Mean-Variance Analysis and Efficient Portfolio Selection in the Nigerian Capital Market

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Abstract: Mean-variance analysis makes possible the choice of an efficient set of security combinations that optimize investment and maximize investor utility. Portfolio returns and variances data of 56 probable portfolio combinations of share investments in eight sampled firms show that only seven probable two-asset portfolio combinations are advisable efficient combinations: two involving manufacturing firms, brewery and petroleum marketing, food and beverages and building materials, as there exists negative covariances between them; and five combinations involving investment in bank shares with investment in either shares in a firm in the petroleum marketing, brewery, food and beverages and building materials sectors. Further results show that combination of two bank securities in a portfolio is not advisable as there exists positive covariances between the four sampled banks. Portfolio combinations inclusive of a bank give high returns necessitating investors' inclusion of investment in bank shares in their portfolio selections. To select efficient portfolio combinations maximizing portfolio returns and minimizing portfolio risks, Nigerian investors should not combine share investments in two banks as they bear the similar risks: industry and systematic risks with expected high volatility in earnings, but combine investment in either two manufacturing firms or a manufacturing firm with share investment in a bank in a two-asset portfolio.

Key words: mean-variance analysis, efficient portfolio, portfolio means and variances and investor utility.

1.0 Introduction

The combination of two or more securities in a portfolio aims to maximize the return from the combination for a given amount of risk. Markowitz (1952) assuming normality or elliptically distributed random variable nature of distributed returns from this combination, concluded that combination of negatively co-varying securities in a portfolio increases total returns for a given level of risk; where risk of the portfolio is measured using variance and standard deviation of the return flows. Determination of the return on a portfolio of negatively co-varying securities and its variance for optimal portfolio selection is the explanation of the mean-variance

analysis. Investors are known to have utility functions that may be sensitive to higher moments of the distribution of the returns. The use of the mean-variance optimization requires the assumption that the combination of utility and returns make the optimization of utility problem similar to the mean-variance optimization problem. The use of probabilities in explaining lottery choices are supported by the prospect and cumulative theories of utility. The decision utility theory according to Kontek (1993) presents an alternative solution not making use of the concept of the former theories; distinguishing between decision and perception utility, postulating a double S-shaped decision utility curve as hypothesized by Markowitz (1952), applying expected decision utility similar to the Von Neumann-Morgenstern (1944) theory. The decision utility theory to Kontek (1993) proposes a straight forward risk measure, presents a simple explanation of risk attitudes by using the aspiration level concept. The dilemma of investors in decision making to satisfy their utility arising from conflicting objectives of high profits versus low risks were seen to be fairly resolved by the parametric optimization model (mean-variance analysis) which was both sufficiently general for a significant range of practical solutions and simple enough for theoretical analysis and numerical solution. Investors to Zhao and Ziemba (2002) can use the mean-variance analysis as an investment criterion under which investors minimize the variance of the total portfolio return by setting the portfolio expected return to a prescribed target as in the static case; differences existing when there is an allowance of the portfolio to be traded dynamically. Doing this, they added, make possible the development of an efficient frontier for the investors; arguing that the global efficient frontier is a straight line in mean-standard deviation of the portfolio return space.

Investors being rational economic agents desire economic returns enough to meet their physiological needs ensuring the constancy of such returns' inflows (Koontz et al, 1980). Investment decisions making this feasible are possible using the mean-variance analysis in appraising the securities.

1.1 Objective of the Study

This study aims to determine probable cross-industry combinable securities producing efficient portfolio combinations in the Nigerian capital market.

2.0 Review of literature and theoretical framework

2.1 Theoretical framework

Markowitz (1952) developed the mean-variance model to aid investors in choosing an efficient portfolio of securities giving the maximum level of return for a given level of risk. Pulley (1981) proved that an expected-utility-maximising investor could do well for himself with a

mean-variance formulation based on the local relative-risk aversion coefficient. A growth of final portfolio is instantaneously mean-variance efficient when asset prices are lognormal (Zhao and Ziemba, 2002). Corporate finance theory posits that decision on possible profitable portfolio combinations is feasible using the mean-variance analysis model.

