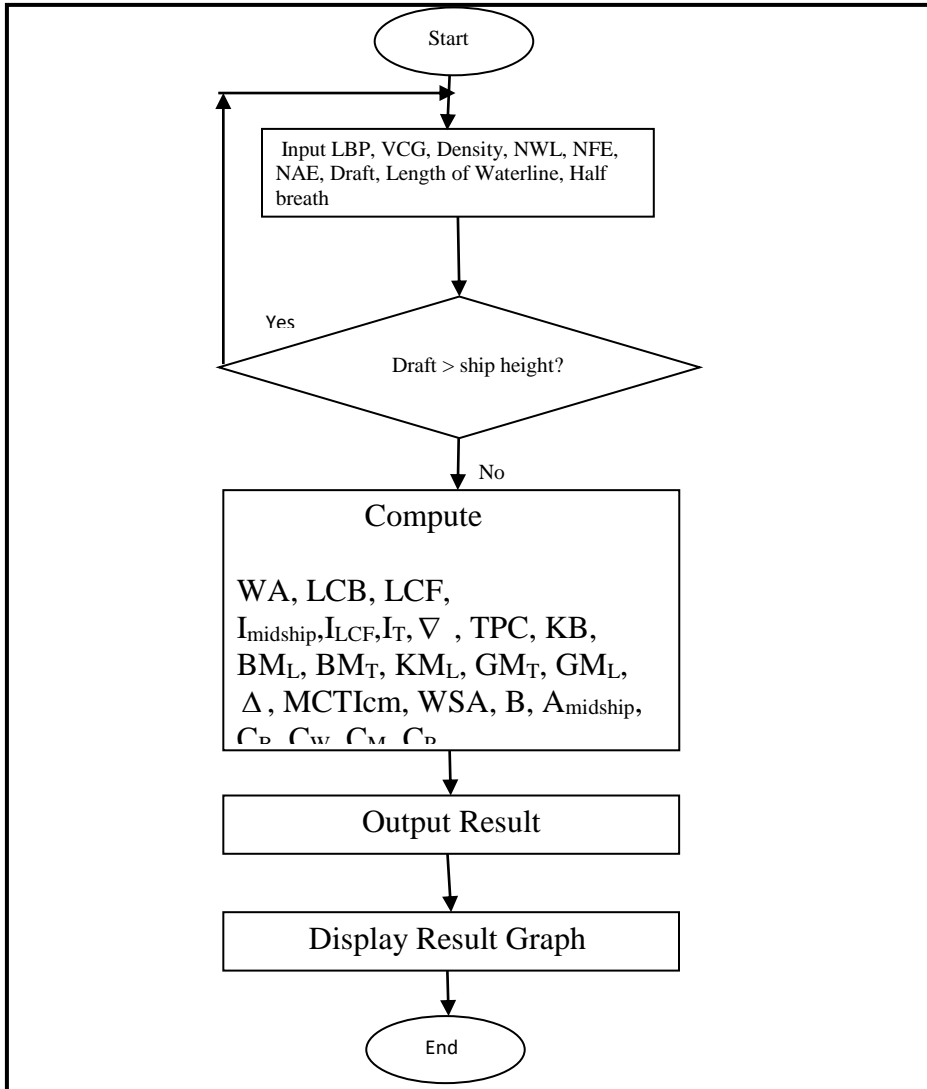(3)   End

Figure 3: Flow Chart for Implementing the Analysis of Ship Hydrostatic Parameters

3.17. Scale Factors for Graph Plotting by the Developed Hydrostatics Software
Some Ship hydrostatics parameters values use to be very high in magnitude, for instance parameters like longitudinal and transverse moments of inertia, mass and volume displacements, area of ship waterplane etc., while other parameters like the coefficients of form, centers of buoyancies and

floatation etc. use to be very small in magnitudes. So, if these range in parameter values are plotted in the same graph, many of the curves will be fused together causing a poor plot, therefore, the software program is developed to give scale factors to the individual parameter curves to enable the curves spread out clearly on the graphs. This is archived by separating the parameters into two graphs with respects to their individual magnitudes and taking a parameter each for the graphs as a reference upon which others are scale as seen below.

3.17.1 Scale Factor Generation for Graph 1

For graph 1, the Ship hydrostatics parameter: moment causing one centimeter trim (MCTicm) was used as the reference parameter upon which others are scaled.

3.17.2. Scale Factor Generation for Graph 2

For graph 2, the ship hydrostatic parameter: moment of Inertia about midship section was used as the reference parameter upon which others are scaled.

3.18. Procedure for Exploring the Developed Hydrostatics Software

The procedure for exploring the developed hydrostatics software is described as follows:

3.18.1. Enter Ship Particulars

The software prompts users to enter vessels' particulars such as; Length between perpendiculars (LBP), Vertical center of gravity (KB) etc. then with the dialog box, users can select the density of fluid (water). Figure 3 shows an illustration of how to enter the ship particulars into the developed program.

3.18.2. Enter Drafts and Waterlines

The software also prompts users to enter numbers and values for drafts, waterlines, stations fore and aft of fore perpendicular (FP) and aft perpendiculars (AP) respectively as the case may be and then save the data. This can be done in two ways: Either by entering the values manually into the software using the keyboard or the software allows users to navigate or call the data which must have been saved already in excel platform in the computer to the software. This is possible because the software has been developed incorporating Java-Excel API to permits the interactions between both platforms. The procedure of doing this is illustrated in Figure 5.
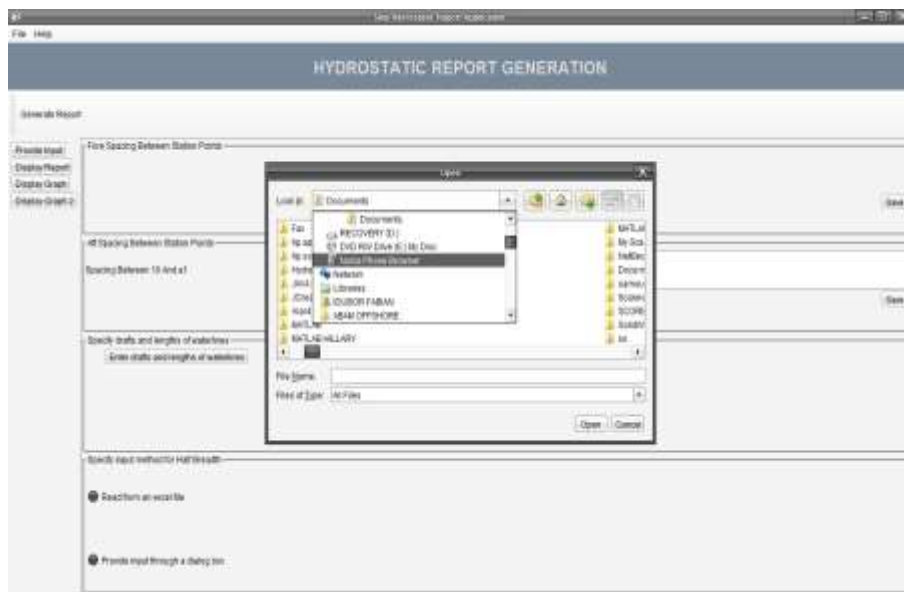
Figure 5: User Interface for Entering Drafts and Waterlines into the Developed Software

3.18.3. Calculate hydrostatics

When the ship particulars are entered, to calculate the hydrostatics, click on the icon 'Display result' on the top left side of the graphical user interface (GUI) of the software. At this point, the entire hydrostatics parameters results are displayed in rows and columns.

3.18.4. Graph Plotting of Ship Hydrostatics Parameters

After calculating the ship hydrostatics parameters, click on the icon 'Display Plot' on the top left side of the GUI of the software, the graphs of the parameters are plotted against their corresponding drafts. Figure 6 shows a sample of the graphs plotted by the developed program for a given set of ship hull offset data. This graph is a plot of the hydrostatics parameters against draft as described above.