2.2 Review of Literature

2.21 The Mean-Variance Analysis and Efficient Portfolio Selection

Mean-variance is a technique that uses expected returns, variances and covariances of individual investments to analyse risk-return trade-off of combinations of these assets within an investment portfolio (Fanelli and Lee, 2011). Though the technique makes investor decisions possible in the short term and long term (where the portfolio may be readjusted several times during the planning horizon), Steinbach (2001) contended that these considerations used a utility function based on consumption of wealth overtime rather than mean and variance of the final wealth recoverable by the investor. The future returns of investors using this model to Steinbach (2001) are overcome by the use of probabilities in determining the expected returns of combinable portfolio securities. The dilemma of investors: conflicting objectives of high profits versus low risks were seen to be fairly resolved by the parametric optimization model (mean-variance analysis) which was both sufficiently general for a significant range of practical solutions and simple enough for theoretical analysis and numerical solution. The choice of a set of portfolios seen as efficient to maximize the expected utility of investors as proposed by Von Neumann-Morgenstern (1947) are according to Cochrane (2012), made possible using the mean-variance analysis; arguing that portfolio theory makes the most sense when the assumed return distribution is characteristic of what a market of investors with the given preferences will produce allowing for defined dimensions of investor heterogeneity; concluding that lognormal returns and mean-variance analysis do not work well together as the long tail of a lognormal distribution adds variance without adding much mean. This argument was extended by Campbell and Viceira (2005) while investigating the mean-variance properties of long-run log returns which seem normal providing no solution to portfolio decisions; and Martin (2012) expanding on the long-run pathologies of lognormal models.

The criticisms of the model in recognizing the existence of perfect market assumption were extended by Pogue (1970), Chen et al (1972), Perold (1984), Karatzas et al (1991), and Merton and Pliska (1995) using both discrete and continuous time frames. Risks to Markowitz (1952) are measurable using variances of security returns. Bawa (1978, 1982),

Fishburn (1977) and Levy (1992) criticized this necessitating the development of asymmetric risk measurement expected losses and semi-variances as alternatives to variances. Coherent risk identified Artzner and Embrechts (1999, 1995) adds another dimension to the problem of variables measurement. Comparing maximum expected utility of the mean-variance efficient portfolio to the maximum expected utility derived from direct optimization both with three types of utility functions with different degrees of absolute and relative risk aversions from Thai securities market, Maharakkhaka (2011) found that picking portfolios on the basis of mean-variance criteria does not lead to the maximization of expected utility; arguing that the performance of mean-variance appropriation is similar to selection of naïve portfolios, adding that investors with various utility functions are found to require significant optimization premium to bring up their welfare to the level achieved by holding expected utility maximization portfolios.

The mean-variance (MV) analysis to Jaaman and Lam (2012) seems faulty as it relies strictly on the assumptions that the returns on assets follow normal distribution and on the existence of a quadratic function of the investors' utility, as these conditions do not hold in reality; proposing the mean-Gini (MG) model to overcome these limitations as findings from analysed data from Malaysian share market using both models show that the portfolio combinations selected using the MG model outperforms those selected using the mean-variance model; as the former is not restricted to normal distribution and quadratic functions, making it possible for investors to develop second degree stochastic dominance (SSD) efficient portfolios. Investors as the mean-variance analysis posits are not in reality concerned only about mean and variances; μ and Σ are difficult to estimate; market returns are to Goodman (2009) not linear functions of investment weights and neither are investments simple as depicted by the analysis.

The prospect theory of efficient portfolio selection proposed by Kahneman and Tversky (1979) was supported by Barberis and Thaler (2003), De Bondt (1998) and Camerer (1995) as it tend to provide a better description of investor choices than the MV model. Barberis and Thaler (2003) and Barberis et al (2006) used the prospect theory to explain the low level of participation of investors in the equity market; Shefrin and Statman (1985) to explain the disposition effects of investors in investing; Barberis et al (2006) to explain insufficient diversification of investments by investors; Gomes (2005) to explain high trading activities in equity trading; and Barberis and Huang (2008) to explain investors' preferences for positively skewed pay-off distributions.

A comparative analysis of state by state optimal values obtained from both mean-variance and utility models using the S&P 500 data by Zhao and Ziemba (2002) showed that the MV analysis is superior. This they argued holds if the outcome of the contingent state price is near its mean and inferior to the expected utility model if the outcome is in the tails. Despite these criticisms, De Giorgi and Hen (2009) observed that the mean-variance analysis is still the industry standard in wealth management as the prospect theory is associated with irrational decisions, more complicated and possess standard simplifying assumptions of normality of distributed returns similar to the mean-variance analysis.