4. Results and Discussion

The developed ship hydrostatics parameters analysis software was run and its accuracy validated using offset data of a single screw, cargo-passenger vessel from the SNAME. Offset tables (tables that contain a vessel's half breadth values at different water lines) as seen in Table 1, was taken from the SNAME Ship Design manual [8]. This offset table contain seven waterlines in addition to the half siding, bottom tangent, main deck and forecastle deck with their respective values of draft. The vessel is a single screw cargo-passenger ship having some other properties listed as follows:

Length between perpendiculars (LPP) = 154.99 meters

Equal Interval (Spacing) S = LPP/10 = 15.499 meters

Density of sea water = 1.025 tons/meters^3

Vertical center of gravity = 9.50 meters

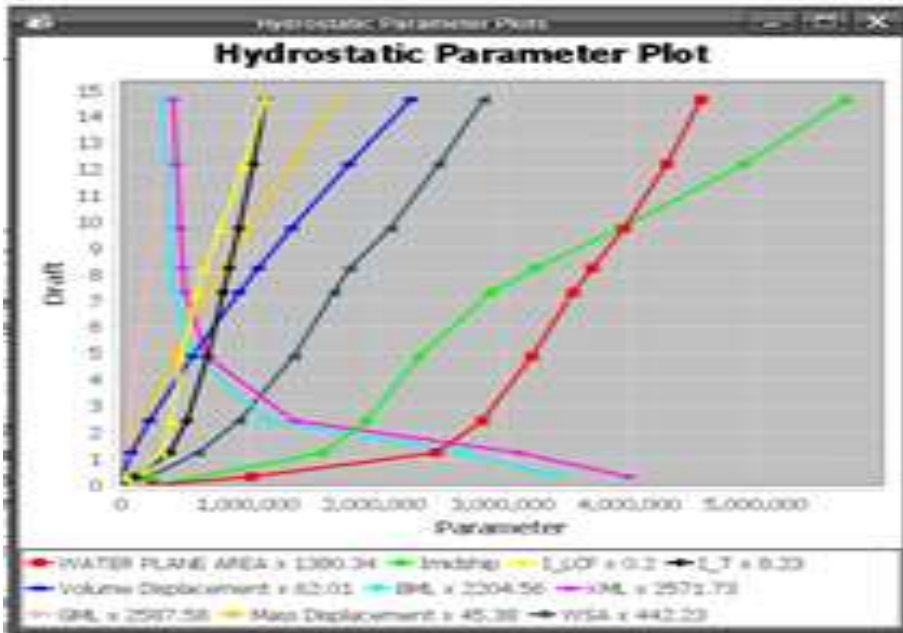LPP = 154.99 meters, Spacing S=15.499 meters,

Figure 6: Graph Plots from the Developed Software

Table 1: Offset Table for the given Cargo-Passenger Vessel (Lewis, 1988)

STATIONS	HALF SIDING	BOTTOM TANGEN	WL, 1.219 m	WL, 2.438 m	WL, 4.877m	WL, 7.315 m	WL, 8.230m	WL, 9.754 m	WL, 12.192 m	MAIN DECK, 14.64 m	FOC'SLE DECK
0	0	-	0.759	0.581	0.108	-	-	0.133	0.879	2.337	4.477
0.5	0.394	-	1.308	1.432	1.27	1.172	1.245	1.613	2.775	4.483	6.674
1	0.483	-	1.968	2.438	2.73	2.962	3.1	3.613	4.823	6.518	8.477
1.5	0.571	-	2.978	3.848	4.626	5.102	5.359	5.886	6.988	8.404	9.934
2	0.66	-	4.324	5.534	6.575	7.315	7.597	8.093	8.979	9.966	11.011
3	0.66	0.86	7.509	8.909	10.173	10.792	10.956	11.195	11.484	11.716	11.944
4	0.66	3.832	10.293	11.208	11.83	11.986	12.007	12.033	12.039	12.039	-

5	0.66	9.144	11.417	11.916	12.039	12.039	12.039	12.039	12.039	12.039	-
6	0.66	6.268	10.344	11.338	11.983	12.039	12.039	12.039	12.039	12.039	-
7	0.66	2.324	6.833	8.49	10.627	11.703	11.899	12.033	12.039	12.039	-
8	0.66	0.679	3.314	4.423	6.788	9.458	10.271	11.246	11.932	12.039	-
8.5	0.66	0.66	2.207	2.896	4.518	7.306	8.417	9.976	11.389	11.89	-
9	0-66	-	1.445	1.778	2.508	4.677	5.962	7.973	10.252	11.37	-
9.5	0.432	-	0.549	0.568	0.6	1.553	3.057	5.41	8.236	10.001	-
10	-	-	-	-	-	-	-	2.13	4.861	6.826	-
a1	-	-	-	-	-	-	-	-	2.658	4.553	-

Point 'a1' is station point 1, a distance of 'Sa1' aft from station point 10,

So, 'Sa1'=3.048 meters (Given)

Therefore,

From equation (10), internal multiplier at station point 10 is 0.795

Also, from equation (10), internal multiplier at station point 'a1' is 0.295

From equation (11), Lever at station point 'a1' is 5.197

4.1. Results from the Ship Hydrostatics Parameters Analysis Software Developed

The ship hydrostatics parameters software developed in this research work was used to analyse the hydrostatics parameters of the Passenger-Cargo ship presented in Table 1. The tabular results are displayed in APPENDIX A as printable hydrostatics parameters reports from the developed software and the graphs are in two plot-categories. One is fitted with power curve to smoothen out the points in the curves, while the other is the actual value from the analysis without curve fitting. These plot-categories contains two graphs in each, the first graph plots hydrostatics parameters such as KB, TPC, BMT, KMT, GMT, MCT1cm, CB, CW, CM and CP against drafts as seen in Figures 7 and 9. The second graph plots the waterplane area, moment of inertia about midship, LCF, transverse moment of inertia, volume displacement, BML, KML.GML, mass displacement and WSA against drafts.

4.2. Graphs of Hydrostatics Parameters without Curve Fitting

The graphs of the ship hydrostatics parameters or curves of form in Figure 7 and 8 below are graphs that are not fitted with curves, these curves may not be too smooth in slope but are of better accuracies when compare with those curve fitted. The ship hydrostatic parameters plotted in the graphs of Figures 7 and 8 are the same with those of Figures 9

and 10 respectively. Though the difference is that the later are curve fitted while the former are not curve fitted. The discussion of these parameters and their curves are done in groups considering the different graphs together. The legends of the graphs are displayed to show the hydrostatic parameters, the colour, legend sign, the units of the parameters and the factor multiplying each of the parameters. Tons per centimeter immersion (TPC) is a hydrostatic parameter that is dependent on mass displacement and density of water the vessel is moving in. From the graphs of Figure 8 and 9, its curves (green colour legend) is seen to increase gradually from the keel of the vessel to a draft of 1.5 meters with an estimated graph value of 80, after which it increased sharply up to the deck. This implies that the vessel has a large mass such that when there is an increase in draft, the effect on the vessel's mass displacement is little. This ship hydrostatics parameter, TPC is very significant in ship stability because it shows to what extent a ship can be submerged under the addition of load and to what extent the ship is buoyed at the removal of the load. This parameter is utilized by Naval architects in determination of the stability of vessels as it can be used to add or remove weights/loads from vessels during ship design.

From the curves of form displayed in the graphs of Figure 7 and 9, it is seen that the curve of water plane area (the area of the plane surface of a vessel's hull at a particular waterline) increases as the draft of the vessel increases, the slope of the curve increases up to a draft of 1 meter, estimated to have a graph value of 2,000,000 which when divided by its curve scale factor of 1,380.34 gives an actual value of 1,448.92 m2 and suddenly increase steeply up to the deck of 14.64 meters. This implies that from the keel (base) of the vessel to the 1 meter draft, the ship has a small hull shape and deadrise (rake at the sides of the ship bottom), then from the 1 meter draft to the deck indicates the parallel and full body of the ship where a little change in draft will produce a significant change in area.

The Volume displacement of a ship indicates the hull type of the ship, whether the ship is a plaining hull (type of ship hull where a small portion is submerged in water) or a displacement hull (where a large portion of the hull is submerged in water). From the volume displacement curve in Figure 7 and 9 with a dark blue colour legend, the slope of the curve increased steeply as draft increase, this implies that the ship has a displacement hull and will obviously be faced with frictional resistances from the water body in which it moves, therefore a proper matching of high capacity propulsion system will be required for this vessel. This is to enable the vessel overcome the frictional resistance from the water.