3.0 Methodology

3.1 Study Samples and Sampling Technique

Samples for this study are quoted firms paying the highest naira dividend in each NSE sector categorization (industry leader in value of naira dividend paid) with actively traded stocks and constant dividend paying history. Nwidobie (2010) concluded that firms in each sector of the NSE categorization follow the dividend paying patterns of the industry (sector) leader: increasing, stabilizing or decreasing naira dividend when the leader increases, stabilizes or decreases naira dividend. Thus, naira dividends paid and yields of firms each industry is a reflection of that of the industry leader and are similar in pattern. Data on the industry (sector) leader is thus a reflection of the data of all firms in that sector.

To determine possible cross-industry combinations of two securities each from the nine sampled firms in six sectors with actively traded stocks from the thirteen sector categorizations of the Nigerian Stock Exchange (NSE), we use permutation model:

possible combinations $P^n_k = n!/(n-k)!$

where n= available number to be chosen from

k= number of combinations

Thus, portfolio combination outcomes will be 56.

3.2 Data Description and Analysis

Data for this study are mean and variance values from possible combinations of securities using secondary data on dividend yield of sampled firms: Zenith Bank Plc, First Bank Nig Plc, UBA Plc, Lafarge Nig Plc, Guinness Nig Plc, Oando Nig Plc, Unilever Nig Plc and Nestle Foods Nig Plc from 2007 to 2011 from six of the thirteen sector categorization of the Nigerian Stock Exchange with actively traded stocks with equal probable investments in the combinable securities. For securities to be combinable in a portfolio with expected stability of portfolio returns and reduced risks, financial theory posits that there must exist negative covariances between such securities. From the 46 probable portfolio combinations, only 7 combinations have negative

covariances existing between them. Actual two-asset portfolio returns, variances and covariance values of probable combinable securities computed from actual dividend yields of the 8 sampled firms within the study periods are shown in table 1.

Table 1: Portfolio Returns, Variances, Correlation Coefficients and Covariances of Combinable Securities

Probable portfolio combinations	Portfolio returns(R_{pi})	Portfolio variances (σ^2_{Pi})	Correlation coefficients of combinable securities	Covariance of combinable securities
Zenith Bank Plc and Lafarge Nig Plc	4.154%	1.215	-0.81	-0.697
First Bank Nig Plc and Lafarge Nig Plc	2.419%	3.428	-0.496	-0.375
First Bank Nig Plc and Oando Nig Plc	2.441%	3.575	-0.427	-0.844
UBA Plc and Guinness Nig Plc	3.651%	3.550	-0.284	-2.487
UBA Nig Plc and Lafarge Nig plc	1.035%	1.991	-0.208	-0.135
Guinness Nig Plc and Oando Nig Plc	2.666%	1.752	-0.657	-0.088
Lafarge Nig Plc and Nestle Foods Nig Plc	2.533%	0.114	-0.324	-0.102

Source: computed from annual reports of sampled firms' data in tables 2 and 3.

The portfolio means of 56 combinations of returns from the seven sampled firms were computed using the model:

$R(P_{xy}) = [E(R_x)][W_x] + [E(R_y)][W_y]$;

where $R(P_{xy})$ = return on portfolio comprising securities x and y;

$[E(R_x)][W_x]$ = expected return on security x into proportion of total investments in security x;

$[E(R_y)][W_y]$ = expected return on security y into proportion of total investment in security y;

and portfolio risks(variances) computed using using the model:

$\sigma^2 p_{xy} = \sigma^2_x W^2_x + \sigma^2_y W^2_y + 2W_x W_y Cov_{xy}$

where σp_{xy}= variance of portfolio comprising securities x and y;

$\sigma^2_x W^2_x$ =variance of returns from security x into square of proportion of total investment in security x;

$\sigma^2_y W^2_y$= variance of returns from security y into square of proportion of total investment in security y; and

$2W_xW_yCov_{xy}$ = 2 into proportion of total investment in securities x and y into covariance of securities x and y.

Cov_{xy} is computed using the model: $corr_{xy}\ \sigma_x\sigma_y$

Where $corr_{xy}$= correlation coefficient of securities x and y;

σ_x=standard deviation of security x;