4.3. Graphs of Hydrostatics Parameters with Curve Fitting

Power curve fitting technique was preferred over other types of curve fitting techniques. For instance the least square, B-spline, cubic spline curve fitting techniques etc., were applied, but the Power curve produced a better faired curves for the parabolic lines of ship hulls.
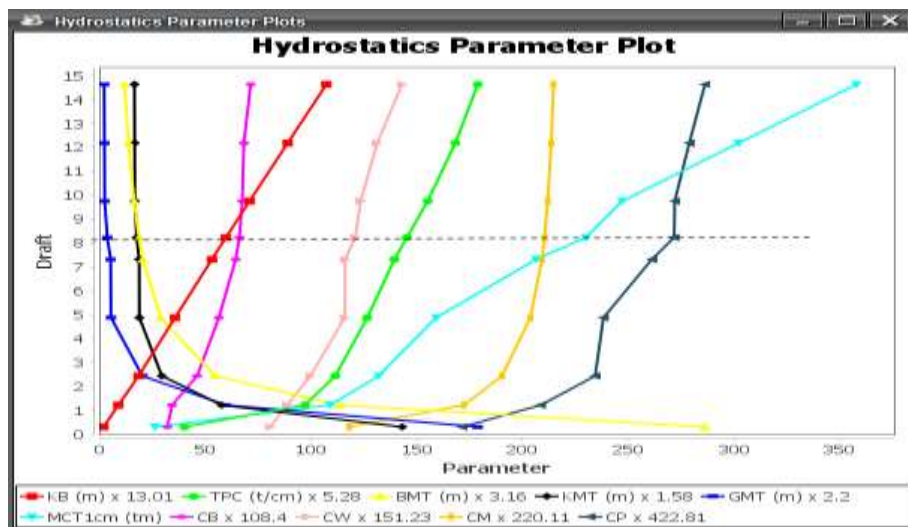

Figure 7: Variations of Parameters; KB, TPC, BM_T, KM_T, GM_T, MCT1cm, CB, CW, CM, CP with Drafts (without Curve Fitting).

The graphs in Figures 9 and 10 are the graphs fitted with power curve. The centers of buoyancy are another very important ship hydrostatics parameters because they are the points in the submerged area of the ship where the upthrust, upward or buoyant forces act to balance the vessel. In this research, two centers of buoyancy such as vertical center of buoyancy (*KB*) and longitudinal center of buoyancy (*LCB*) are considered. The former is the vertical center of gravity of the ship's immersed volume. From the graph of Figures 7 and 9, it is seen that the *KB* curve plotted with red colour legend is almost a straight line graph which indicates that the vessel is in stable equilibrium throughout the analyses at the various drafts. This is in line with what is obtainable in practice.
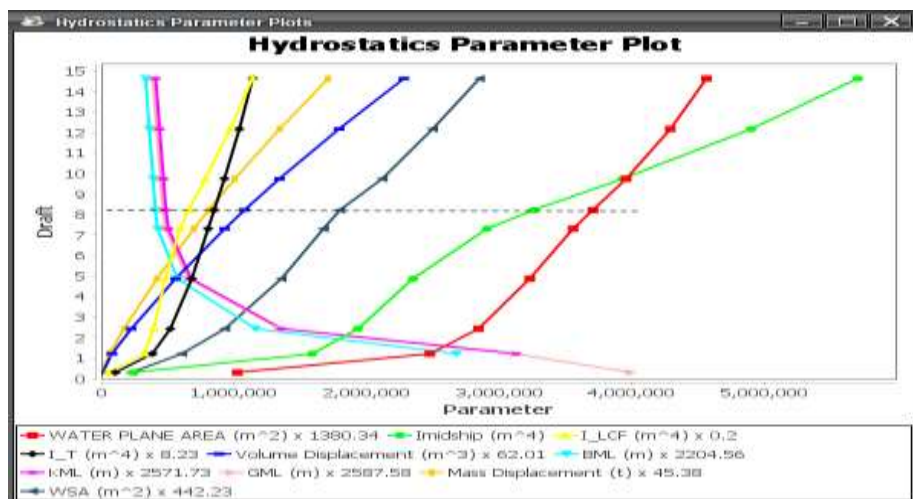
Figure 8: Variations of Waterplane Area, Inertia about Midship, LCF, Transverse Moment of Inertia, Volume Displacement, BM_L, $KM_L.GM_L$, Mass Displacement, WSA with Drafts (without Curve Fitting).
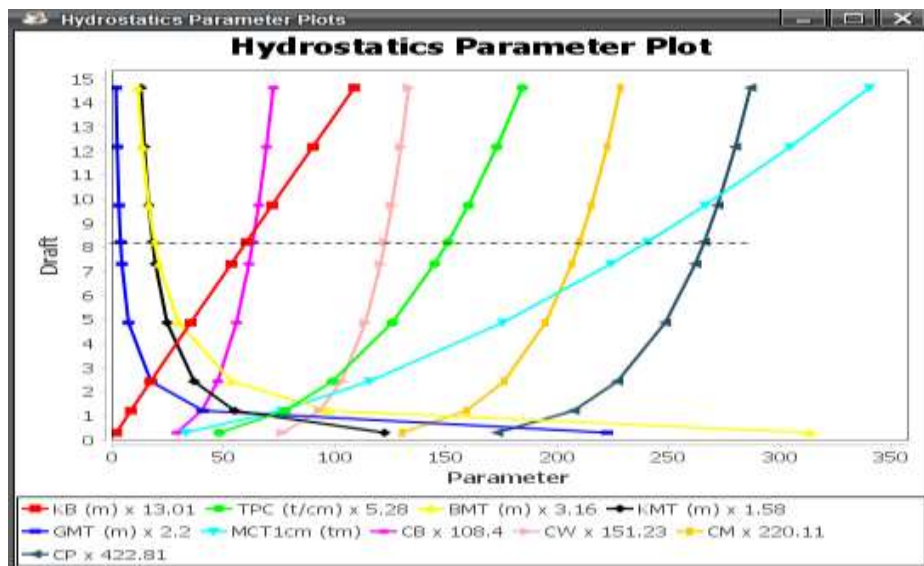

Figure. 9: Variations of Parameters; KB, TPC, BM_T, KM_T, GM_T, MCT1cm, CB, CW, CM, CP with Drafts (with Power Curve Fitting).

The height of metacenter above the keel of a ship (*KM*) and the metacentric height (*GM*) are another important ship hydrostatics parameters that influences the stability of a ship in water. They can be transverse or longitudinal. From the curve of *KM* and *GM* in

Figures 7 and 10 for transverse case; it is seen that the curves falls steeply with increase in draft at shallow drafts and leveled out at higher drafts. The *KM* depends and is directly proportional to the ship's metacentric radius (*BM*), which in turn is inversely proportional to volume displacement. From the Figure 8 and the software report printout in APPENDIX A, it is seen that at small draft of about 0.305 meters, the volume displacement is 141 meters^3, which make the metacentric height (*GM*) to be high about 81.2 meters and *KM* of about 90.62 meters. At the parallel body of the ship, their values leveled out as *GM* reduced from 2.44 meters at draft 4.877 meters to 1.036 meters at draft 8.23 meters and increased slightly to the deck of value 2.51 meters with draft of 14.64 meters. These changes are as a result of the geometry of the ship's hull. These parameters form the initial stability condition of vessels. At these waterlines, *GM* values are all positive values. It implies that the vessel is in stable equilibrium at the individual waterline, but when the values become negative, it means that the vessel is in an unstable condition. This may lead to rolling or capsize, if the negativity increases to a certain critical value. Also, when the *GM* value is zero, it implies that the vessel is in a neutral equilibrium, In such situation, the vessel will continue in the tilted or heeled condition until an external righting moment restores it to stable equilibrium. However, all these conditions other than stable equilibrium comes into play when there is loading and unloading exercise on the ship, which is outside the scope of this research work.
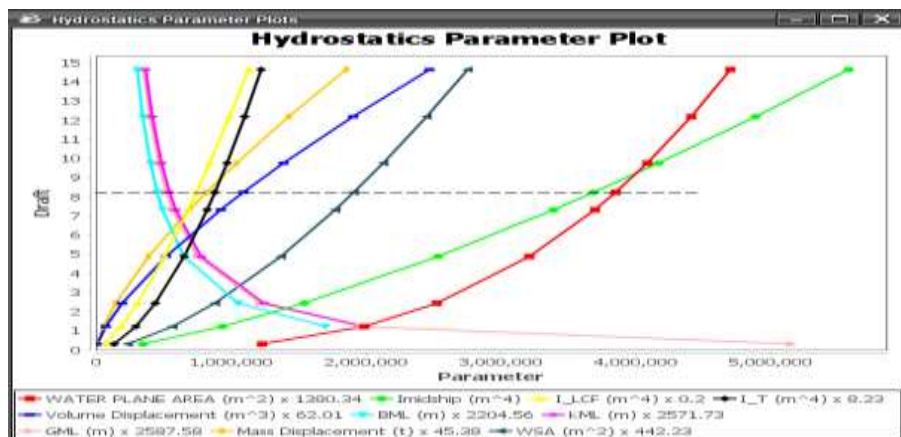