σ_y=standard deviation of security y;

with securities having equal weights

4.0 Discussion of Findings, Conclusions & Recommendations

Research results show that of the seven probable two-asset portfolio combinations, five involve investment in shares of a bank in combination with either investment in shares of a firm in the petroleum marketing, brewery, food and beverages and building materials sectors. Further results show that combination of two bank securities in a portfolio is not advisable as there exists positive covariances between returns of the four sampled banks. Advisable combinations from research results are two manufacturing firms in any sector: brewery, food and beverages, petroleum and building materials as there exists negative covariances between them; or combination of investment in bank shares with investment in shares in a manufacturing firm. Unilever Nig Plc seems not combinable with the sampled firms as there exists positive correlation coefficients and covariances between the returns from share holding in the firm and other sampled firms. For risk taking investors, combinations of investment in shares in a new generation bank and a manufacturing firm as represented by the samples: Zenith Bank Plc and Lafarge Nig Plc is an ideal choice as it gives the highest portfolio return of 4.15%; followed by investment in an old generation bank represented by UBA Plc and a manufacturing company in the brewery sector represented Guinness Nig Plc with a portfolio return of 3.651%; then an old generation bank represented by First Bank Nig Plc and an oil marketing firm represented by Oando Nig Plc with a portfolio return of 2.441%. Portfolio returns from two manufacturing firms' combinations are relatively low at 2.666% and 2.533% for Guinness Nig Plc and Oando Nig Plc, and Lafarge Nig plc and Nestle Foods Nig Plc respectively , both also have the lowest levels of risk respectively at 1.752 and 0.114 (both recommendable to risk averse investors) as shown in table 1. Portfolio combinations inclusive of a bank give high returns necessitating investors' inclusion of investment in bank shares in their portfolio selections. Instability in total investors' income in the Nigerian capital market may be caused by combinations of positively covarying securities: two banks as findings from covariance results from sampled quoted firms confirm existing arguments in financial

theory literature that there exists uniform risk factors among firms in the same sector bearing similar systematic and unsystematic risks causing positive covariances among firms within a sector. To select efficient portfolio combinations maximizing portfolio returns and minimizing portfolio risks, Nigerian investors should not combine share investments in two banks as they bear the similar risks: industry and systematic risks with expected high volatility in earnings, but combine investment in either two manufacturing firms or a manufacturing firm with share investment in a bank in a two-asset portfolio.

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Table 2: Correlations between Returns of Sampled Firms

		ZB	FB	UBA	GUNNS	UNIL	NESTL	LARFG	OAND
Pearson Correlation	ZB	1.000	.396	.791	.109	.950	.775	-.081	.286
	FB	.396	1.000	.006	.743	.403	.653	-.496	-.427
	UBA	.791	.006	1.000	-.284	.581	.337	-.208	.176
	GUNNS	.109	.743	-.284	1.000	.207	.190	.104	-.657
	UNIL	.950	.403	.581	.207	1.000	.830	.111	.393
	NESTL	.775	.653	.337	.190	.830	1.000	-.324	.382
	LARFG	-.081	-.496	-.208	.104	.111	-.324	1.000	.219
	OAND	.286	-.427	.176	-.657	.393	.382	.219	1.000
Sig. (1-tailed)	ZB	.	.255	.056	.431	.007	.062	.449	.320
	FB	.255	.	.496	.075	.251	.116	.198	.237
	UBA	.056	.496	.	.322	.152	.289	.368	.389
	GUNNS	.431	.075	.322	.	.369	.380	.434	.114
	UNIL	.007	.251	.152	.369	.	.041	.429	.256
	NESTL	.062	.116	.289	.380	.041	.	.297	.263
	LARFG	.449	.198	.368	.434	.429	.297	.	.362
	OAND	.320	.237	.389	.114	.256	.263	.362	.
N	ZB	5	5	5	5	5	5	5	5
	FB	5	5	5	5	5	5	5	5
	UBA	5	5	5	5	5	5	5	5
	GUNNS	5	5	5	5	5	5	5	5
	UNIL	5	5	5	5	5	5	5	5
	NESTL	5	5	5	5	5	5	5	5
	LARFG	5	5	5	5	5	5	5	5
	OAND	5	5	5	5	5	5	5	5

Table 3: Descriptive Statistics of Samples' Returns

	N	Minimum	Maximum	Mean	Std. Deviation	Variance
ZB	5	4.10	15.50	8.2800	4.32169	18.677
FB	5	.05	9.60	4.8100	3.80335	14.466
UBA	5	.06	7.60	2.0420	3.26674	10.672
GUNNS	5	3.40	10.00	5.2600	2.68104	7.188
UNIL	5	1.00	7.20	4.0400	2.20068	4.843
NESTL	5	2.90	6.60	5.0200	1.57861	2.492
LARFG	5	.01	.06	.0276	.01992	.000
OAND	5	.00	.12	.0710	.05248	.003
Valid N (listwise)	5					