Figure 10: Variations of Waterplane Area, Inertia about MidshiP, LCF, Transverse Moment of Inertia, Volume Displ. BM$_L$, KM$_L$.GM$_L$ Mass Displ. WSA witht Drafts (with Power Curve Fitting)

Mass displacement is the ship hydrostatics parameter that is dependent on volume displacement and the density of water in which the ship floats. It is directly proportional to the volume displacement. From its curve in Figures 8 and 10, it is seen to increase at increase in draft, similar to the volume displacement, though its values are somewhat bigger than that of the corresponding volume displacement because the vessel is floating in sea water of density 1.025 tons/meters^3. Mass displacement as a hydrostatic parameter is very significant for ship stability because it is equal to the weight of the ship in water in accordance to Archimedes' principle.

The Moments of inertia of vessels are important hydrostatics parameters as well because they are the measure of the ship's resistance to rotation. In this research, three moments of inertia such as moment of inertia about the ship's transverse section, moment of inertia about the longitudinal center of floatation, and moment of inertia about midship section were considered. From the graphs of Figures 8 and 10, the curves of the moments of inertia increased gradually for smaller drafts and then sharply for higher drafts. For instance, it is seen that from the keel of the vessel to small draughts of about 1.219 meters, after converting their graph values, their individual factors or when read from the software printout in APPENDIX A, have actual values of 46,782.42 meters^4, 1,579,317.19 meters^4 and 1,589,031.08 meters^4 respectively.

These values increased greatly up to the deck of draft 14.64 meters, where they became 138,586.19 meters^4, 5,561,378.67 meters^4 and 5,700,058.8 meters^4 respectively. This implies that displacement ships are not stable at very little draughts, but they increase in stability as the vessel's draft increases and it is maximum at the maximum designed waterline.

The wetted surface area is a ship hydrostatics parameter that enables ship designers to determine the quantity of paint required to coat the portion of the ship's hull exposed to corrosion. From the curve of wetted surface area in Figure 9, it is seen that the curve increases with increase in draft. It means that the more the ship's hull is submerged, the more the area prone to corrosion is exposed. Additionally, the more the quantity of paint require to protect the vessel's hull from corroding is revealed.

4.4. Verification of the Hydrostatic Parameters Analysis Software Program Developed against Results from SNAME Ship Design Manual

SNAME carried out an analysis on the Passenger-cargo ship, and displayed the results of hydrostatics parameters for just one waterplane, that is the waterplane of draft 8.23 meters as seen in [8]. Then the results from this waterplane was specifically compared graphically and statistically with the results produced by the developed software in this research work as presented in the verification Table 2.

The verification of results was done by superimposing a

horizontal line upon the draft value 8.23 meters on the graphs, then the desired hydrostatics parameter values were read-off at sight. In comparing the results from SNAME's analysis and the results produced by the developed ship hydrostatics analysis software in this research, at the specific waterplane of draft 8.23 meters, the validation table containing the required hydrostatics parameters, the figures where each parameter curve is located, the estimated graph values when curve fitted and without curve fitting, the curves' scale factors, the actual value which is the quotient of the estimated graph value and the curves' scale factor when curve fitted and without curve fitting, SNAME values and the percentage deviations were drawn as seen in the Table 2. When these individual hydrostatics parameter values were statistically compared, percentage deviations were determined for each hydrostatic parameter. It was observed that these two sets of results have little differences because of the application of power curve fitting for smoothening out the points in the curves. However, both results are in good agreement with the results produced by SNAME analysis for the vessel.

5. Conclusion

This research centers on the development of a software for ship hydrostatic parameters analysis. The various methods of formulating ship hydrostatics parameters were considered in the

course of the research. Mathematical models were developed for the ship hydrostatics parameters using piecewise integration, Trapezoidal and Simpson's rules to approximate the ship hull form. Algorithms in the forms of pseudo codes and flow chart were then developed to carry out the analyses on the various ship hydrostatics parameter models in a chronological order. The software was designed as a stand-alone application using Netbeans Integrated Development Environment of Java programming language, and my structured query language data base management system. MySQL serves the purpose of creating, receiving, accessing, managing, searching, retrieving, structuring, replicating, storing and deleting of data. Java matrix application programming interface, J-Free Chart API and Java Excel API were also incorporated to enable the software perform matrix computations, plot graphs and read files from excel platform respectively. The developed ship hydrostatics parameters analysis software was run and its accuracy verified using offset data of a single screw, cargo-passenger vessel from the Society of Naval Architects and Marine Engineers.
The offset table of the single screw, cargo-passenger vessel contains seven waterlines in addition to the half siding, bottom tangent, main deck and forecastle deck with their respective values

of draft. The ship has a length between perpendiculars (LPP) of 154.99 meters in sea water. SNAME carried out the analysis on the vessel and displayed the results of hydrostatics parameters for just one waterplane draft of 8.23 meters (27ft). The ship hydrostatics parameters software developed in this research work

was used also to analyse the same cargo-passenger ship and the results were displayed in both tables and graphs. The graphs are in two categories. One is fitted with power curve to smoothen out the points in the curves, while the other is the actual value from the analysis without curve fitting.

Table 4.1: Verification of Results with that of Society of Naval Architects and Marine Engineers

S/N	Hydrostatics Parameters	Figure	Estimated Graph Values without Curve Fitting	Estimated Graph Value with Curve Fitting	Curves' Scale Factors	Actual Value without Curve Fitting	Actual Value with Curve Fitting	SNAME Results	% Deviation without Curve fitting	% Deviation with Curve fitting
1	Awp (m^2)	8.0,10.0	3700000	3800000	1380.34	2680.499	2752.944	2683.77	0.12	2.5
2	LCF (m)					4.096		4.1	0.24	
3	Imidship (m^4)	8.0,10.0	3250000	3600000	1	3250000	3600000	3257400	0.23	10.5
4	Ilcf (m^4)	8.0,10.0	650000	700000	0.2	3250000	3500000	3212300	1.17	9.0
5	I_T (m^4)	8.0,10.0	855000	855000	8.23	103888.21	103888.21	103390	0.48	0.48
6	TPC (t/cm)	7.0,9.0	145	150	5.28	27.46	28.41	27.51	0.17	3.3
7	Vol. Disp. (m^3)	8.0,10.0	1100000	1106000	62.01	17739.07	17835.83	17845	0.59	0.05
8	BM_T (m)	8.0,10.0	400000	400000	2204.56	181.44	181.44	180	0.8	0.8
9	BM_T (m)	7.0,9.0	18.5	19	3.16	5.85	6.01	5.79	1.11	3.8
10	Cwp	7.0,9.0	120	120	164.81	0.728	0.728	0.719	1.3	1.3

The results of the referenced water plane at draught 8.23 meters obtained from the developed software and SNAME data for the same vessel were specifically compared graphically and statistically. The percentage deviations as determined for the individual hydrostatics parameters were as follows: Area of water plane 0.12% and 2.5%, Moment of Inertia about midship section 0.23% and 10.5%, Transverse moment of Inertia 0.48% and

0.48%, Moment of Inertia about longitudinal center of floatation 1.17% and 9.0%, Tons per centimeter immersion 0.17% and 3.3%, Volume displacement 0.59% and 0.05%, Longitudinal metacentric radius 0.8% and 0.8%, Transverse metacentric radius 0.11% and 3.8%, and Coefficient of waterplane 1.3% and 1.3% for graphs not curve-fitted and those with curve fitting respectively. It was observed that these two sets of results have little differences

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because of the application of power curve fitting for smoothening out the points in the curves. However, both results are in good agreement with the results produced by SNAME analysis for the vessel. In future work, this software can be developed further to incorporate ship stability analysis, since the results produced from hydrostatic calculations serves as initial conditions for ship stability.

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